Numerical Analysis in the Design of Urban Tunnels

Lecture Outline

1. Characteristics of urban tunnels
   - Need to control ground deformations
   - Numerical analyses to predict ground deformations

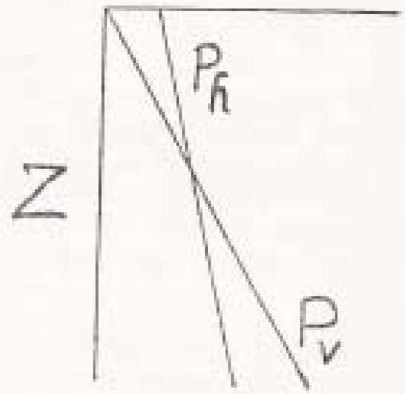
2. Tunnelling methods in urban areas (to control settlements)
   - Emphasis on pre-convergence and face pre-treatment

3. Methods of numerical analysis
   - Continuum / discontinuum modelling
   - Continuum 3-D modelling :
     Analysis of pre-convergence & face pre-treatment (for design)
     Estimation of ground parameters (E) by monitoring extrusion
   - Continuum 2-D modelling :
     How to model the 3-D problem in 2-D (in a cross-section)

Elastic Stress Distribution

1. Primitive Stresses

$P_h = \lambda P_v$



Terzaghi

$\lambda = \frac{\nu}{1-\nu}$

Heim's rule (1912)

$\lambda = 1$

Hoek & Brown (1980)

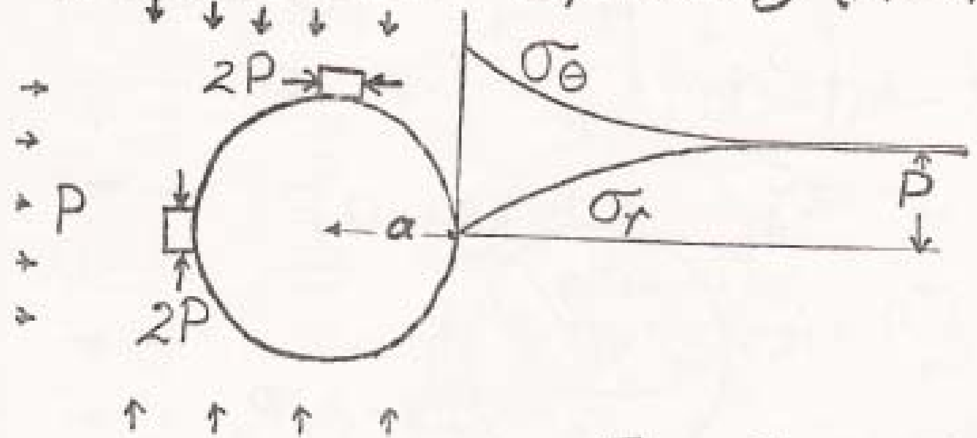
$100/Z + 0.3 < \lambda < 1000/Z + .5$

or $270 + .3\gamma Z < P_v \lambda < 2700 + .5\gamma Z$

$P_v = 2.7Z \quad t/m^2$

For $Z < 500m$, $\lambda > 1$

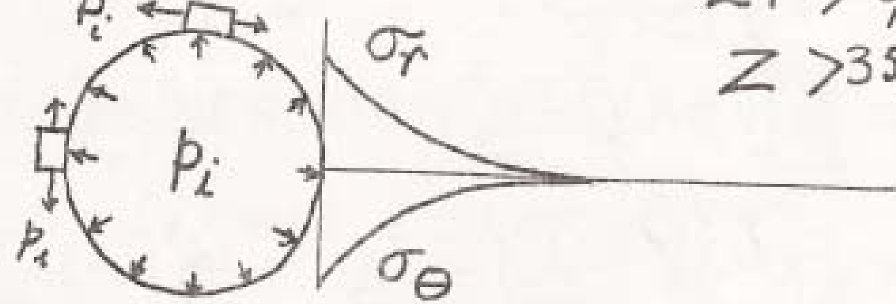
2. Circular Opening ($\lambda = 1$)



For Squeezing

$2P > q_c$

$Z > 350 Q^{1/3}$

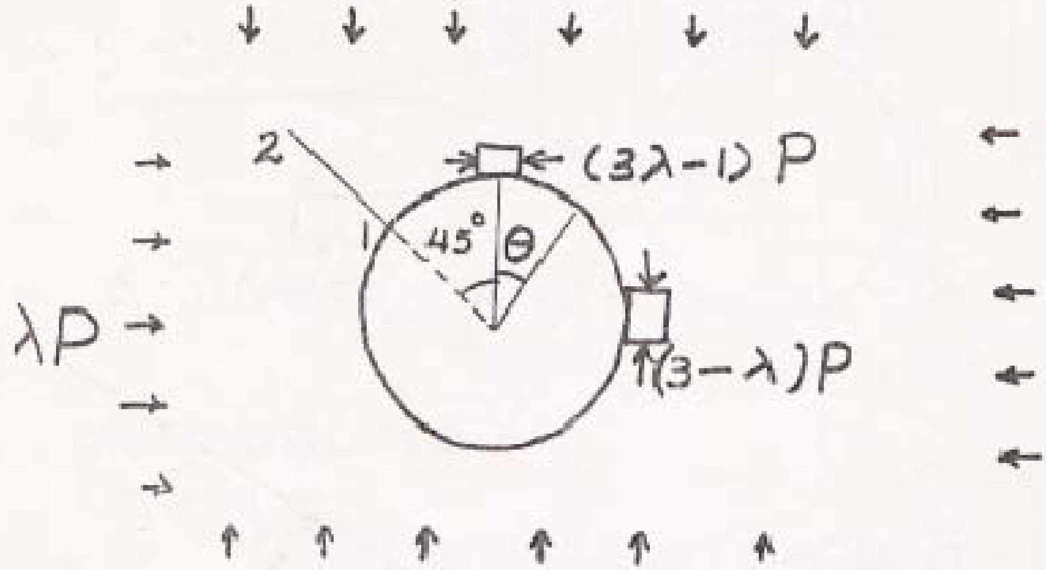


$$\sigma_r = P\left(1 - \frac{a^2}{r^2}\right) + p_i \frac{a^2}{r^2}$$

$$\sigma_\theta = P\left(1 + \frac{a^2}{r^2}\right) - p_i \frac{a^2}{r^2}$$

$$\sigma_r + \sigma_\theta = 2P$$

3. Circular Opening ($\lambda \neq 1$)



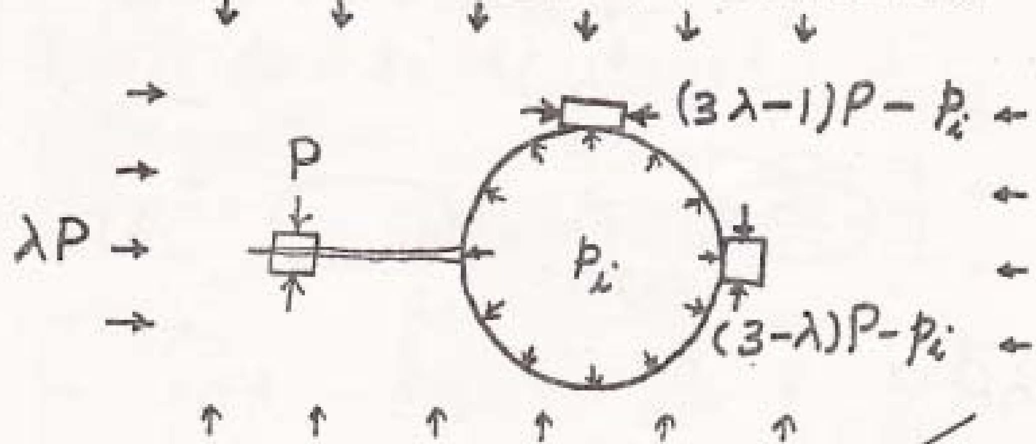
$$\sigma_r = \frac{P}{2}\left[(1+\lambda)\left(1-\frac{a^2}{r^2}\right)+(1-\lambda)\left(1+\frac{3a^4}{r^4}-\frac{4a^2}{r^2}\right)\cos 2\theta\right]$$

$$\sigma_\theta = \frac{P}{2}\left[(1+\lambda)\left(1+\frac{a^2}{r^2}\right)-(1-\lambda)\left(1+\frac{3a^4}{r^4}\right)\cos 2\theta\right]$$

$$u = \frac{Pa^2}{2Er}\left[(1+\nu)\left(1+\frac{a^2}{r}\cos 2\theta\right)-4(1-\nu^2)\cos 2\theta\right]$$
$$+\frac{P\lambda\, a^2}{2Er}\left[(1+\nu)\left(1-\frac{a^2}{r}\cos 2\theta\right)+4(1-\nu^2)\cos 2\theta\right]$$

$$u_{12} = \frac{P(1+\lambda)}{E}(1+\nu)a\left[1-\frac{a}{r}\right]$$

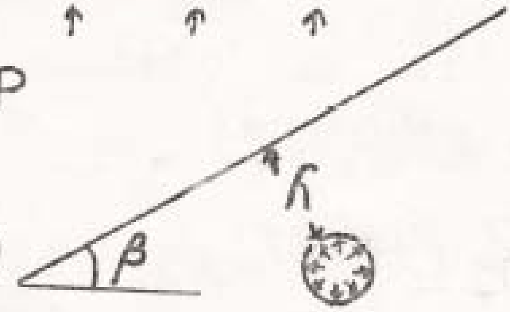
4. Unlined Power Tunnel



No hyd. Fra. P

$p_i < P$

$\gamma h \cos\beta > p_i$ (NGI)

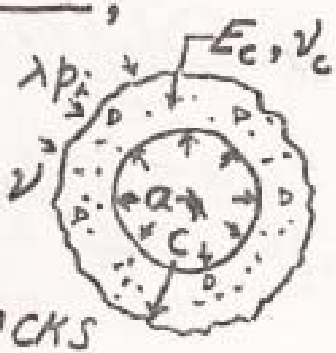


5. Lined Power Tunnel

$$\lambda = \frac{\dfrac{2a^2}{E_c(c^2-a^2)}}{\dfrac{1+\nu}{E} + \dfrac{(1-\nu_c)c^2 + (1+\nu_c)a^2}{E_c(c^2-a^2)}};$$

$$\sigma_\theta = -\frac{c^2 + a^2 - 2\lambda c^2}{c^2 - a^2}\, p_i ; \quad E, \nu$$

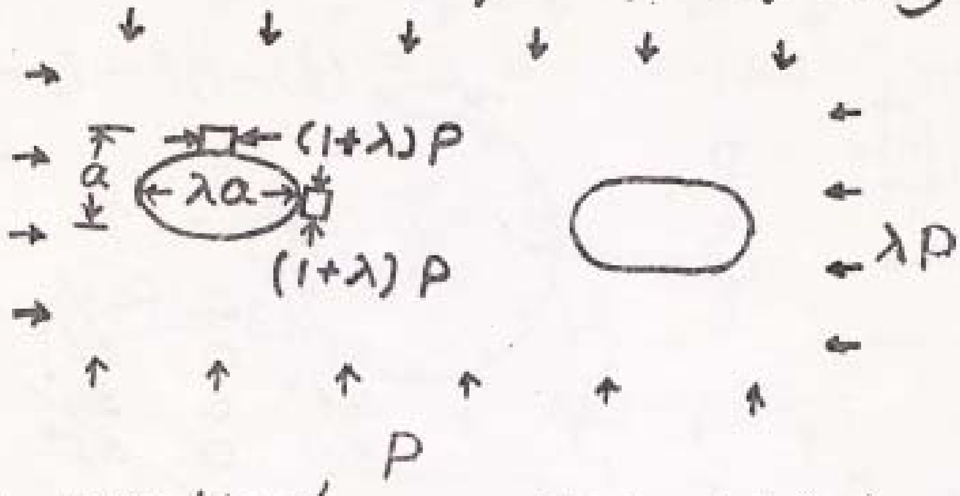
at $r = a$.



Current trend

− R.C.C. LINING WITH CRACKS

6. Better Shape of Opening



a. Elliptical

(b) Ovaloidal (Better)

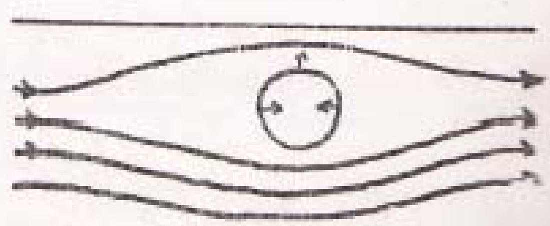
7. Pillar

$W > .4 - .7(B_1 + B_2)$

$\sim 1 - 1.5(B_1 + B_2)$

8. Opening Near Surface

Roof may go up, $\lambda > 1$



Elasto-plastic Stress Distribution ($\lambda = 1$)

1. Parameters

RMR	C_p (kg/cm^2)	ϕ_p	C_r	ϕ_r
90	3	45°	0	40°
70	2	40°	0	35°
50	1·5	35°	0	30°
25	1	30°	0	25°

$b/a = 3$ to 5 for conv. tunn.

