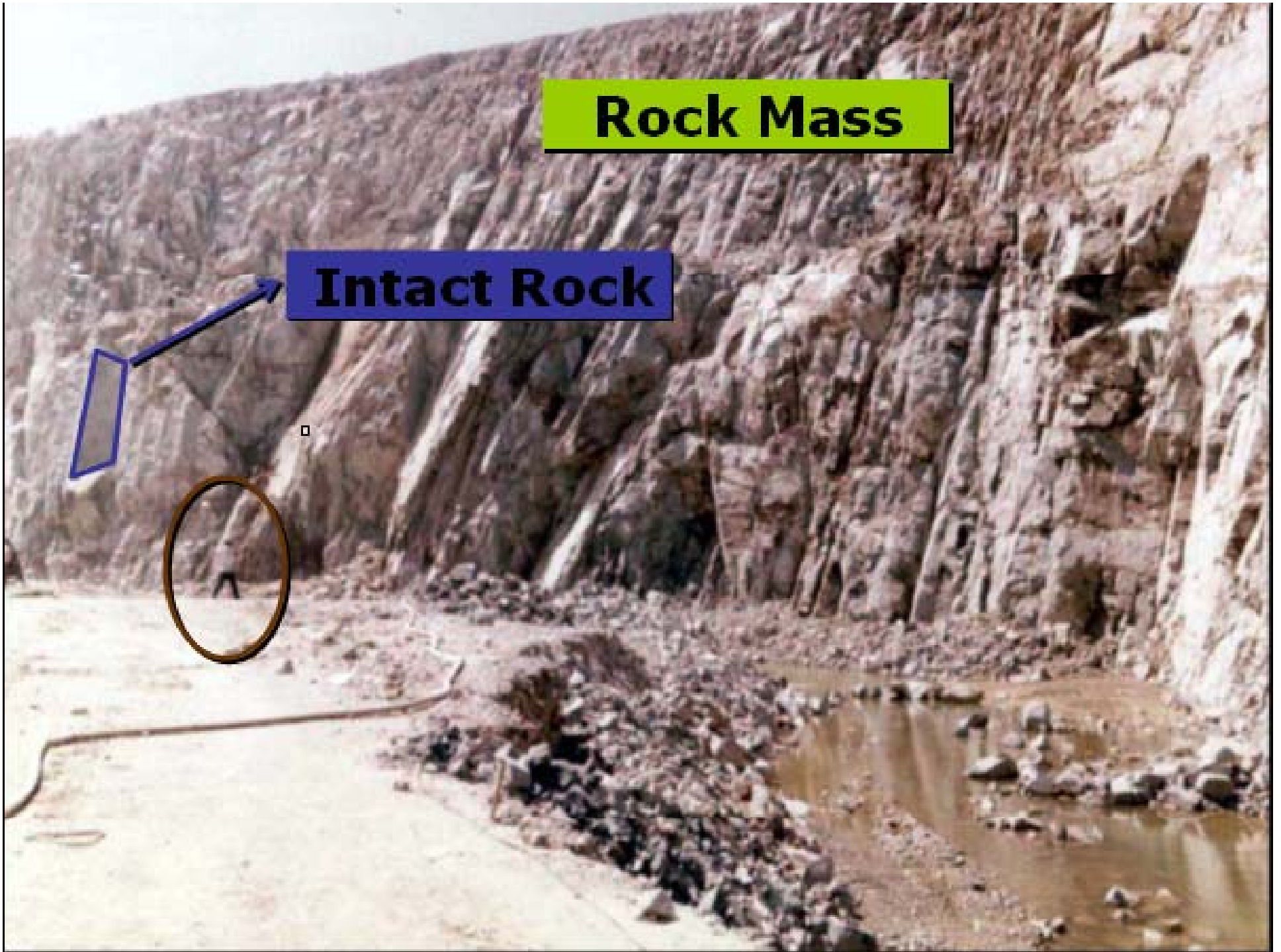
