PHY 103N: GENERAL PHYSICS 2, (2007-2008, Semester -II)

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Assignment - 1 (1, 2, 6, 8, 9, 10 will be discussed in the tutorials)

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1. Read about and understand a simple device by which you can detect and measure the amount of electric charge on a given conducting object. How would you be able to quantify the amount of electric charge on the object?

2. The Coulomb law of electrostatic forces is an inverse square law, i.e., the force between two point charges is inversely proportional to the squared distance between them. This is an experimental finding expressed as ($f \propto r^{-(2+\varepsilon)}$) where the deviation ε from the inverse square is fixed to some limit. Find out what is the current limit on ε and the experiment where this limit was determined.

3. (a) Estimate the number of electrons in a cow. If you removed 1% of these electrons (somehow!), estimate what would be the forces between two such charged cows at a distance of 1 m. Can you suggest some terrestrial sources where you may encounter forces of similiar magnitude. (The bodies of animals and humans are reasonably conducting and, if you want, you may assume the cow to be spherical!)
   (b) A Pottasium (K^+) ion is across a cell wall (membrane that is 10 nm thick) from a Chloride (Cl^-) ion. What is the force on the K^+ ion due to the Cl^- ion?

4. Consider two identical charges ($+q$) placed on a line. Will a charge ($-q$) placed exactly in between execute simple harmonic motion - along the line ? – and in a direction normal to the line?

5. Consider the vector $4\hat{x} + 5\hat{y} + 6\hat{z}$ - write down the components of this vector in a new co-ordinate system whose y'-z' are rotated with respect to the old y z axes by 60^o.

6. (A) A three dimensional plot of a 2-D function
$$f(x, y) = \exp\left[-\frac{3x^2 + 4y^2 - 4xy + 18x - 28y + 5}{60}\right]$$
   where the height f is plotted along one axis and x and y along the other two orthogonal axes appears like a hill. (a) find the top of the hill, (b) the height of the hill, and (c) the steepness of the slope at the point (2,2) and the direction of steepest ascent.
   (B) Consider two hills of height $2h$ and $3h$ separated by a "pass" (saddle point) with height h in the middle. If the function $f(x, y)$ were to describe the height profile, the peaks were at $(\pm a, 0)$ and the saddle at the origin, draw the constant height contours and the field lines for the vector function $\vec{A} = \nabla f$.

7. Given the vector field $\vec{v} = (3y^2 + 2xz^3)\hat{x} + (6xy + 3z^2)\hat{y} + (6yz + 3x^2z^2)\hat{z}$, compute the line integral between the corners of the unit (0, 0, 0) and (1, 1, 1) along two paths: one directly along the diagonal and another that goes along the edges.

8. A vector field is given by
$$v(x, y) = \begin{cases} \frac{\omega_0 r_0^2 (x\hat{y} - y\hat{x})}{x^2+y^2} & \forall \quad x^2 + y^2 > r_0^2 \\ \omega_0(x\hat{y} - y\hat{x}) & \forall \quad x^2 + y^2 < r_0^2 \end{cases} \tag{1}$$
   Obtain the Curl of this function everywhere. Discuss the result when $r_0 \to 0$ and $\omega_0 \to \infty$ while keeping $r_0^2\omega_0 =$ constant.

9. Find the surface integral $\int_S \vec{f_0} \cdot d\vec{s}$ where $\vec{f_0} = \hat{z}$ is a constant vector and S is the upper part of the surface of:
   (a) The hemisphere $x^2 + y^2 + z^2 = a^2$ for $z > 0$.
   (b) The ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{a^2} + \frac{z^2}{b^2} = 1$ for $z > 0$.

10. Consider the vector field $\vec{A}$ which is constant inside the volume of a cylinder and zero outside. $\vec{A}$ is oriented along the axis of the cylinder. Obtain expressions for the Divergence and Curl of this vector field.