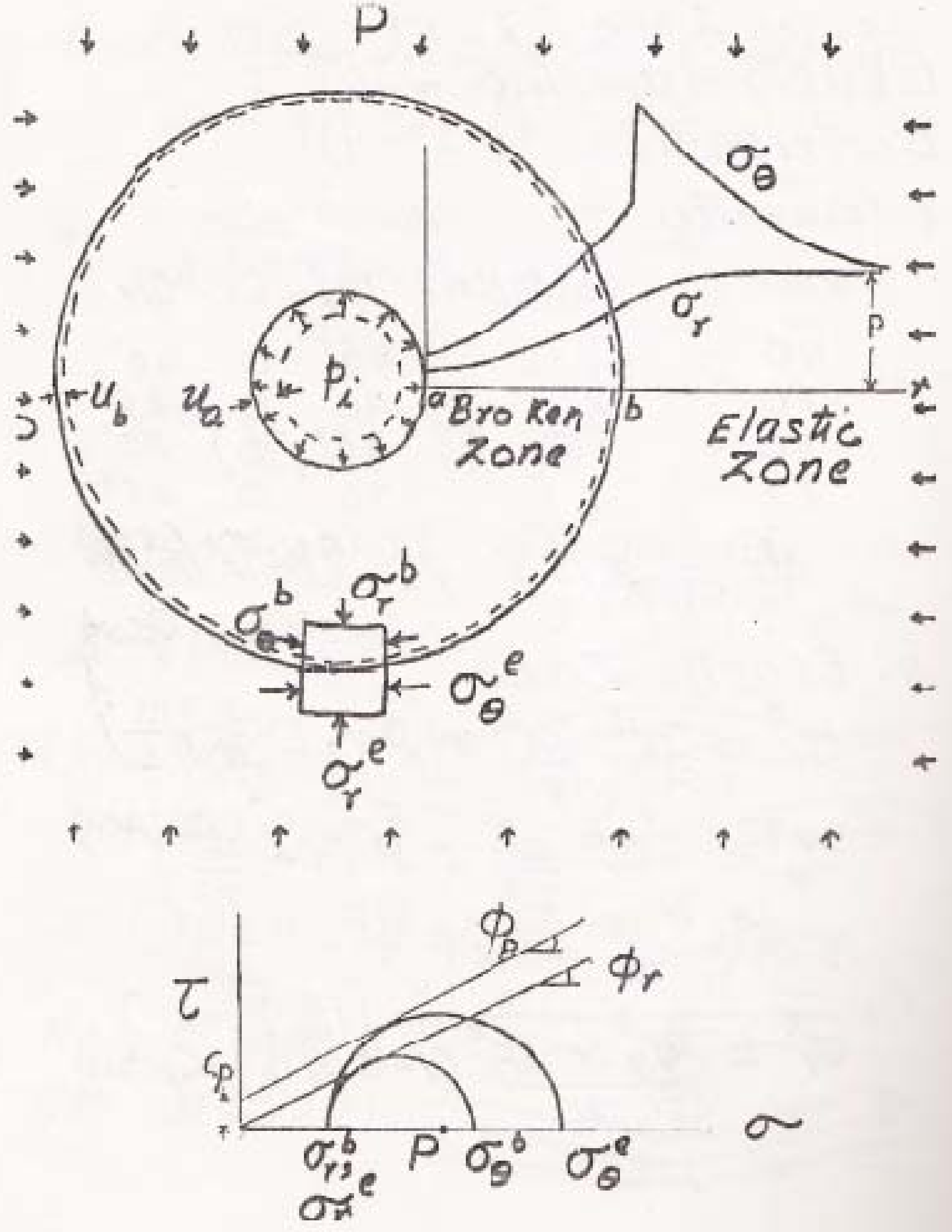
2. Elastic Zone

$$\sigma_r^e = \sigma_r^b \cdot \frac{a^2}{r^2} + P\left(1 - \frac{a^2}{r^2}\right)$$

$$\sigma_\theta^e = -\sigma_r^b \cdot \frac{a^2}{r^2} + P\left(1 + \frac{a^2}{r^2}\right)$$

$$\therefore \sigma_r^e + \sigma_\theta^e = 2P$$

$$\sigma_r^e = \frac{\sigma_\theta^e + \sigma_r^e}{2}(1 - \sin\phi_p) - C_p \cos\phi_p$$


P
σ_θ
σ_r
P
r
p_i
u_b
u_a
a
Broken Zone
b
Elastic Zone
σ_θ^b
σ_r^b
σ_θ^e
σ_r^e
τ
ϕ_p
ϕ_r
c_p
σ
σ_r^b, σ_r^e
P
σ_θ^b
σ_θ^e

3. Broken Zone ($\gamma = 0$)

$$\frac{d\sigma_r}{dr} - \frac{\sigma_\theta - \sigma_r}{r} \pm \gamma = 0$$

$$\sigma_\theta - \sigma_r = \alpha\,\sigma_r$$

$$\alpha = 2\sin\phi_r / (1-\sin\phi_r)$$

Sol. $\sigma_r = p_i (r/a)^\alpha$

$$\sigma_\theta = p_i (1+\alpha)(r/a)^\alpha$$

4. At Broken Zone Boundary ($r=b$)

$$\sigma_r^b = p_i\left(\frac{b}{a}\right)^\alpha = \frac{\sigma_\theta^e + \sigma_r^e}{2}(1-\sin\phi_p) - C_p\cos\phi_p$$

$$\therefore p_i = \left[\frac{\sigma_\theta^e + \sigma_r^e}{2}(1-\sin\phi_p) - C_p\cos\phi_p\right]\left(\frac{a}{b}\right)^\alpha$$

5. Correction for Gravity (Daemen)

$$\pm\,\gamma(b-a)\,M_r$$

$$M_r = \frac{a}{b-a}\cdot\frac{1-\sin\phi_r}{1-3\sin\phi_r}\left[\left(\frac{a}{b}\right)^{\alpha-1} - 1\right]$$

+ for crown

− for floor

6. Non-hydrostatic case

crown $\sigma_r^e + \sigma_\theta^e = (3\lambda - 1) P$

Walls " $= (3 - \lambda) P$

$M_r = 0$

7. Squeezing Displacement

$$\frac{\pi (b - u_b)^2 - \pi (a - u_a)^2 - \pi (b^2 - a^2)}{\pi (b^2 - a^2)} = K$$

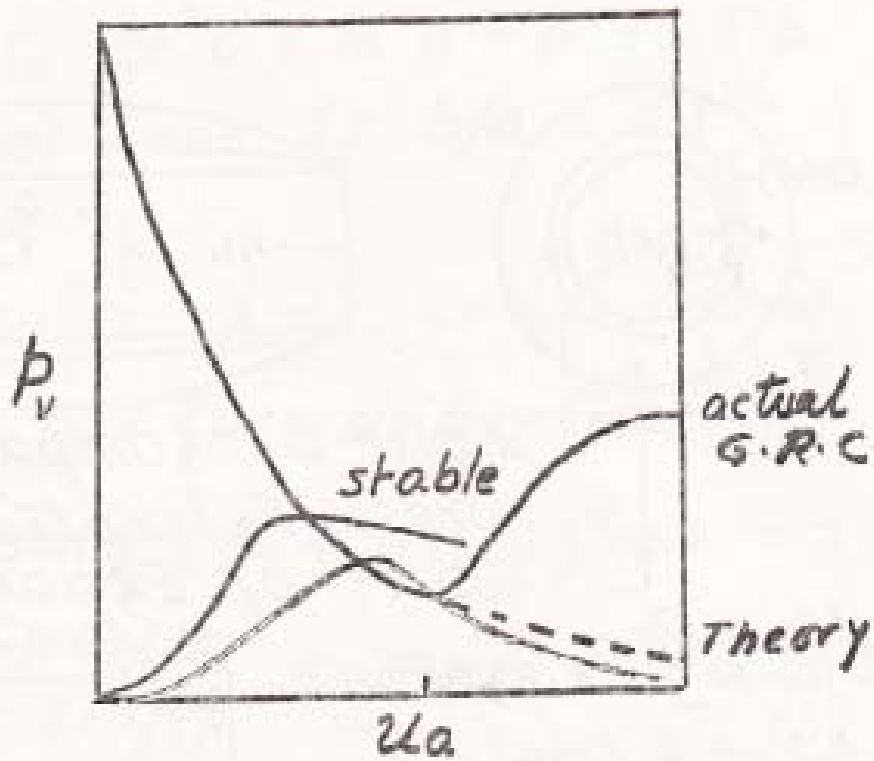
K = coef. volumetric Expansion

$u_b = 0$

$$u_a = \frac{K (b^2 - a^2)}{2a}$$

Rock	K
Phyllites	0·003
clay st. / Silt st.	0·01
High Swelling clay	0·01
Crushed sand st.	0·004
Crushed shales	0·005

8. Ground Reaction Curve

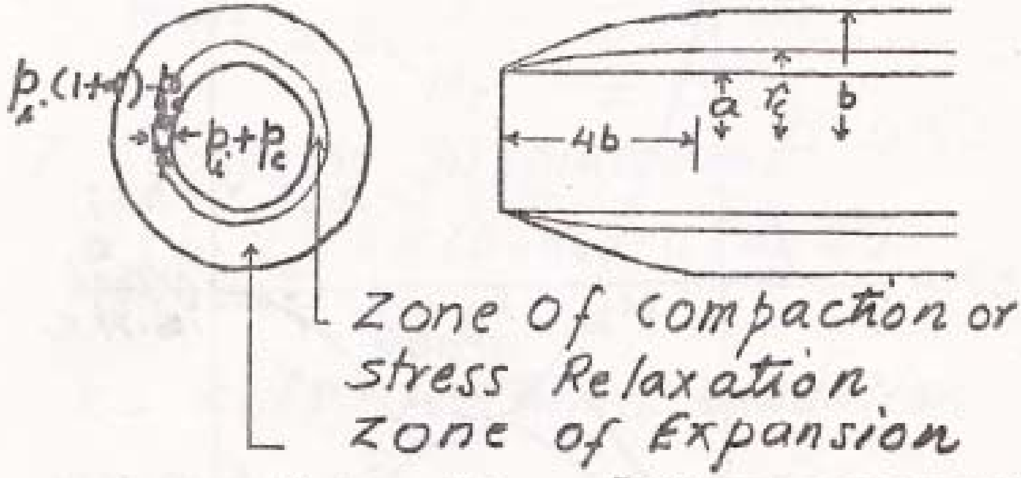


$(u_a/a)_{optimum} = 4 - 6\ \%$

Support
- Not too stiff
- Not too Flexible
- Not too early
- Not too late

Ultimate Rock Pressures

1. Zone of Compaction



During Relaxation

$p_i + p_c \to p_i(1+\alpha) - p_c$

$p_c \to p_i \alpha/2$

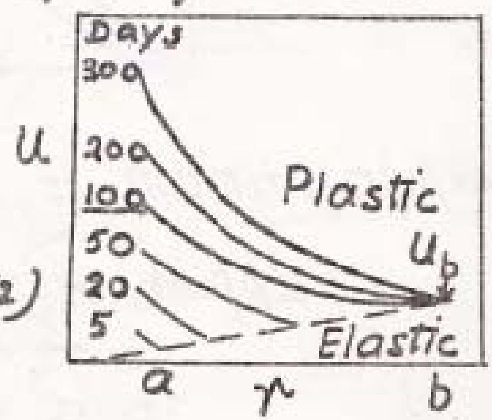
$p_{ult} = p_i + p_c = p_i(1+\alpha/2)$

$p_{ult} = \dfrac{p_i}{1-\sin\phi_r}$

$\therefore \dfrac{p_{ult}}{p_i} \simeq 1.7$ to 2, $\phi_r = 25°$ to $30°$

$= 5^{1/3}$ or 1.7 (Barton et al 1974)

= more for large r_c



Observations

$$p_h \rightarrow p_v$$

$$r_c \simeq 0.3 \text{ to } 0.4b \simeq 0.37b$$

$\therefore$ No Comp. Zone If $b/a < 2.7$

2. Chart

$$\gamma = 0, \quad \lambda = 1, \quad C_r = 0, \quad r_c = \text{small}$$

$$p_v = p_h = p_i = [P(1-\sin\phi_p) - C_p \cos\phi_p]\left(\frac{a}{b}\right)^{\alpha}$$

$$= (1-\sin\phi_p)(P - q_c/2)\left(\frac{a}{b}\right)^{\alpha}$$

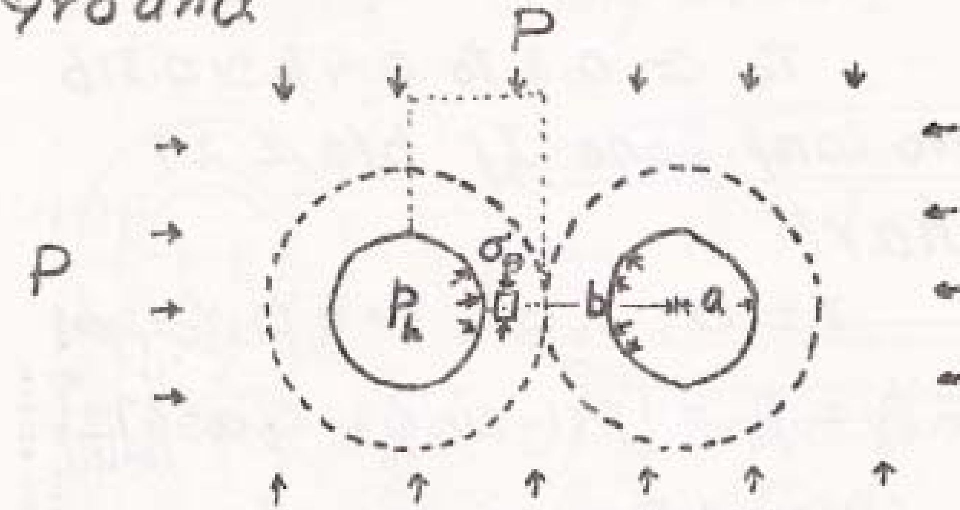
$$q_c = \frac{2C_p \cos\phi_p}{1-\sin\phi_p} = \text{insitu strength}$$

$$\frac{p_{ult}}{P} = \frac{1-\sin\phi_p}{1-\sin\phi_r}\left[1 - q_c/2P\right]\left(\frac{a}{b}\right)^{\frac{2\sin\phi_r}{1-\sin\phi_r}}$$

Quality	$p_{ult}/\gamma Z$	Assumption
Fair	13 %	$b/a \sim 4$
Poor	13 - 22 %	$\phi_p - \phi_r \sim 5°$
Very Poor (clays)	40%	

CONC. LINING – Creep pressure $p_c = p_{ult} - p_i$

Parallel Tunnels in Squeezing Ground



$$\sigma_\theta = (1+\alpha)\, p_h \left(\frac{r}{a}\right)^{\alpha}$$

Capacity of Pillar with F.S. F_p

$$\int_a^b \sigma_\theta \, dr = F_p \cdot P \cdot b$$

$$p_h = \frac{F_p P b a^{\alpha}}{b^{\alpha+1} - a^{\alpha+1}}$$

Pillar width (Practice)
$\simeq 2\text{-}3(B_1+B_2)$
$\simeq 4\text{-}6$ dia.