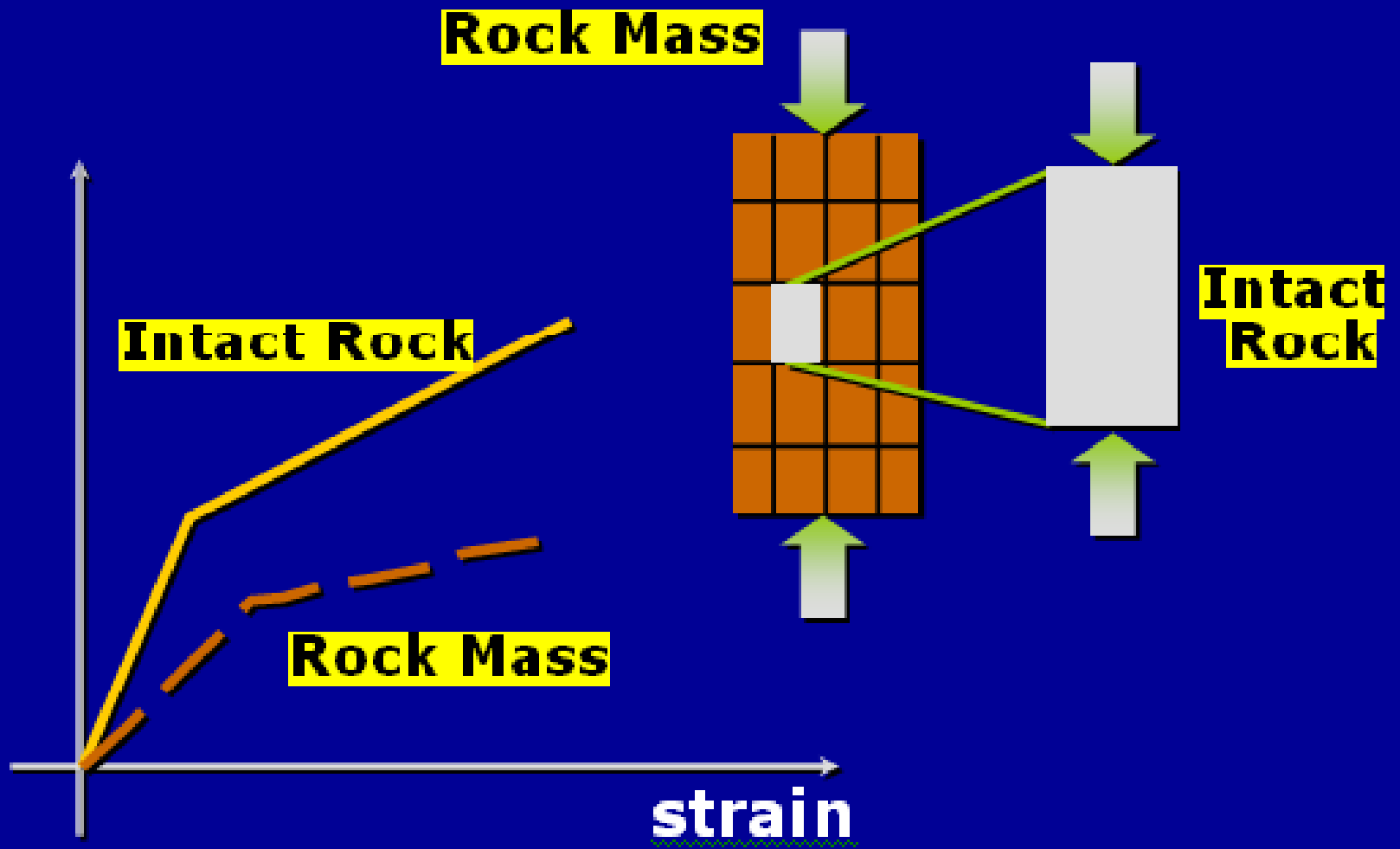


