PHY 552: Electromagnetic Theory - 1, (2011-12, Semester -I)

Department of Physics, I.I.T. Kanpur

Assignment – 1 (On Vector Calculus background)

---

1. Derive the Gauss' divergence theorem and the Stokes theorem for vector fields

2. Obtain the following quantities

$$\nabla \left| \frac{1}{\vec{r} - \vec{r'}} \right|$$

$$\nabla^2 \left| \frac{1}{\vec{r} - \vec{r'}} \right|$$

3. Derive the quantities $(\partial \hat{u}_j)/(\partial u_i)$ for the unit vectors of the spherical polar coordinates (r, θ, ϕ) and the cylindrical coordinates (r, ϕ, z). In each case, express in terms of the unit vectors in the respective coordinate system as

$$\frac{\partial \hat{u}_j}{\partial u_i} = \sum_k \omega_{ijk} \hat{u}_k$$

4. Using the above relations, obtain

   (a) $\nabla \cdot \hat{u}_i$

   (b) $\nabla \times \hat{u}_i$

   for all the unit vectors in the spherical and cylindrical coordinate systems Following this, obtain $\nabla \cdot \vec{A}(\vec{r})$, $\nabla \times \vec{A}(\vec{r})$ and $\nabla^2 f(\vec{r})$, where $\vec{A}(\vec{r})$ is an arbitrary vector field and $f(vecr)$ is a scalar field. Compare your expressions to the expressions in Griffith's book or notes on the course-webpage.

5. Given

$$\begin{aligned} \vec{A}(\vec{r}) &= \frac{\hat{r} \times \hat{z}}{r^2} \\ \vec{B}(\vec{r}) &= \exp\left[-\alpha r^2\right] \hat{r} \\ \vec{C}(\vec{r}) &= x \exp\left[-\alpha r^2\right] \hat{y} \end{aligned}$$

   obtain

   (a) $\nabla \cdot \vec{A}(\vec{r})$

   (b) $\vec{B} \cdot \vec{C}$

   (c) $\nabla \times \vec{B}(\vec{r})$

   (d) $\int \vec{A}(\vec{r}) \cdot \vec{C}(\vec{r}) \mathrm{d}^3 r$

6. Using Dirac delta and Heaviside step functions in appropriate coordinates, express the charge densities in the following cases:

   (a) A charge Q spread uniformly on a circle of radius R

   (b) Charge λ per unit length uniformly distributed over a cylindrical surface of radius R

   (c) A charge Q distributed on the upper hemisphere of a spherical surface only

   (d) The surface of a cone with cone angle α and carrying a surface charge density σ