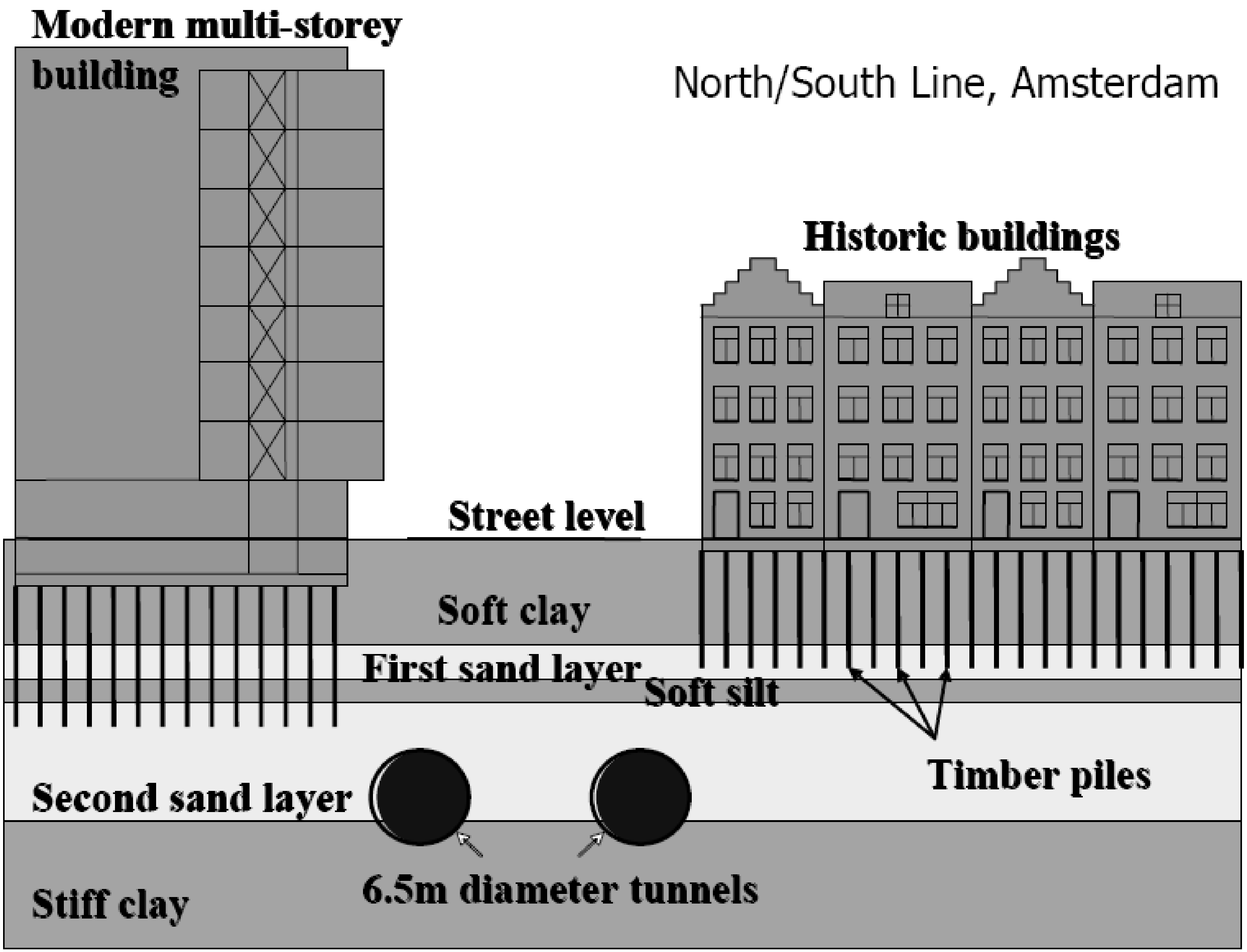
Swelling Ground

Non-Squeezing → Q system

Squeezing → El. Plastic Theory
$K \simeq 0.01$
Low str. parameters

Main characteristics of urban (shallow) tunnels

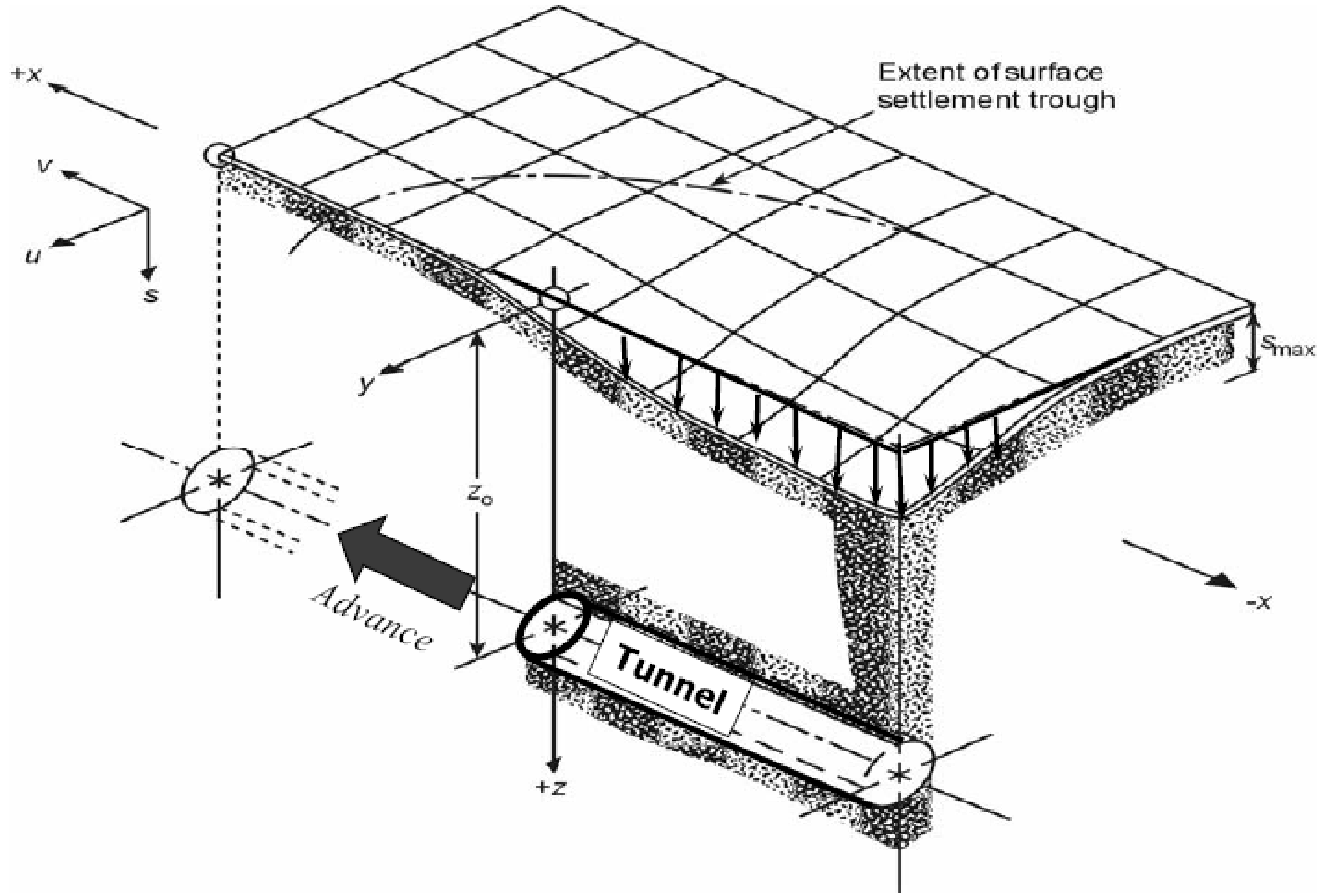
Minimisation of ground surface displacements



Main characteristics of urban tunnels

Minimisation of ground surface displacements

Surface settlement trough above an advancing tunnel

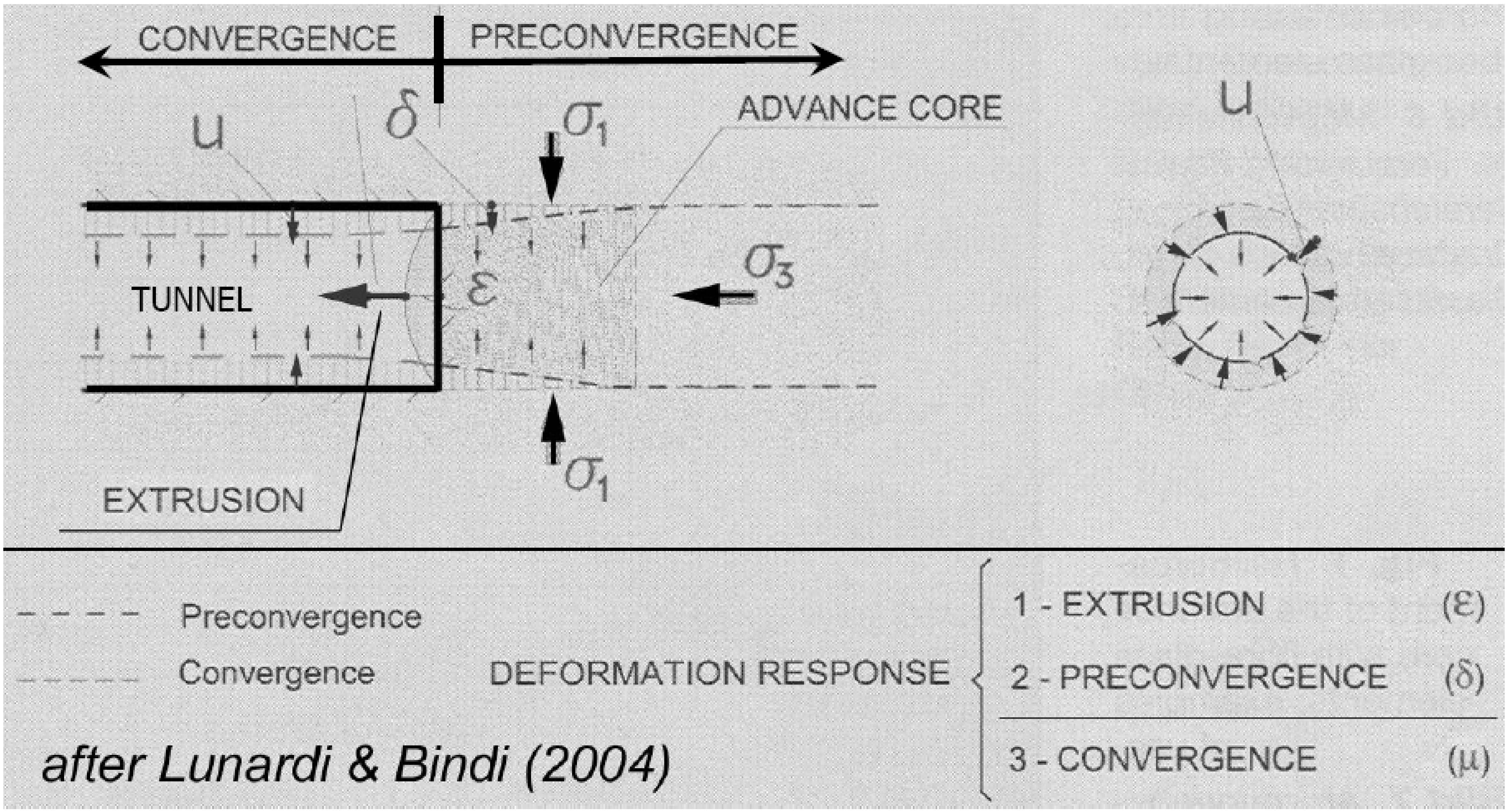


Settlement depends on ground, depth, diameter and excavation method

Causes of ground surface displacements :

1. Ahead of tunnel face : Axial face extrusion (radial pre-convergence)

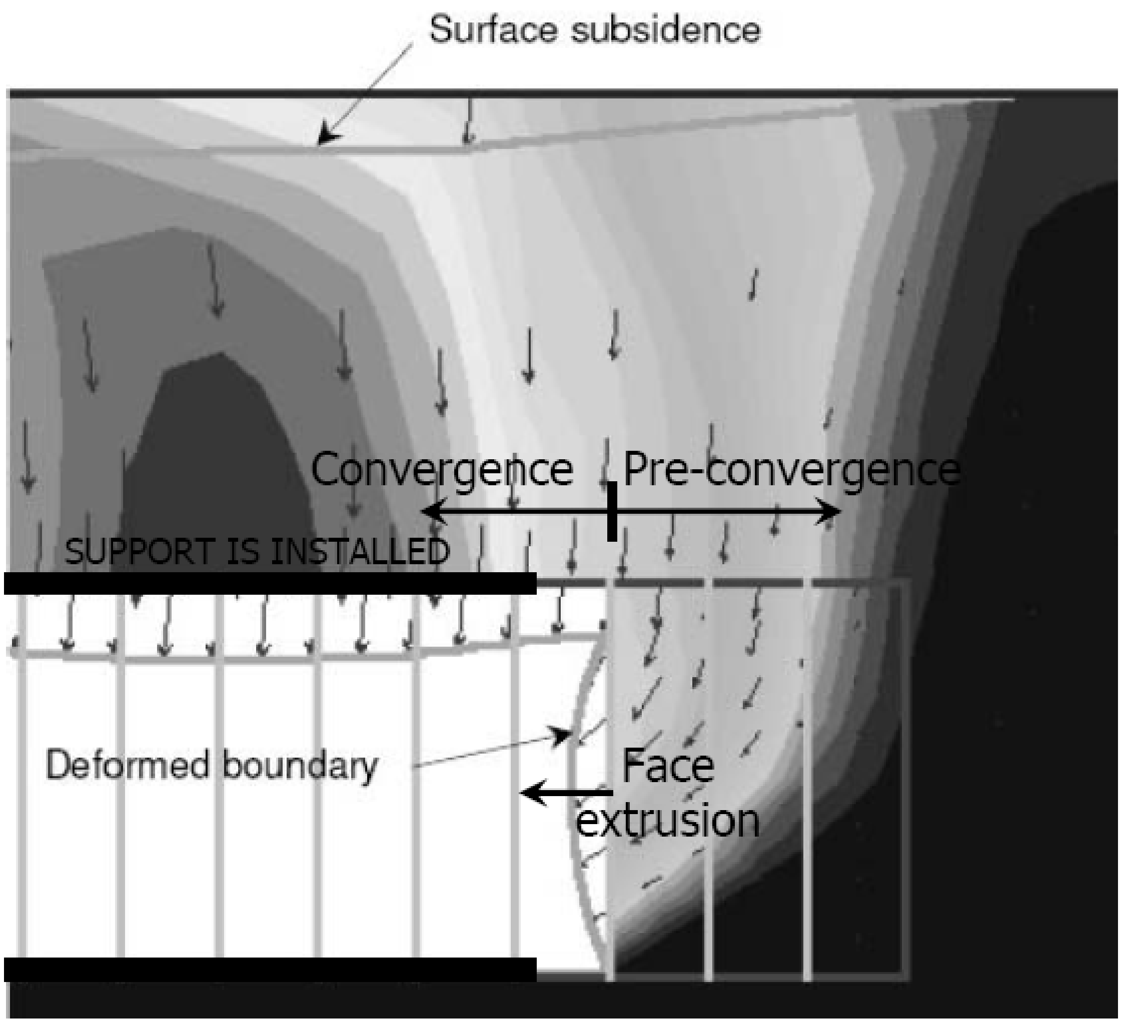
2. Behind tunnel face : radial convergence



after Lunardi & Bindi (2004)

Minimisation of ground surface displacements

Relative contribution of pre-convergence and convergence



In a properly supported non-TBM tunnel, 70-80% of total surface settlement is due to deformations ahead of tunnel face