**Rock Mass**

**Intact Rock**





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# Geometrical Properties of Rock Joints

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## Types of Rock Discontinuities

**Joints** – Most common, normally in sets

**Fractures** – Randomly distributed

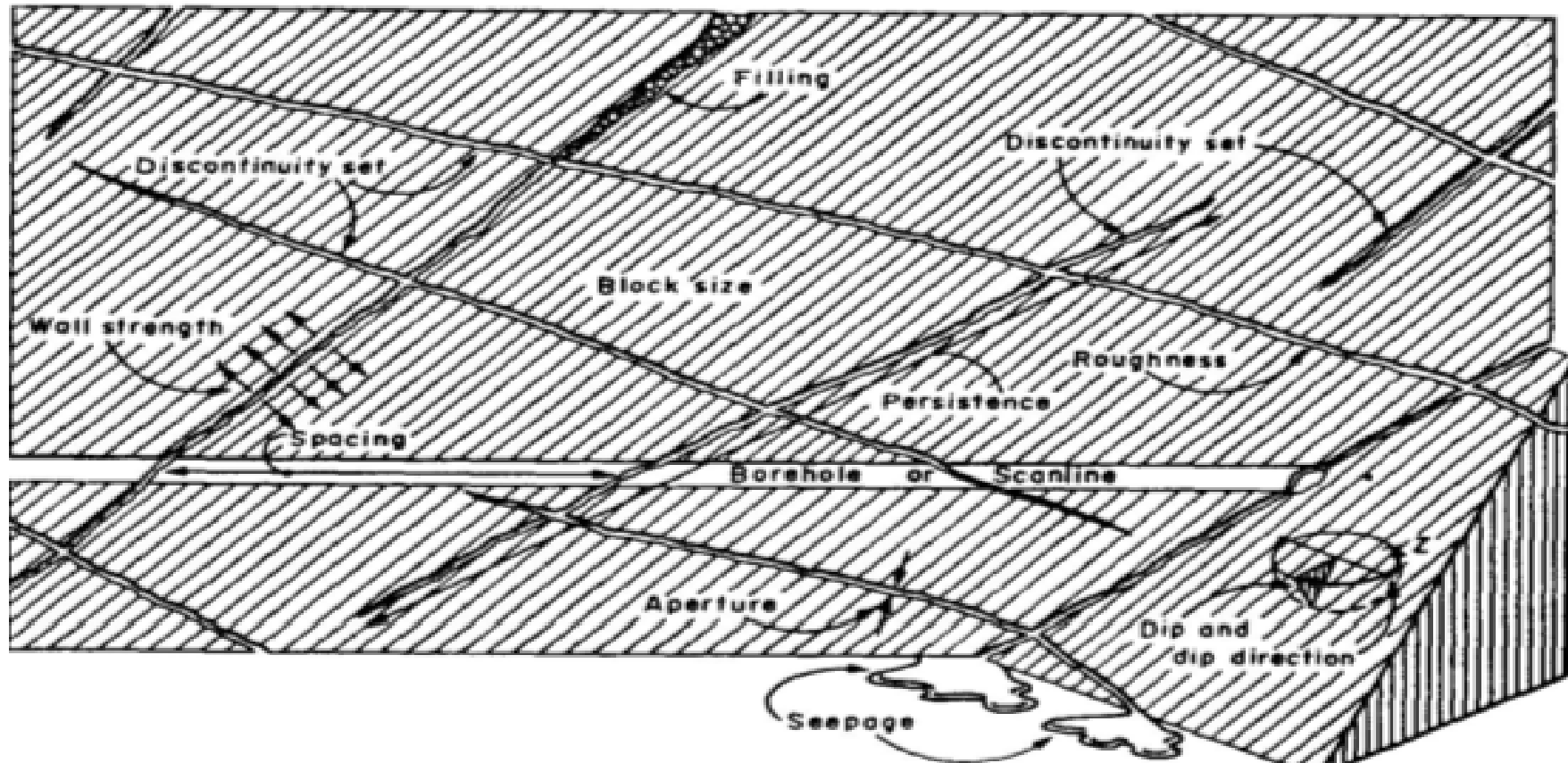
**Faults** – Singular and large scale

**Bedding** - Singular and large scale

**Interfaces** – Singular and large scale

**Joints and fractures are often inter-used. An individual joint is often termed as a fracture.**

# Geometrical Properties of Rock Joints



## Discontinuity Sets

Discontinuities do not occur at completely random orientations: they occur for good mechanical reasons with some degree of "clustering" around preferred orientations associated with the formation mechanisms. Hence, it is convenient to consider the concept of discontinuity set (which consists of parallel or sub-parallel discontinuities) and the number of such sets that characterize a particular rock mass geometry