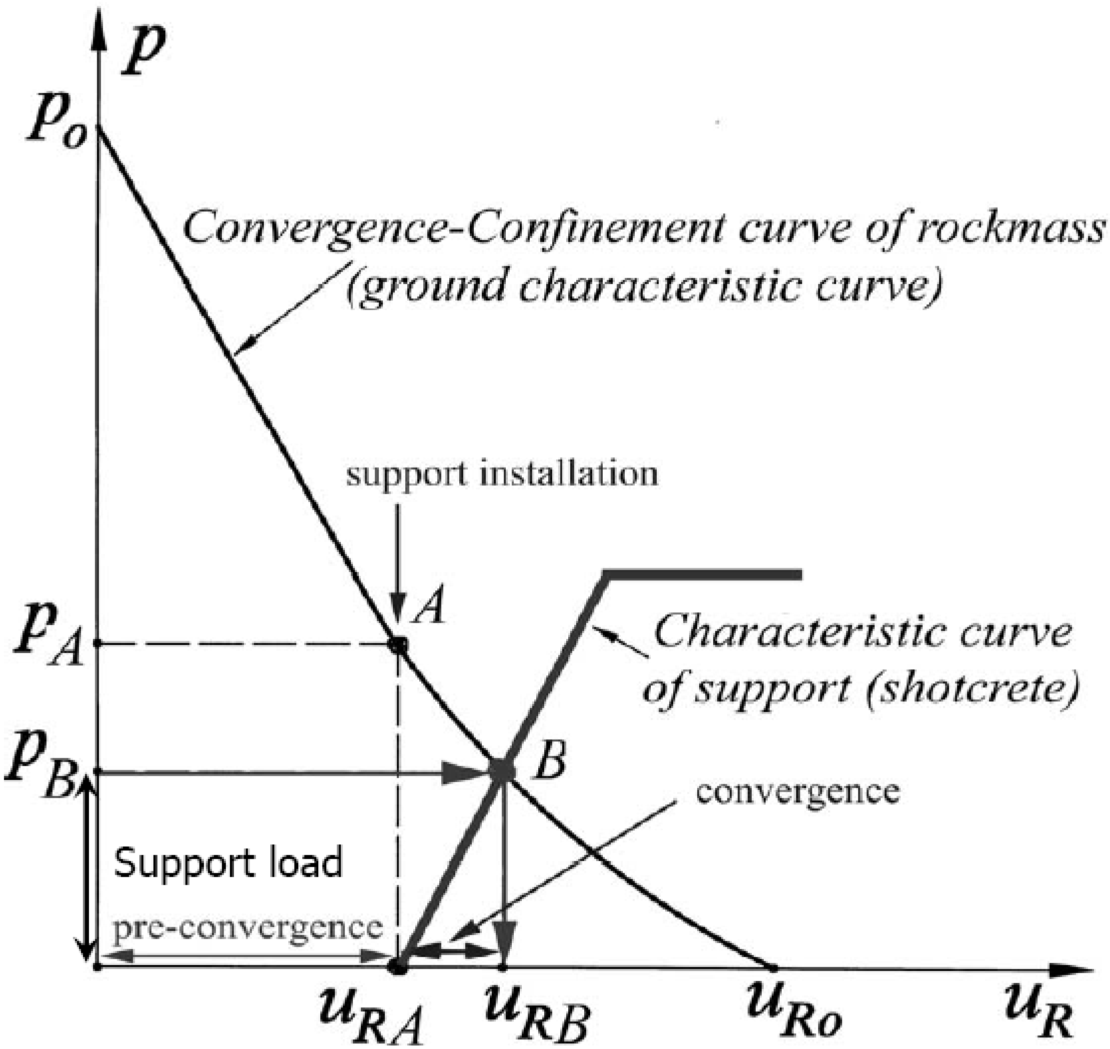
In TBM tunnels the fraction varies significantly (< 70%) depending on the method

Conclusion :

In non-TBM tunnels, control of pre-convergence (face extrusion) is critical in urban tunnelling

Control of pre-convergence is contrary to the basic NATM principle of mobilising rockmass strength by deformation

This NATM principle is mainly applicable in mountain tunnels



Mountain tunnels :

- Stability is critical
- Deformation not critical (usually desirable)

Urban tunnels :

- Deformation critical : to be minimised
- Stability is ensured by controlling deformation

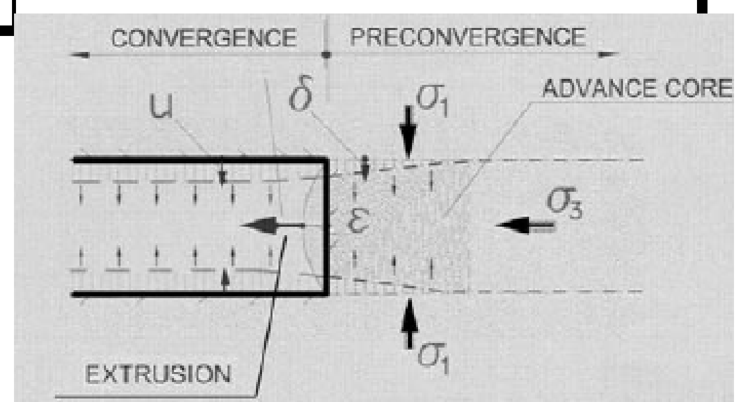
Calculation of deformations requires numerical modelling (important in urban tunnels)

Urban tunnelling methods

Minimisation of pre-convergence & convergence

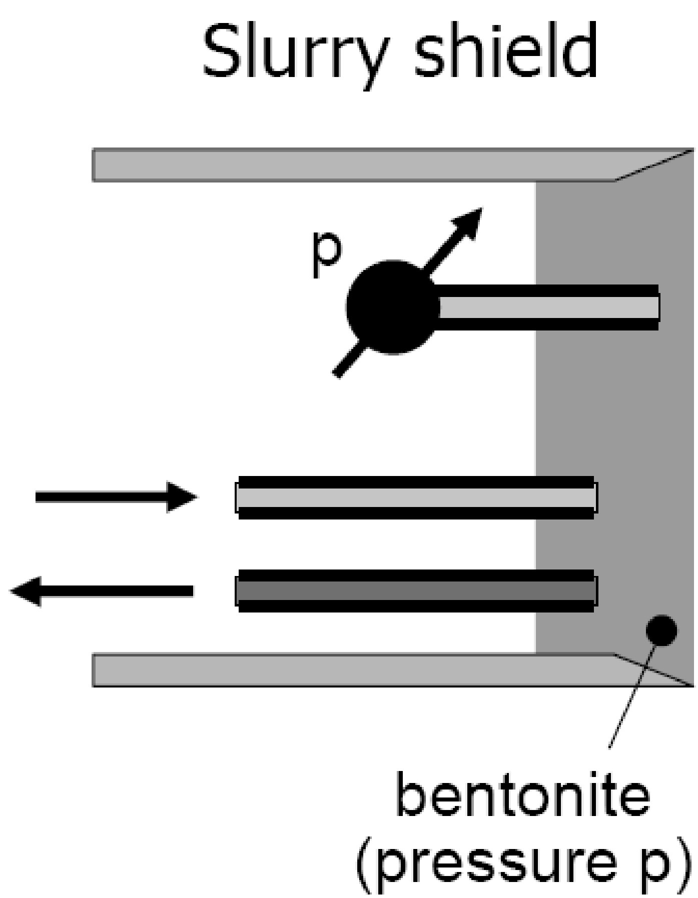
Tunnelling method	Minimisation of pre-convergence	Minimization of convergence
TBM	Adequate face support : Pressure control (closed) Cutter-head openings (open)	Control cutter-head overcut and tail-void grouting
NATM (North of Alps)	Multiple drifts $(u_R \propto D)$	Stiff support Early closure of ring
SATM (South of Alps)	Face pre-treatment	

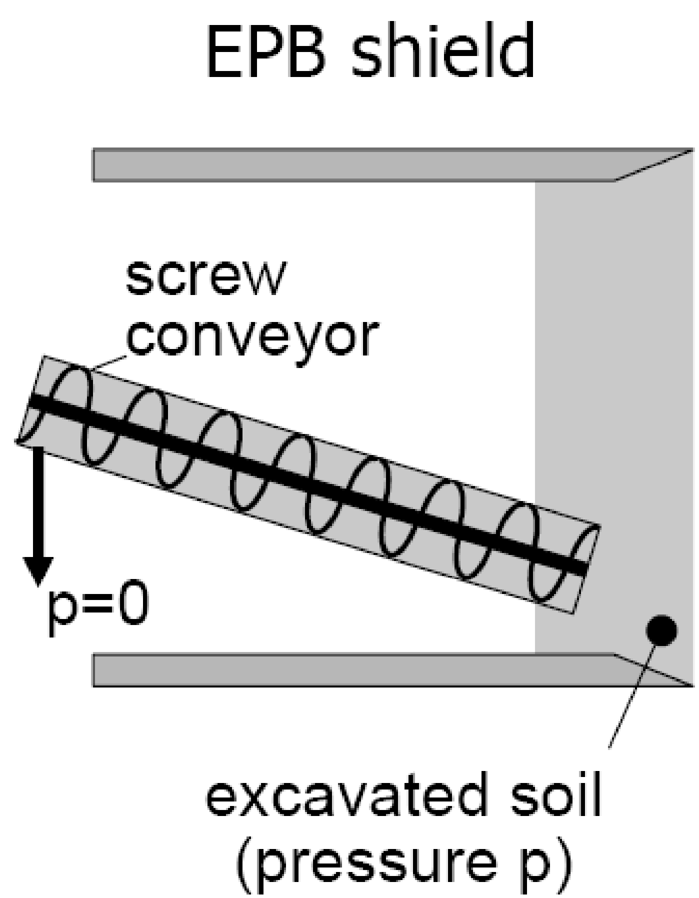
Emphasis on pre-convergence, since it controls 70-80% of total settlement



Urban tunnelling methods : TBM tunnelling

Control of pre-convergence by face pressure and ground conditioning in closed-face machines





Urban tunnelling methods : TBM tunnelling

Control of pre-convergence by the size of cutter-head openings in open face machines

Athens Metro – 9.5m dia. open TBM

Numerical analysis in the design of urban tunnels

1. Characteristics of urban tunnels
2. Tunnelling methods in urban areas (to control settlements)
3. Methods of numerical analysis
   - Continuum vs. discontinuum modelling
   - Continuum 3-D modelling :

     Analysis of pre-convergence & face pre-treatment (for design)

     Prediction of ground parameters (E) by monitoring extrusion
   - Continuum 2-D modelling :

     How to model the 3-D problem in 2-D (in a cross-section)

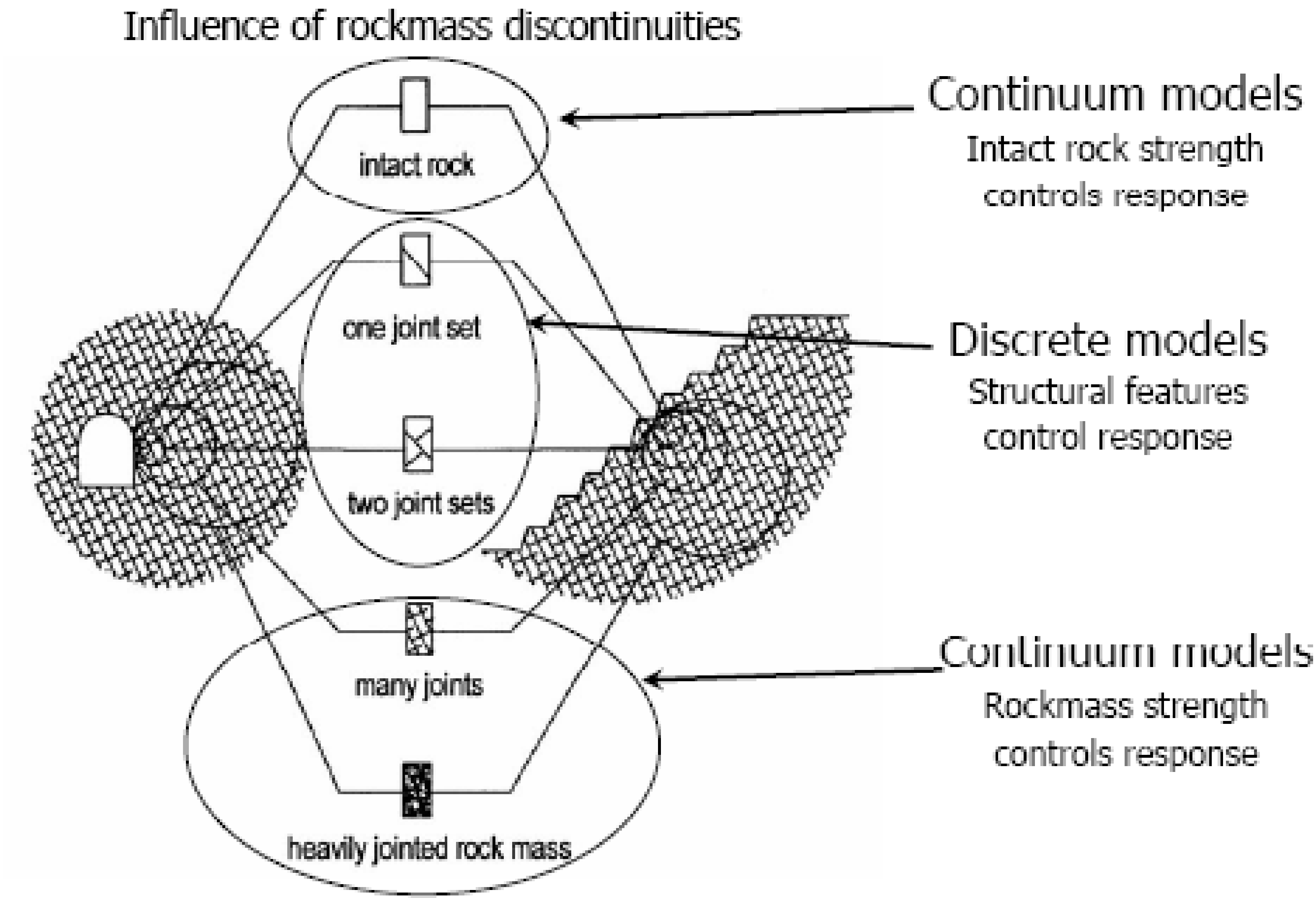
Urban tunnel design using numerical analysis

Tunnel excavation and support is traditionally an empirical art

Numerical analyses are useful in the following cases :

- Calculation of ground surface settlements
- Design of face pre-treatment in difficult ground conditions
  (selection among alternative methods)
- Sensitivity analyses :
  - Effect of locally inferior ground on the support system
  - Comparison of alternative support methods
- Selection of most appropriate corrective action in case of contingency
- Assessment of ground properties ahead of the excavation face using monitoring data (mainly face extrusion)
- "Legal" support of design decisions
  (decisions based on "engineering judgment" rarely stand in courts)

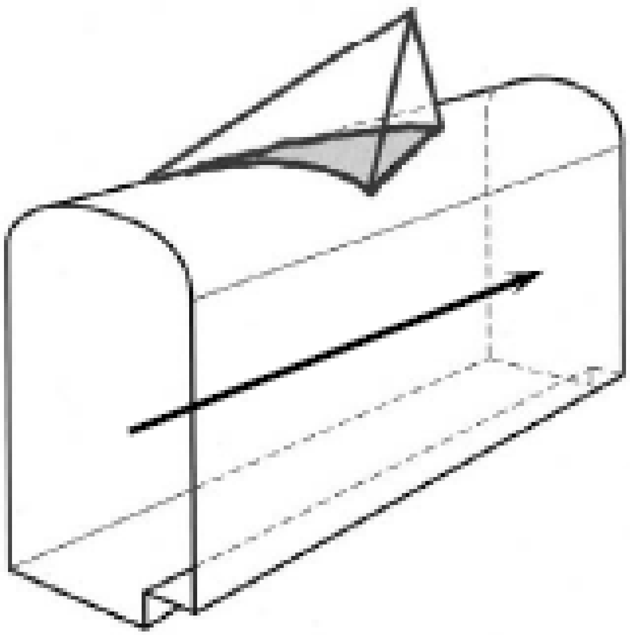
Design using numerical analysis: Continuum / Discontinuum models