# Geometrical Properties of Rock Joints

---

## Principal Geometrical Characteristics of Rock Joints

**Number of joint sets**

**Joint persistence**

**Joint plane orientation**

**Joint spacing, joint frequency, block size, and RQD**

**Joint surface roughness and matching**

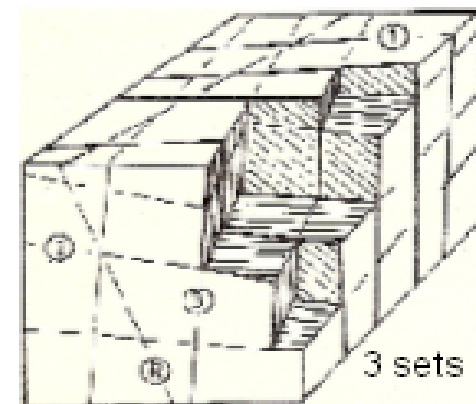
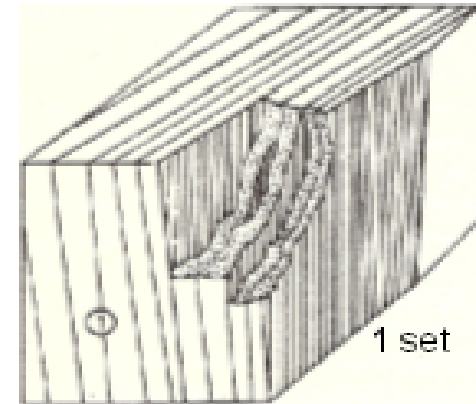
**Joint aperture and filling**

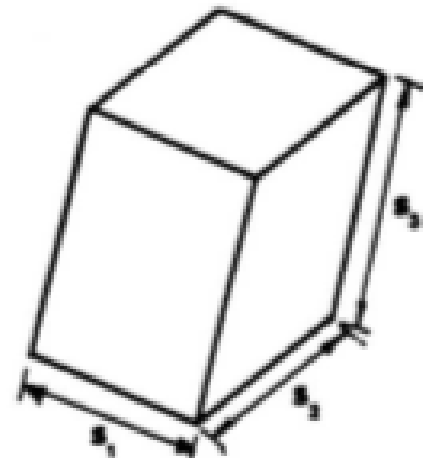
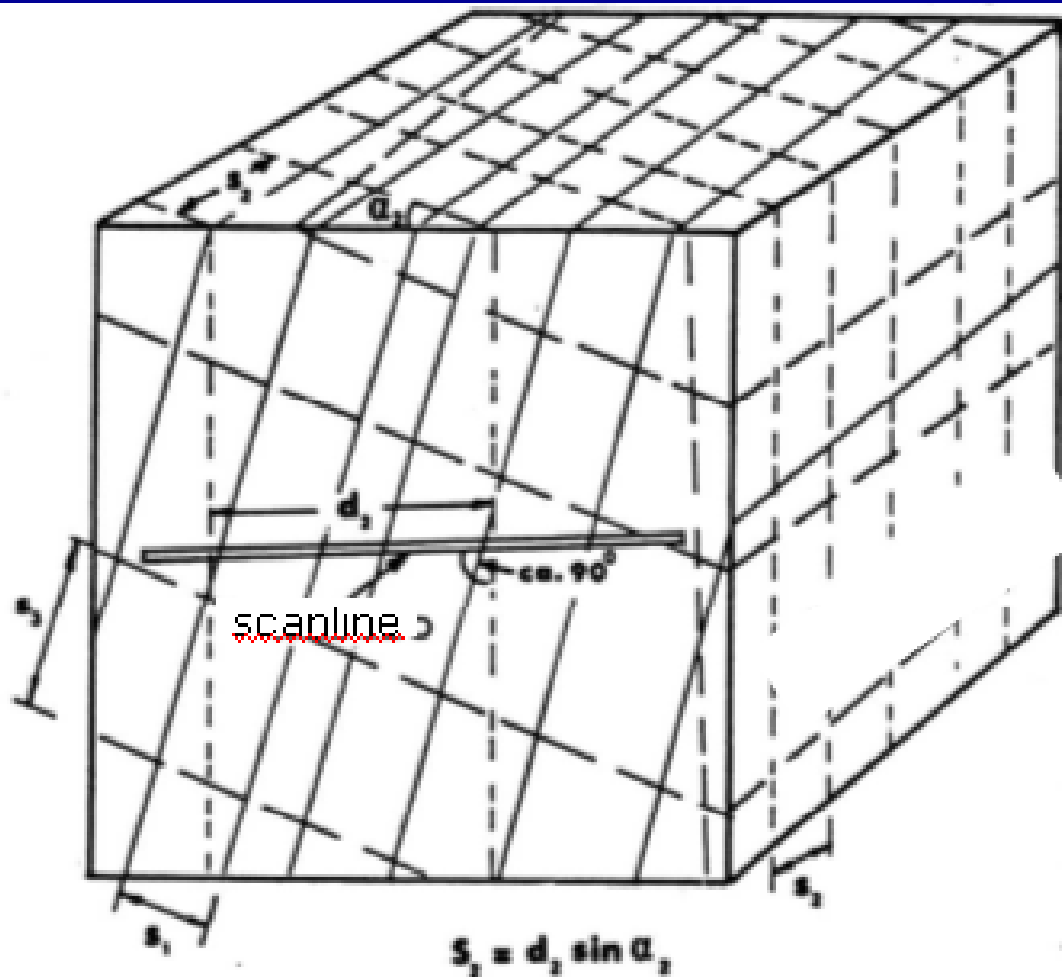
# Geometrical Properties of Rock Joints

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## Number of Joint Sets

Joints are generally in sets, i.e., parallel joints. The number of joint sets can be up to 5. Typically one joint set cuts the rock mass into plates, two perpendicular sets cut rock into column and three into blocks, and more sets cut rocks into mixed shapes of blocks and wedges.