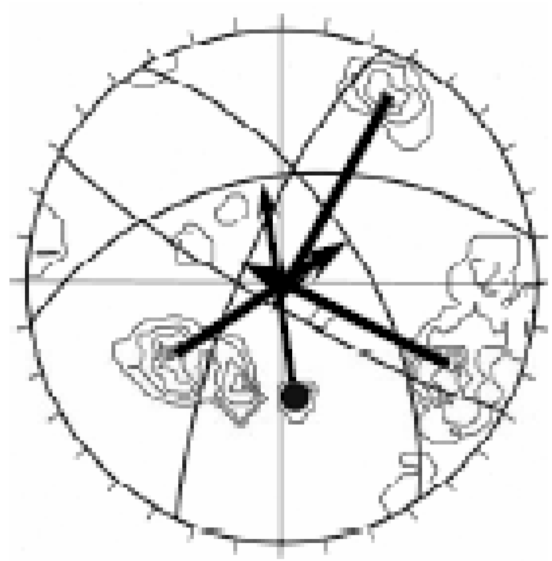


Design using numerical analysis: Discontinuum models

Applicable : mainly in rock where structural features control response

1. Analysis of wedge stability (at roof and sidewalls) :





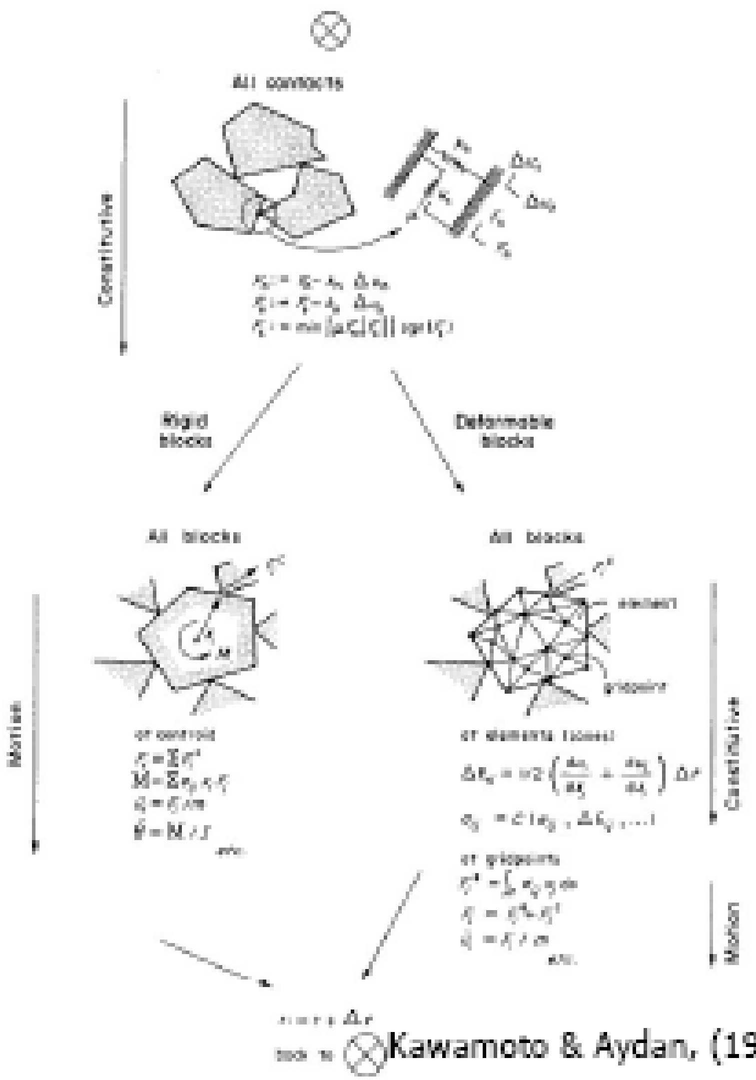
Typical numerical analysis using computer programs :

- UNWEDGE (for tunnels)
- SWEDGE (for slopes)

Design using numerical analysis: Discontinuum models

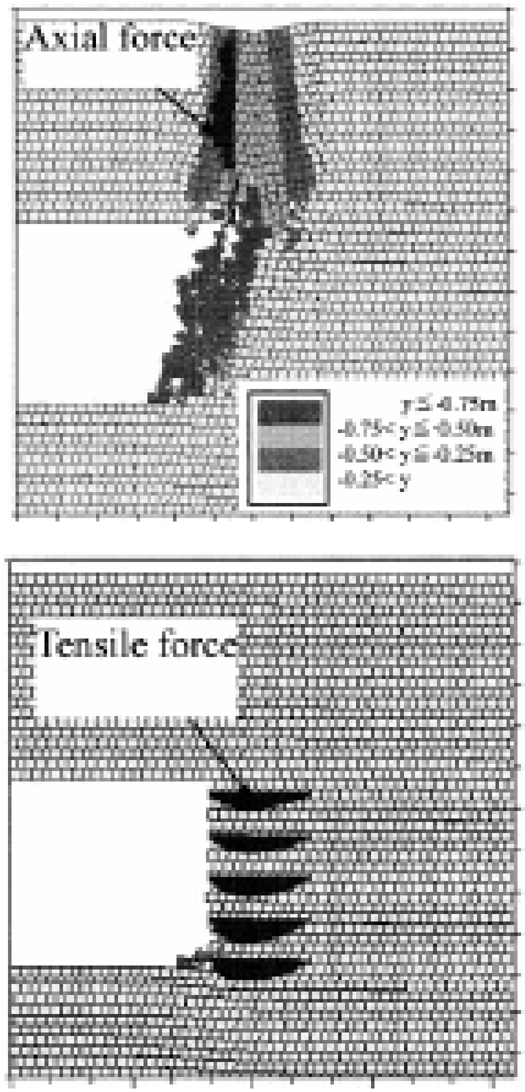
2. Analysis of tunnel excavation and support using discontinuum models :

Discrete Element Method: Calculation scheme



Kawamoto & Aydan, (1999)

e.g. programs UDEC (2-D) , 3-DEC (3-D)

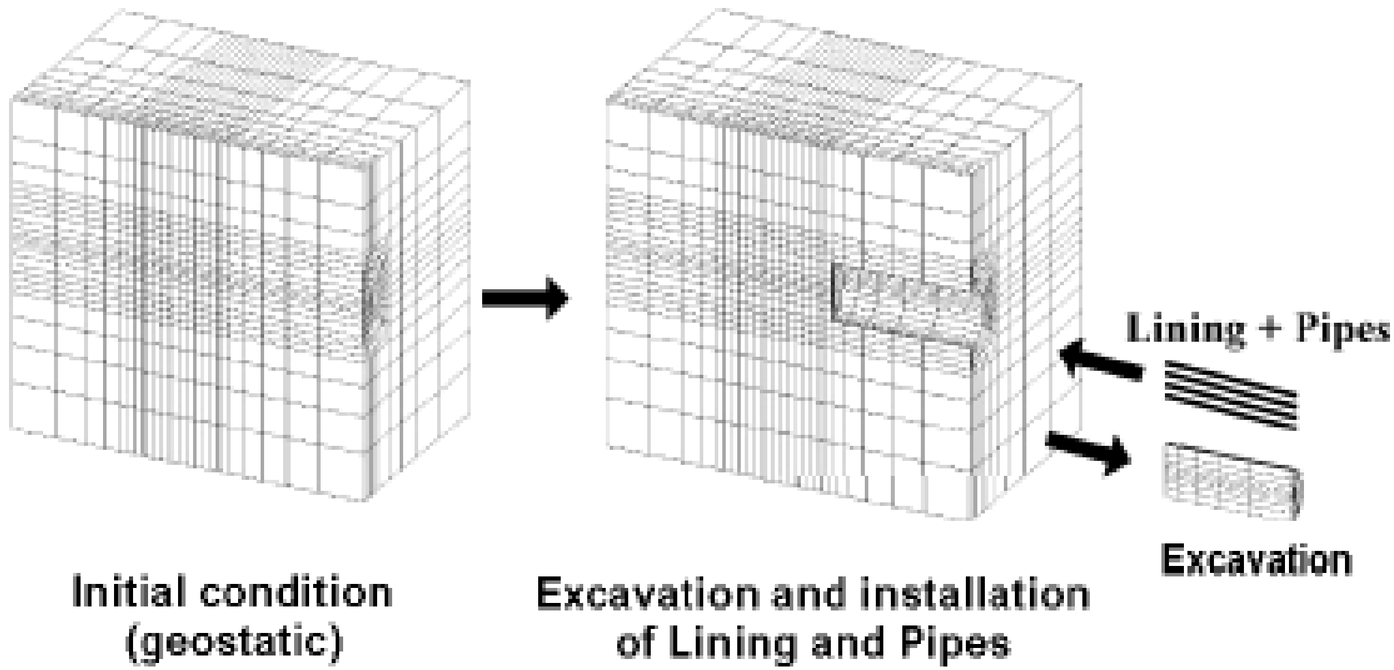


2-D analysis of tunnel face stability: UDEC Results

Kamata & Mashimo (2003)

Design using numerical analysis: Continuum models

3-D models : Check face stability / design face pre-treatment



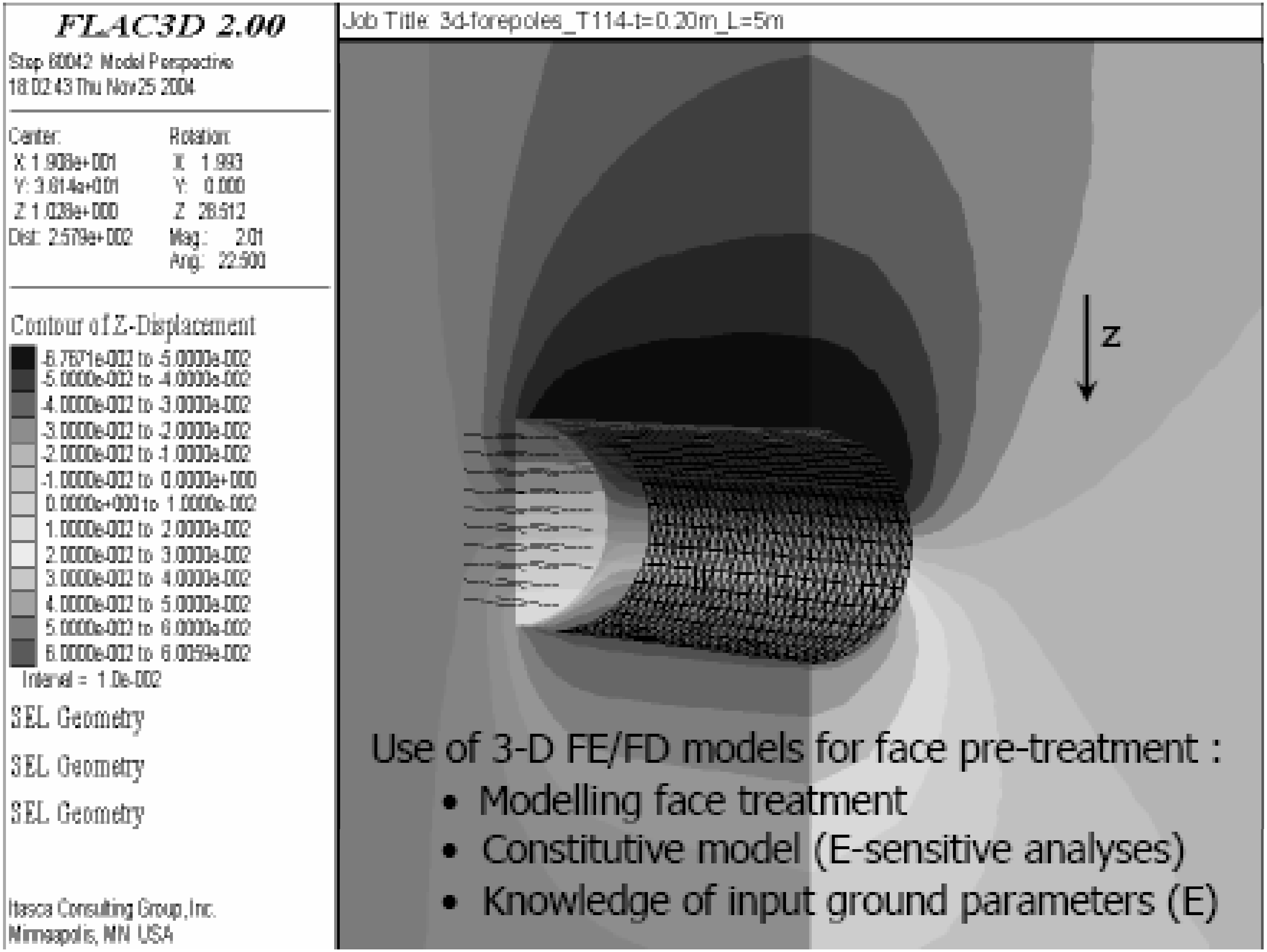
Modelling stages are direct :

1. Geostatic (initial conditions)
2. Installation of face support
3. Advancement of the excavation (one step)
4. Installation of side support
5. REPEAT steps 3–4 until new face support
6. Install face support .....

However :

- Input preparation and output presentation is often complicated
- Analysis is time consuming
- Improved accuracy may be incompatible with the level of knowledge of ground conditions

Design using numerical analysis: Continuum models / 3-D



Use of 3-D FE/FD models for face pre-treatment :

- Modelling face treatment
- Constitutive model (E-sensitive analyses)
- Knowledge of input ground parameters (E)

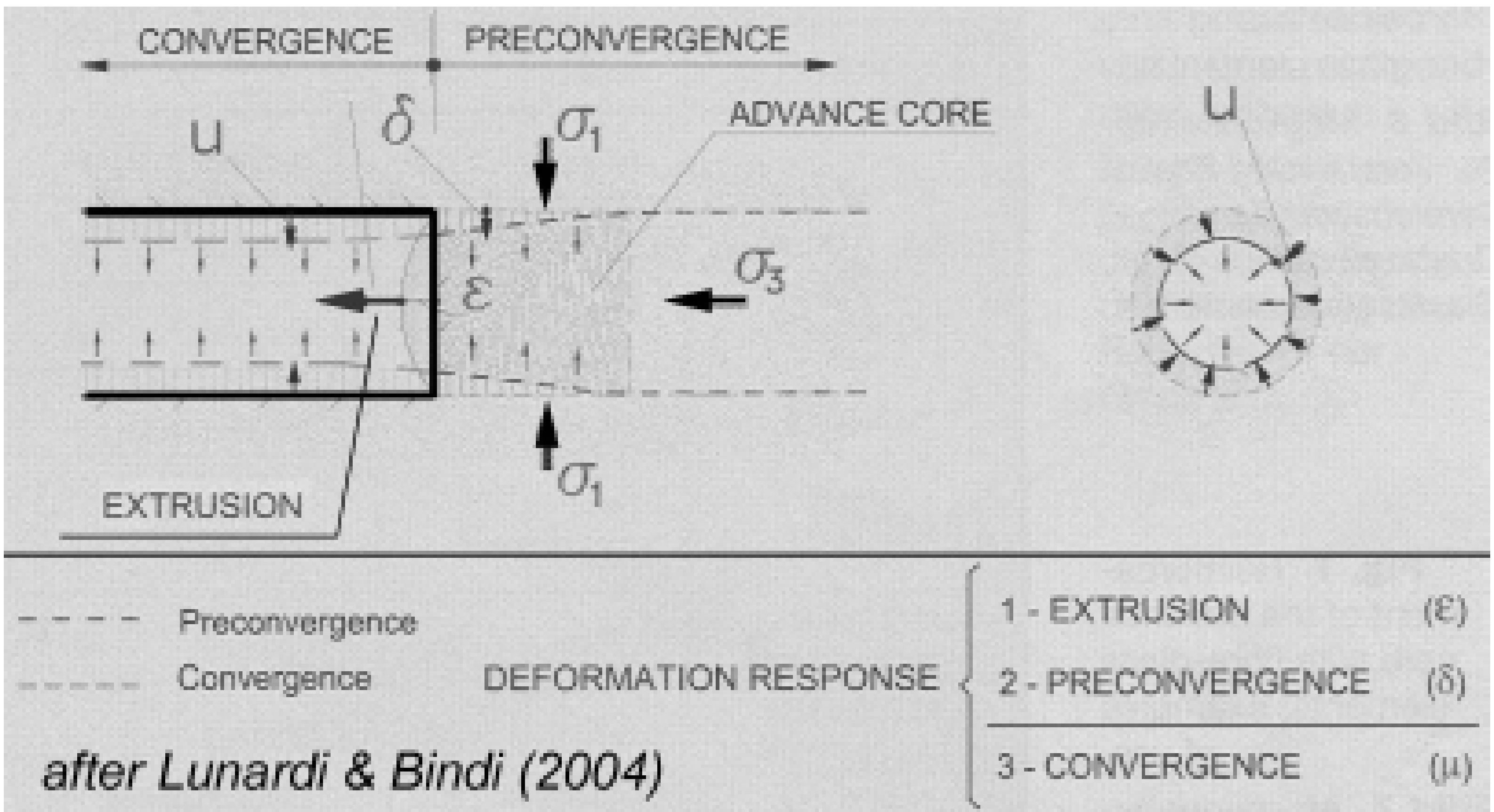
Use of numerical analyses in assessing ground parameters

Ground parameters for tunnelling can be obtained by :

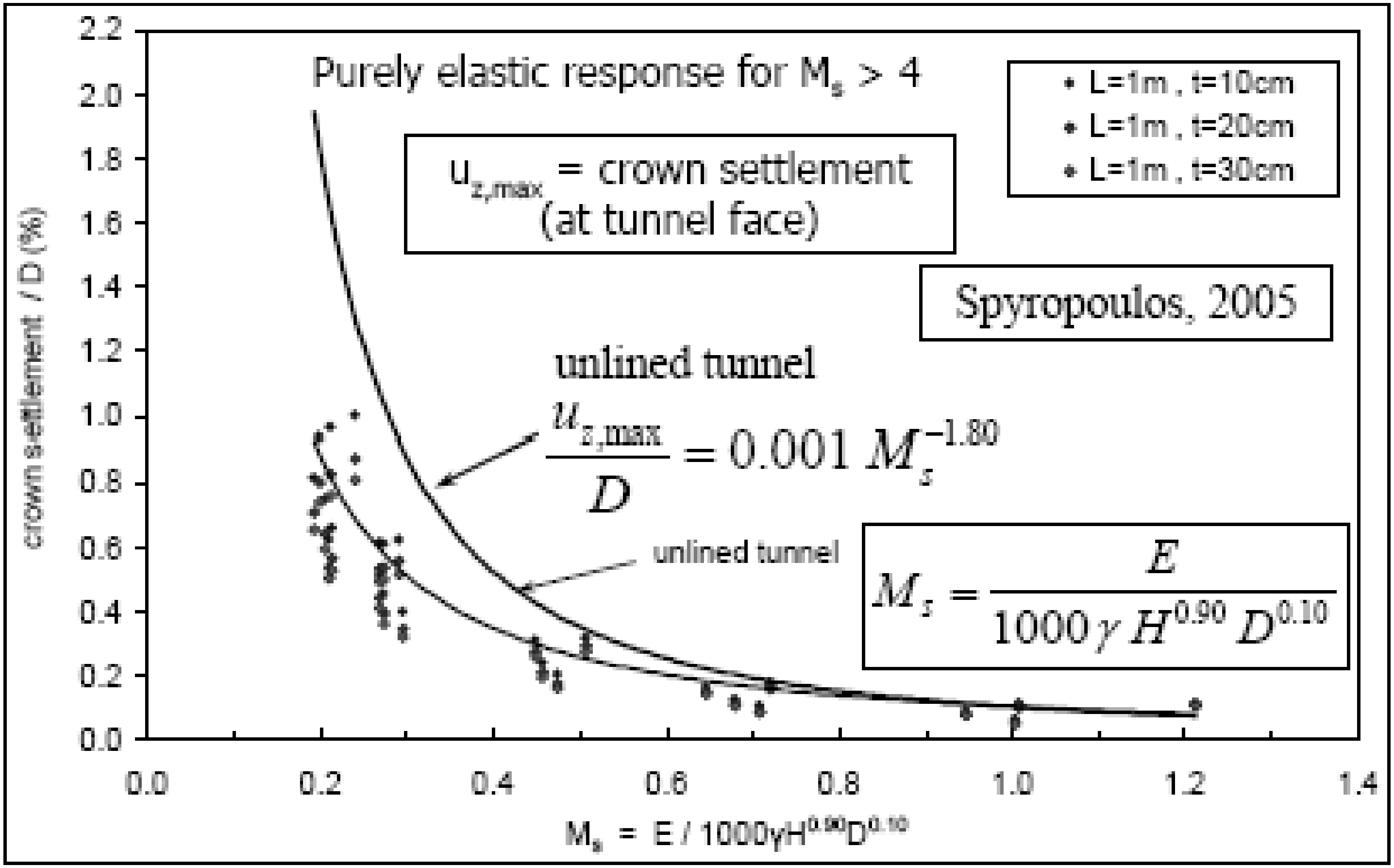
- Boreholes & lab tests : not very relevant
- Field tests (inside the tunnel) : expensive, slow and not very relevant
- Exploitation of excavation data (monitoring)

  Wall convergence (not sensitive)

  Face extrusion (very useful)



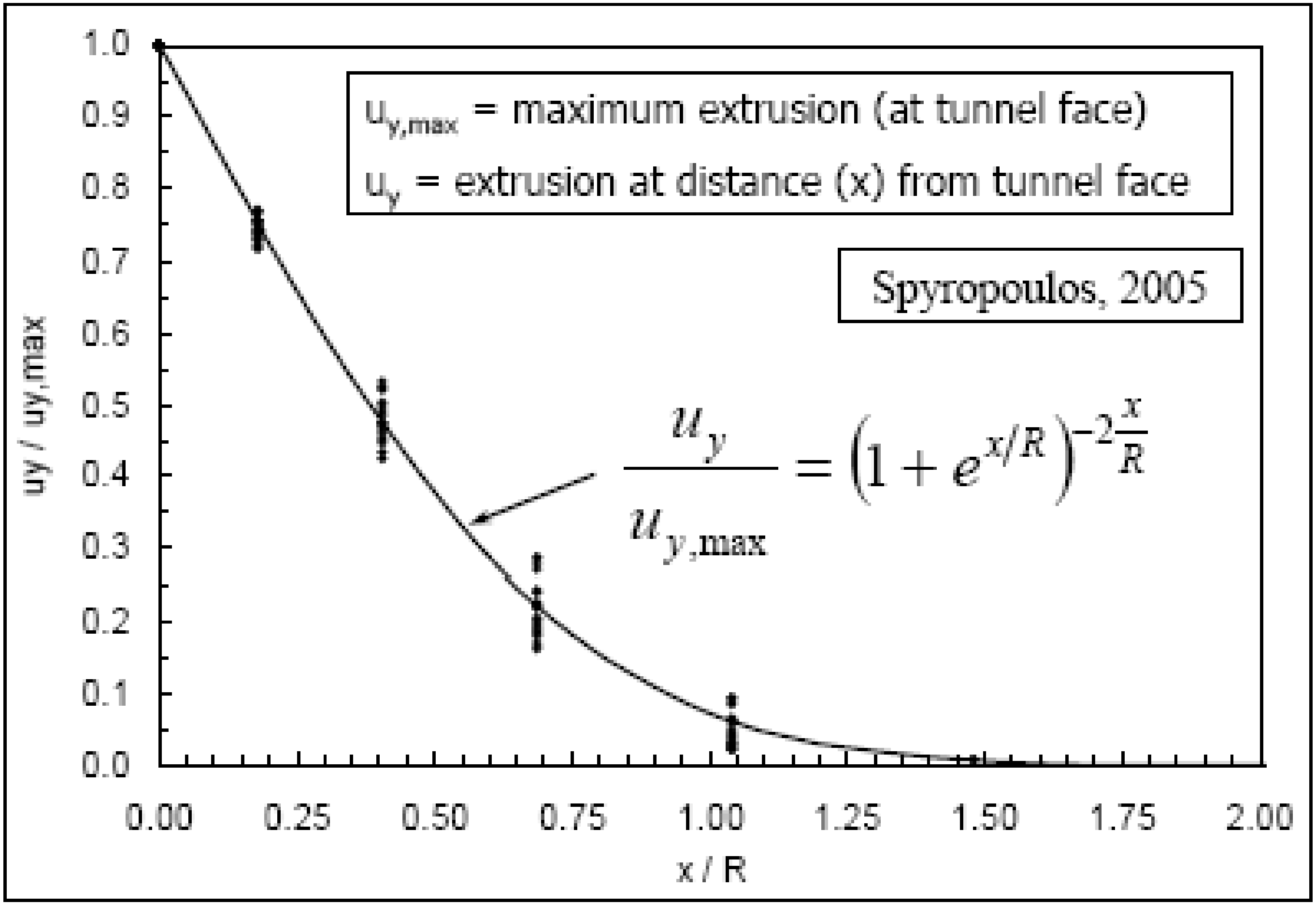
Use of numerical analyses in assessing ground parameters



Crown settlement $u_{z,max}$ (at tunnel face) as a function of the controlling ground parameter M_s. Crown settlement is strongly influenced by the installation of shotcrete lining (thickness t) behind the face (distance L).

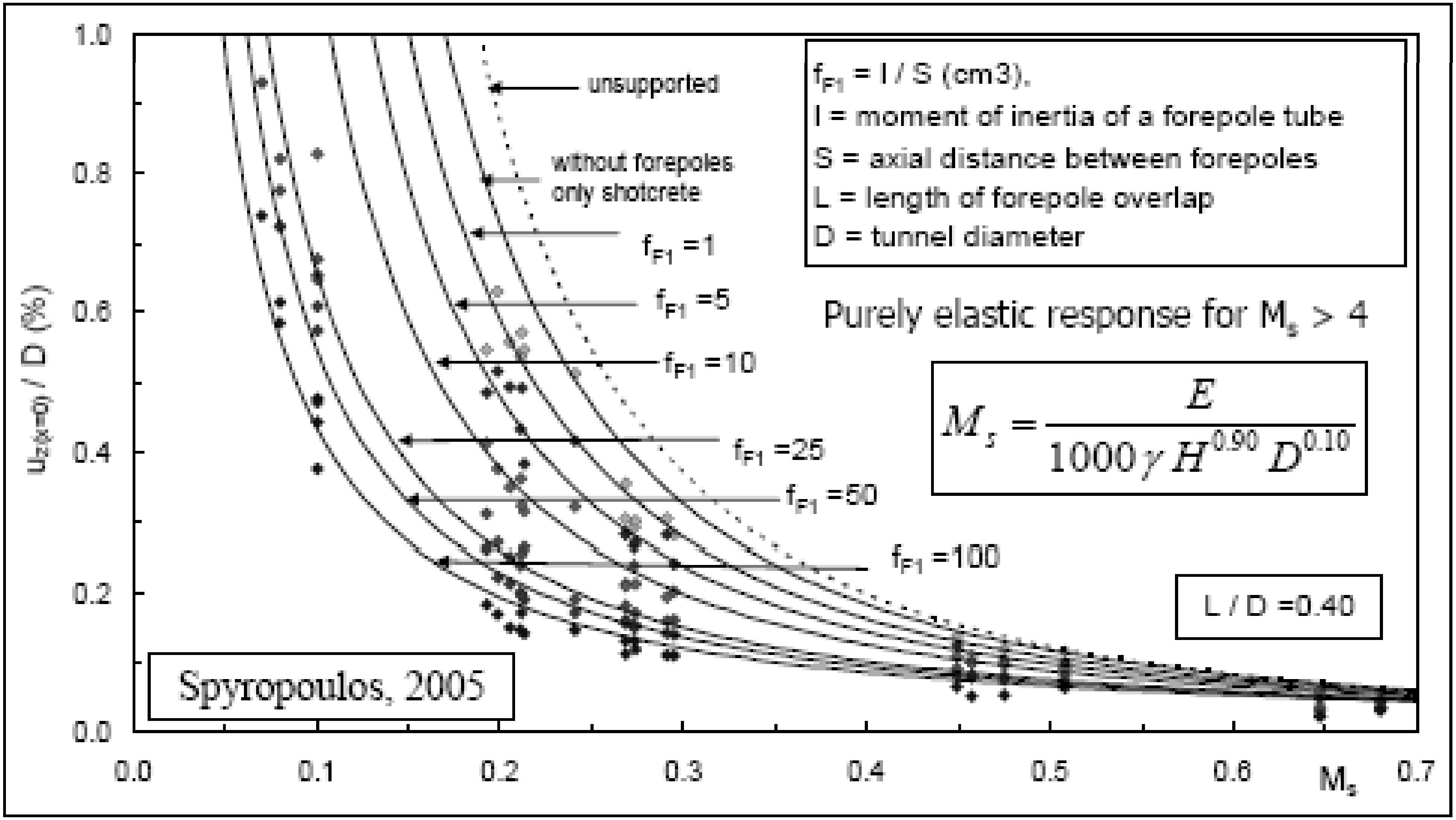
Crown settlement cannot be used to assess the value of M_s ahead of the tunnel face

Use of numerical analyses in assessing ground parameters



Extrusion u_y as a function of the distance from tunnel face. Since the value of $u_{y,max}$ is related to $M_s \Rightarrow$ correlation u_y & M_s (for any x/R) is useful $\Rightarrow$ E