- Set no. 1
- - - Set no. 2
- · - Set no. 3

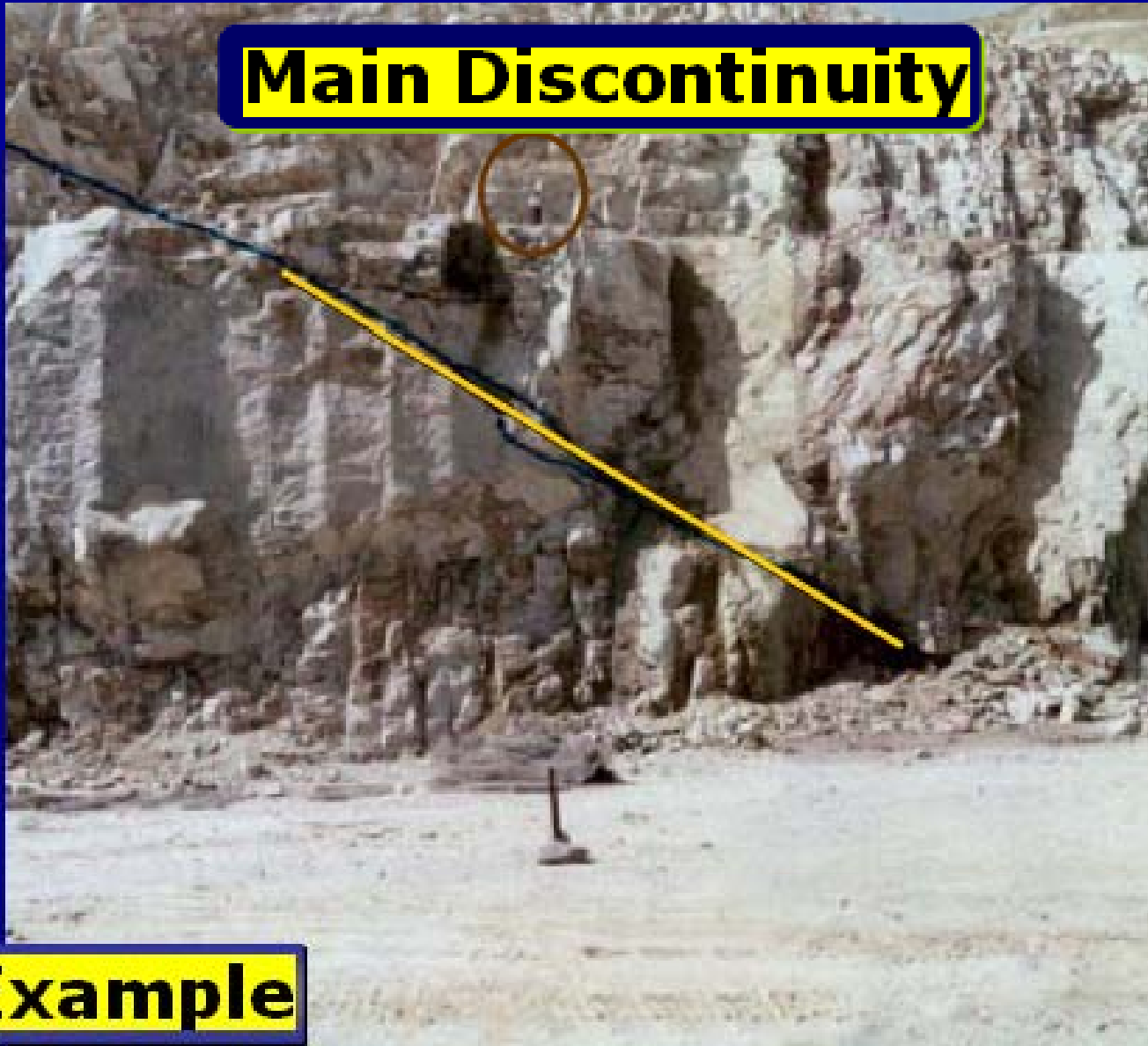






**Rock Mass with  
3 discontinuity sets**

**Main Discontinuity**



**Example**

**A fault, with complex morphology**

# Geometrical Properties of Rock Joints

---

## ISRM suggested description

|      |                                        |
|------|----------------------------------------|
| I    | Massive, occasional random fractures   |
| II   | One joint set                          |
| III  | One joint set plus random fractures    |
| IV   | Two joint sets                         |
| V    | Two joint sets plus random fractures   |
| VI   | Three joint sets                       |
| VII  | Three joint sets plus random fractures |
| VIII | Four or more joint sets                |
| IX   | Crushed rock, earth-like               |

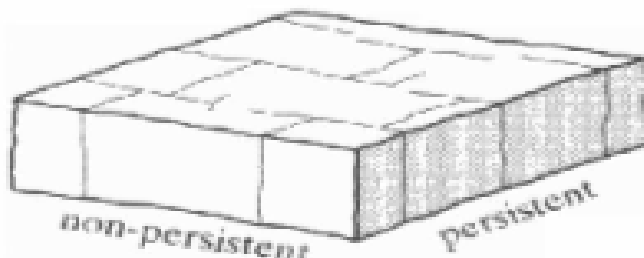
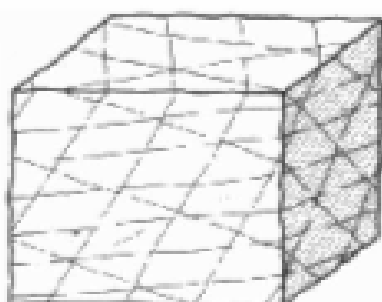
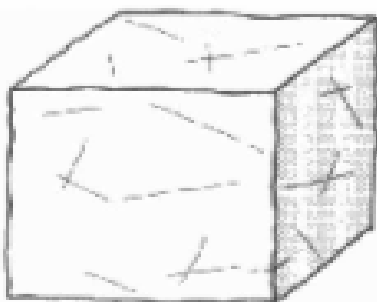
**Mechanical properties of the rock mass is influenced by joint sets. More joint sets provide more possibilities of potential slide planes .**