Reduction of crest settlement (u_z at x=0) by using forepoles

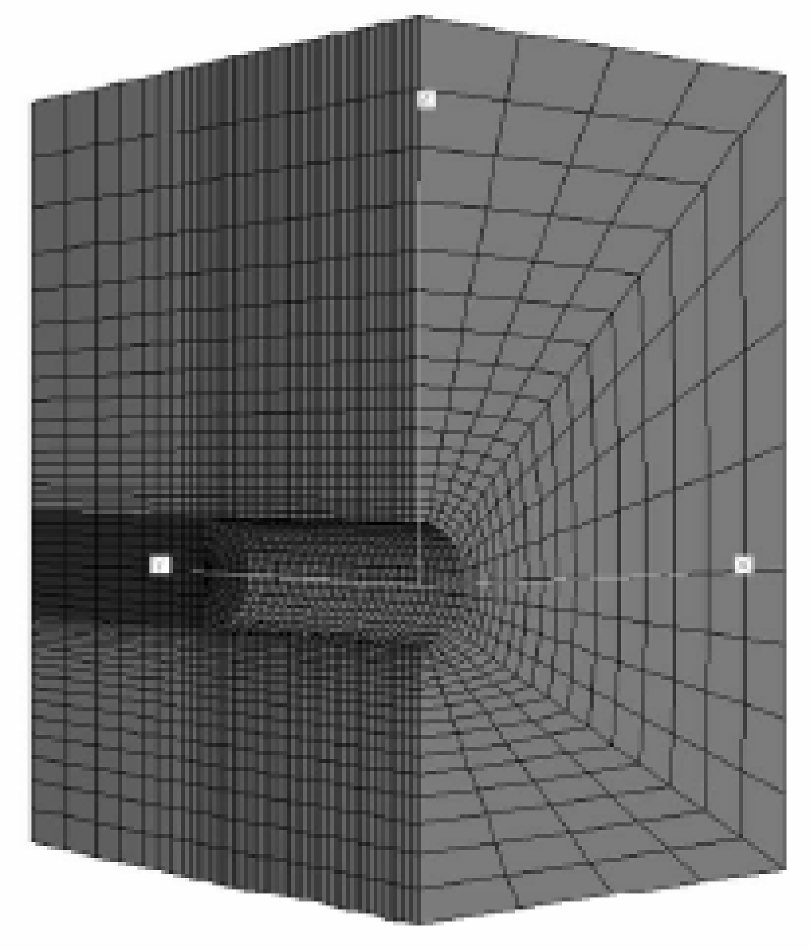


Practical forepoling applications correspond to $f_{F1} < 20$

Design using numerical analysis: Continuum models

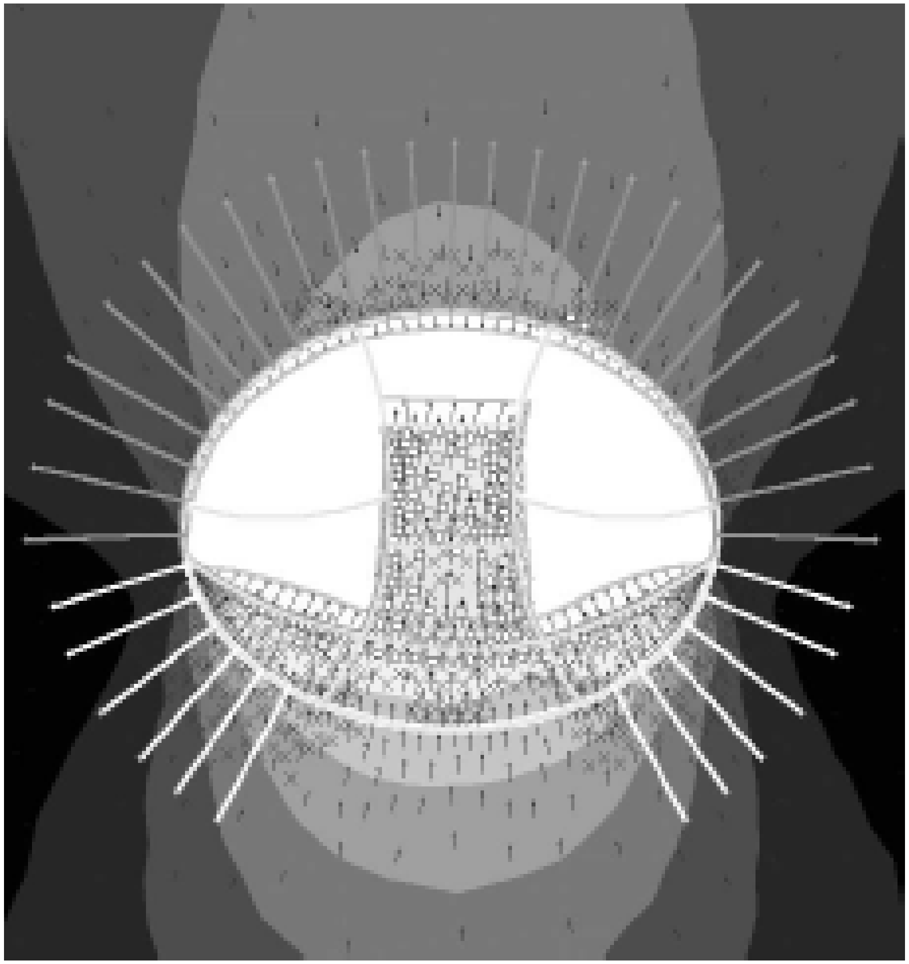
3-D models : Most suitable for face pre-convergence / face pre-treatment

2-D models : Analysis of tunnel cross-section (from 3-D to 2-D)



3-D model using FLAC

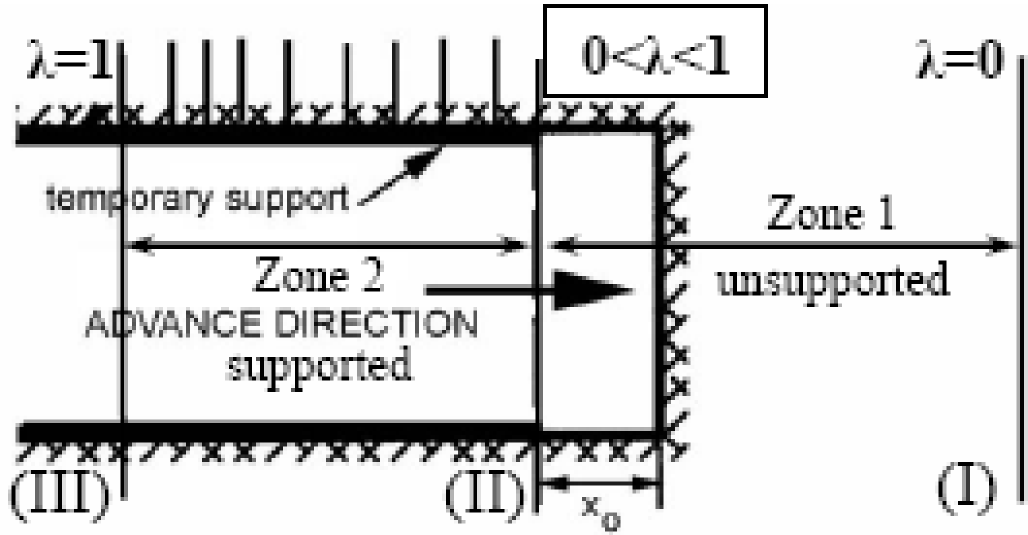
Disadvantage : sophisticated



2-D model using PHASE2

Disadvantage : cannot model face

Design using numerical analysis: Continuum models / 2-D



The analysis is performed by gradually reducing the internal pressure "p"

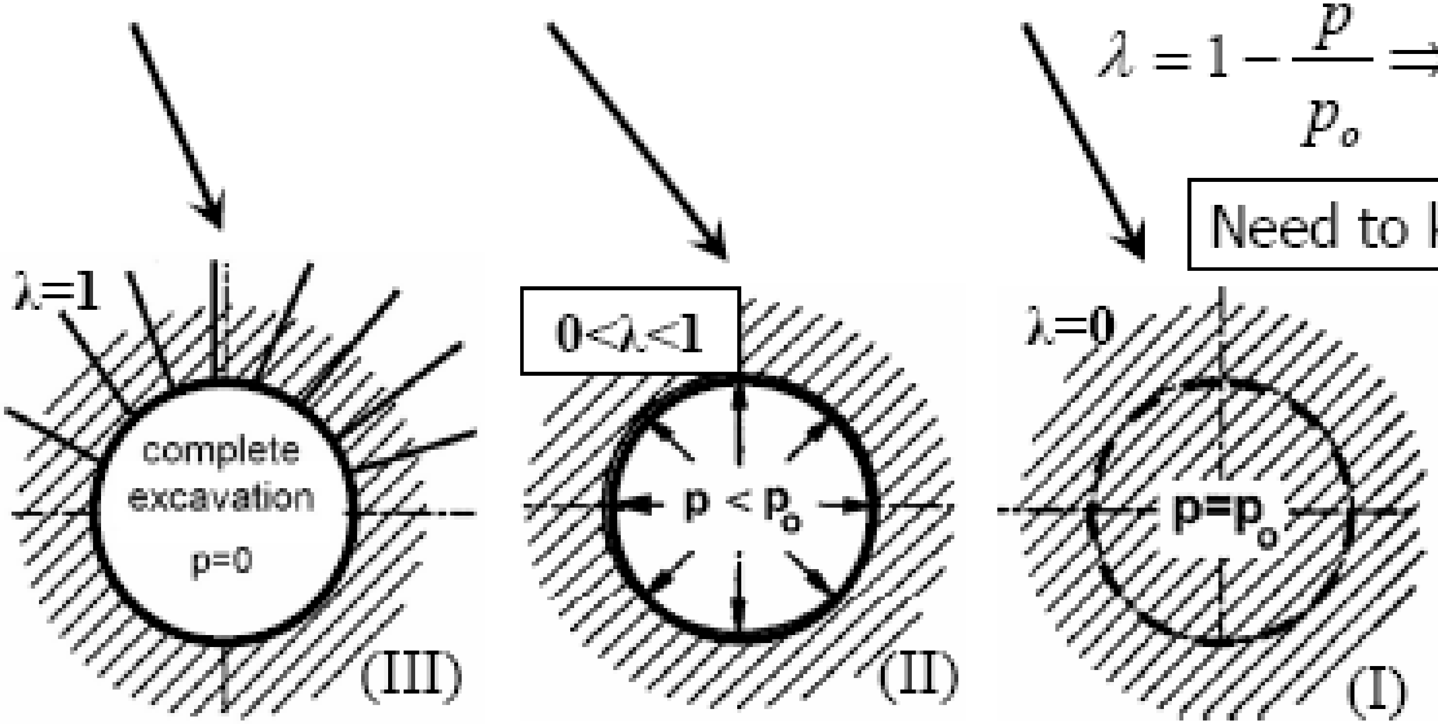
p_o = geostatic stress (isotropic)

p = tunnel "internal pressure"

λ = deconfinement ratio

$$\lambda = 1 - \frac{p}{p_o} \Rightarrow p = p_o(1-\lambda)$$

Need to know λ= λ(x)



Design using numerical analysis: Continuum models / 2-D

Use of deconfinement ratio (λ)

Deconfinement using internal pressure reduction :

$$p = (1-\lambda)\, p_o$$

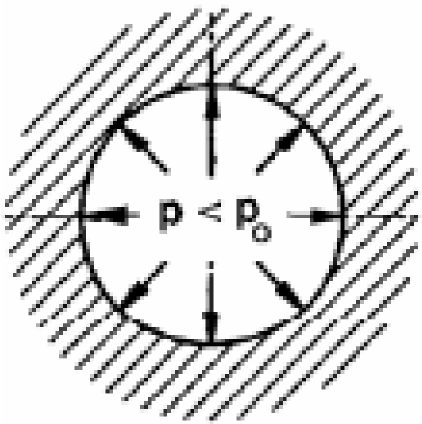
p_o = geostatic stress (isotropic)

⇦ λ ⇨

Deconfinement using section modulus reduction :

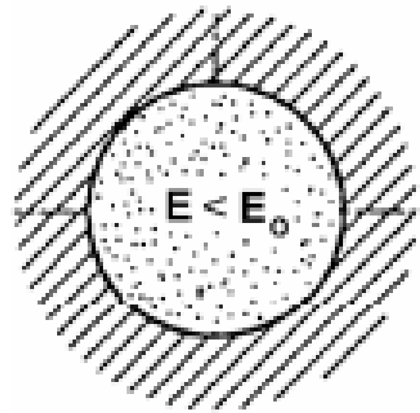
$$E = \left[\frac{(1-2\nu)(1-\lambda)}{(1-2\nu)+\lambda}\right] E_o$$

E_o = ground E-modulus



Example :

$\lambda = 0.70 \Rightarrow p = 30\%\, p_o$



Example :

$\lambda = 0.70 \Rightarrow E = 10\%\, E_o$

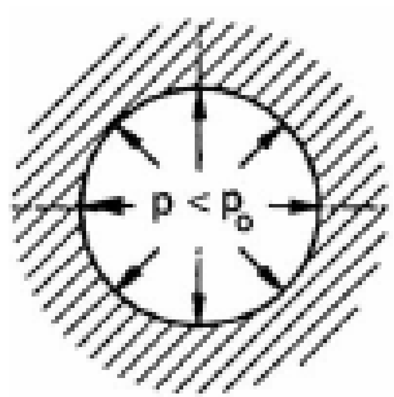
Advantage : Good in anisotropic fields

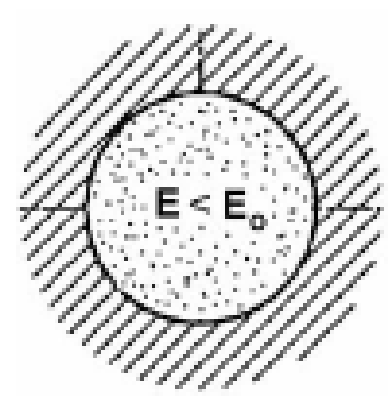
Use of deconfinement ratio (λ) and equivalent "reduced modulus" E

λ	p/p_o	Values of E/E_o for		
		$\nu = 0.25$	$\nu = 0.30$	$\nu = 0.35$
0.20	0.80	0.571	0.533	0.480
0.30	0.70	0.438	0.400	0.350
0.40	0.60	0.333	0.300	0.257
0.50	0.50	0.250	0.222	0.187
0.60	0.40	0.182	0.160	0.133
0.70	0.30	0.125	0.109	0.090
0.80	0.20	0.077	0.067	0.054
0.90	0.10	0.036	0.031	0.025

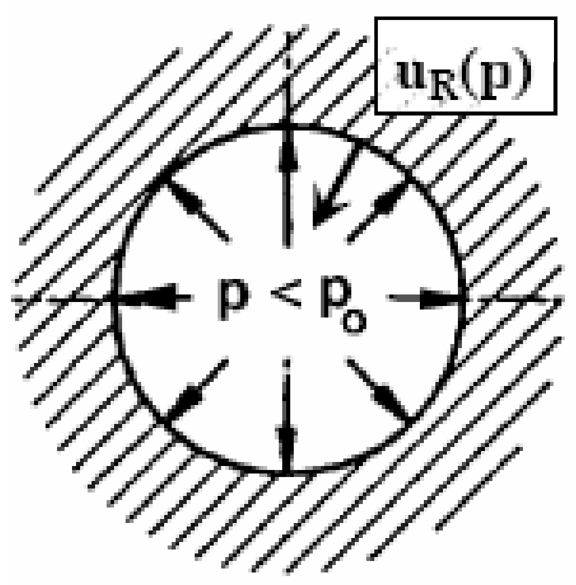
$$\lambda = 1 - p/p_o$$

$$\frac{E}{E_o} = \frac{(1-2\nu)(1-\lambda)}{(1-2\nu)+\lambda}$$



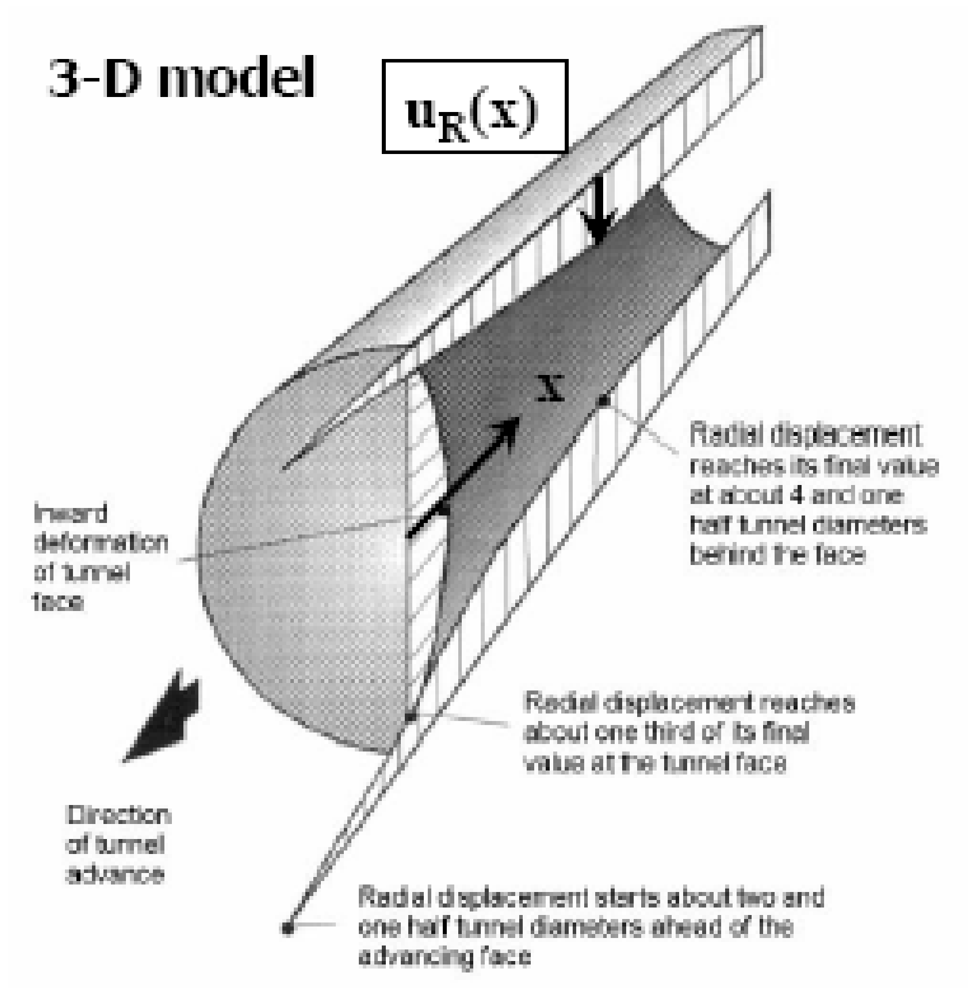


Determination of the deconfinement ratio (λ) along the tunnel axis



2-D model

Tunnel wall displacement (u_R) varies along the tunnel axis



Calculation method :

3-D model : $u_R = u_R(x)$

2-D model : $u_R = u_R(p)$

or $u_R = u_R(\lambda)$

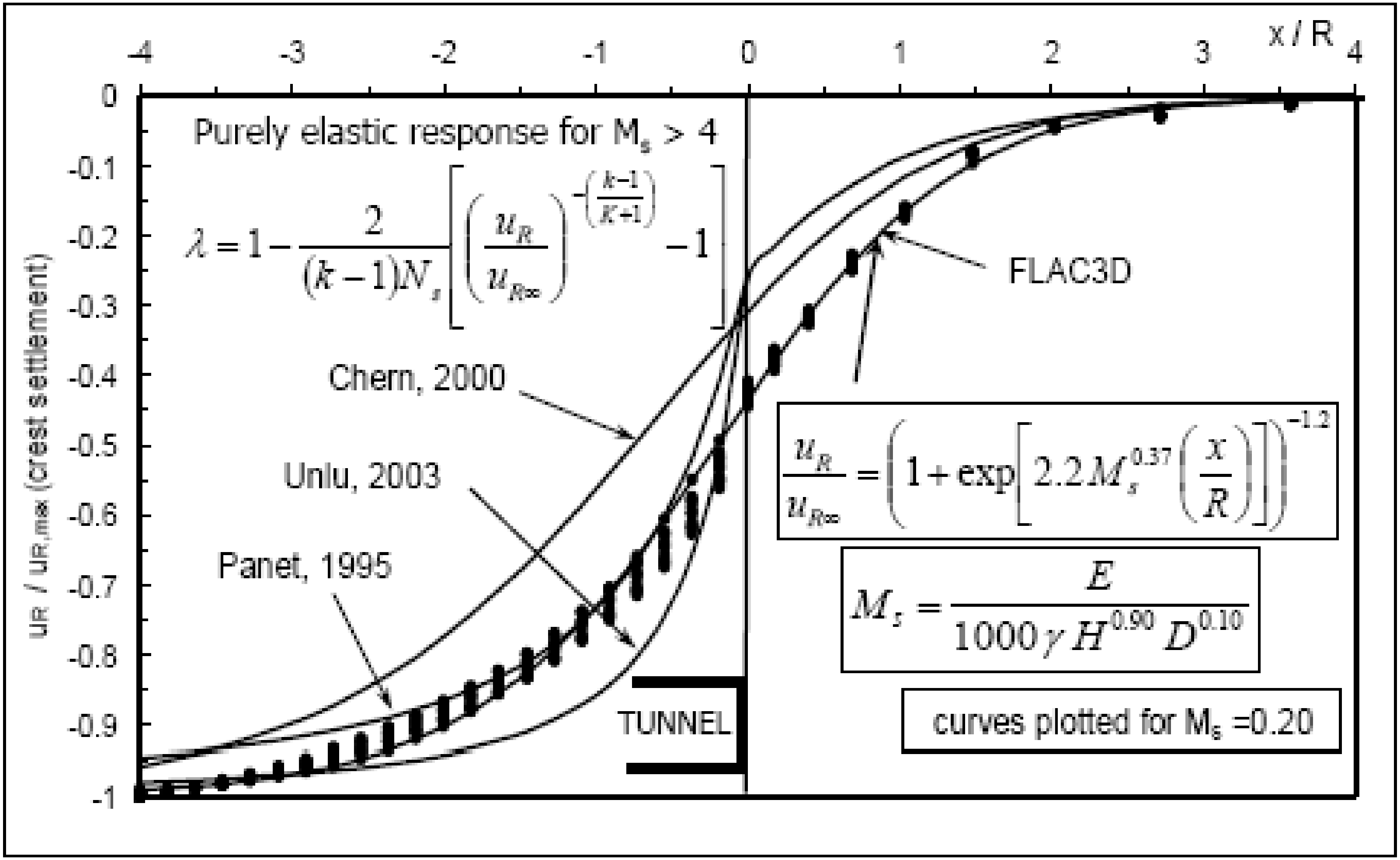
Thus : $\lambda = \lambda(x)$

Standard diagrams are available

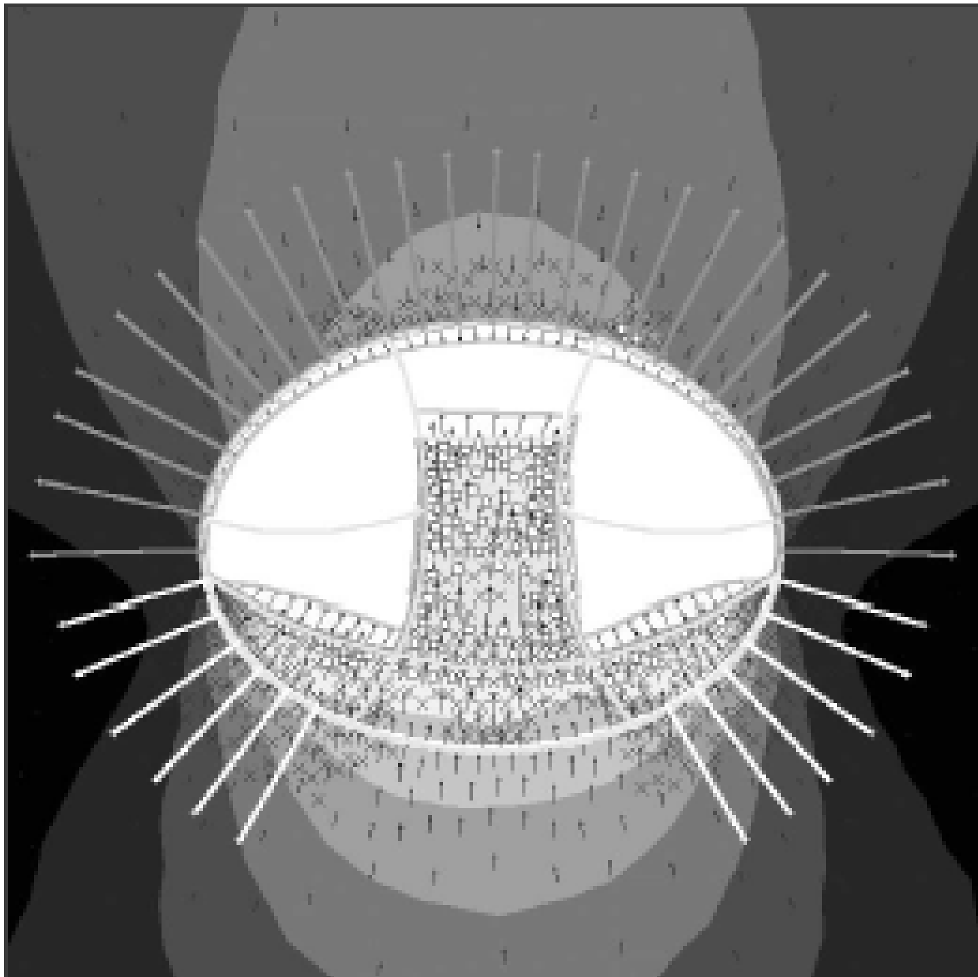
Determination of the deconfinement ratio (λ) along the tunnel axis

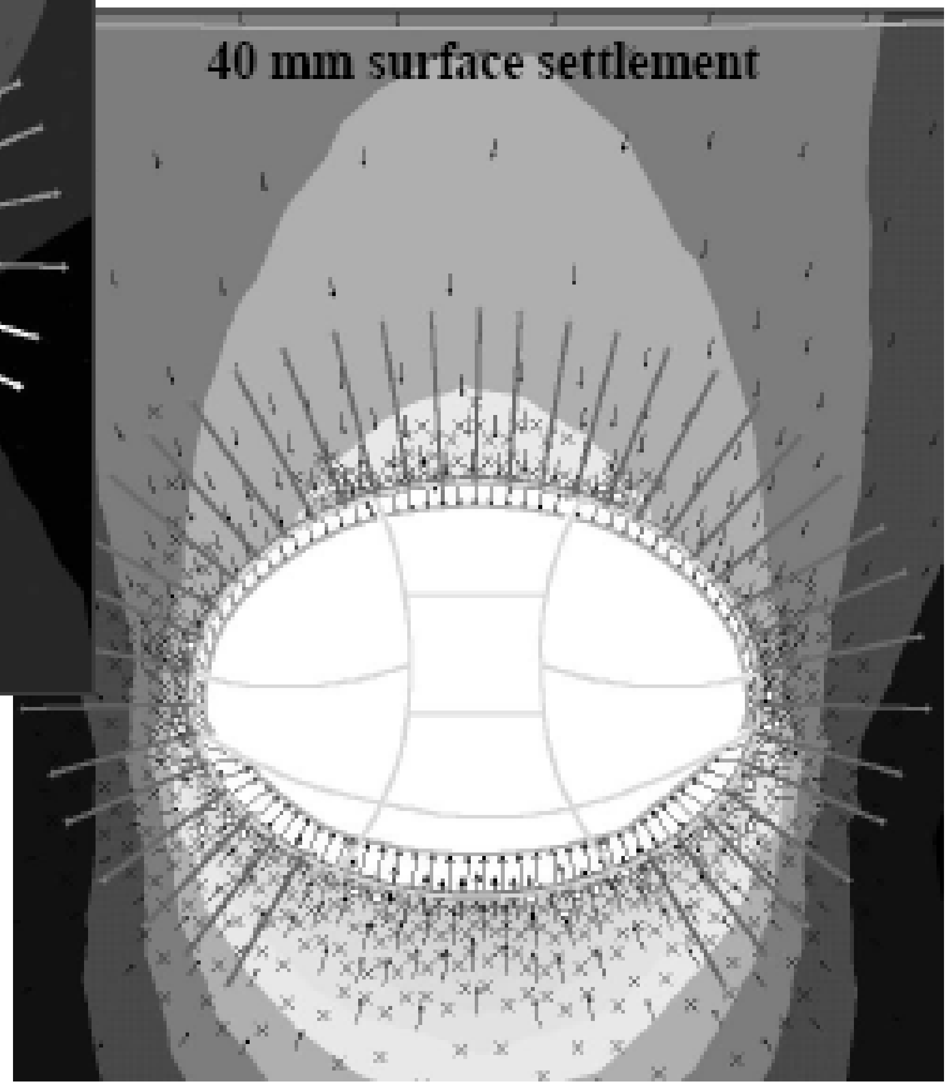
FLAC-3D : Spyropoulos, 2005

$$\lambda = f\left(\frac{x}{R};\ M_s\right)$$



Excavation with side-drifting and central pillar





Athens Metro : Acropolis Station
excavation in"schist" (phyllite)

Conclusions

1. Ground deformations are critical
2. Estimates of ground deformations require 3-D numerical analyses (+ ground model + ground properties)
3. Relevant ground properties (mainly E) can be obtained by measurement of face extrusion & numerical back-analyses (or use of the normalised graphs)
4. For many tunnel designers, 3-D analyses may seem too sophisticated :
   - Methods exist to analyse the problem in 2-D using the "deconfinement method (λ)"
   - Normalised graphs are available to estimate (λ) in tunnels without / with face pre-treatment