# Geometrical Properties of Rock Joints

---

## Joint Persistence

Persistence is the areal extent or length of a discontinuity, and can be crudely quantified by observing the trace lengths of discontinuities on exposed surfaces. The persistence of joint sets controls large scale sliding or 'down-stepping' failure of slope, dam foundation and tunnel excavation.



| ISRM Suggested Description | Surface Trace Length (m) |
|----------------------------|--------------------------|
| Very low persistence       | < 1                      |
| Low persistence            | 1 – 3                    |
| Medium persistence         | 3 – 10                   |
| High persistence           | 10 – 20                  |
| Very high persistence      | > 20                     |

# Geometrical Properties of Rock Joints

---

## Joint Plane Orientation

Orientation of joint sets controls the possibility of unstable conditions or excessive deformations. The mutual orientation of joints determines the shape of the rock blocks.

Orientation is defined by dip angle (inclination) and dip direction (facing) or strike (running).



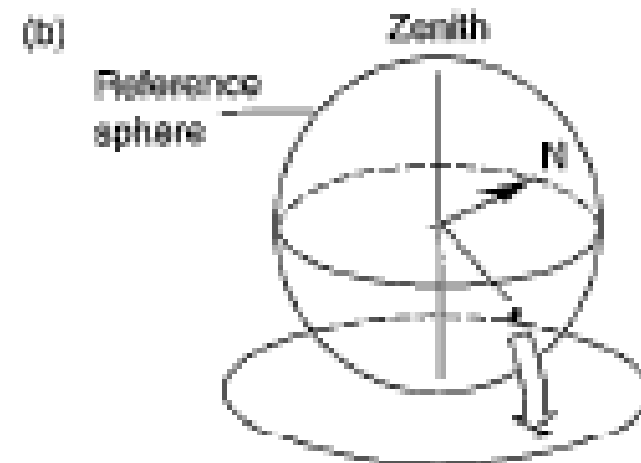
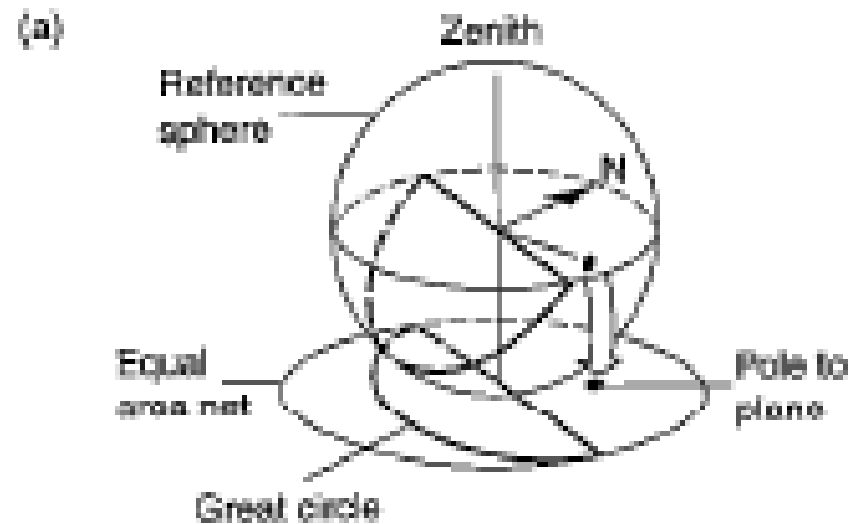
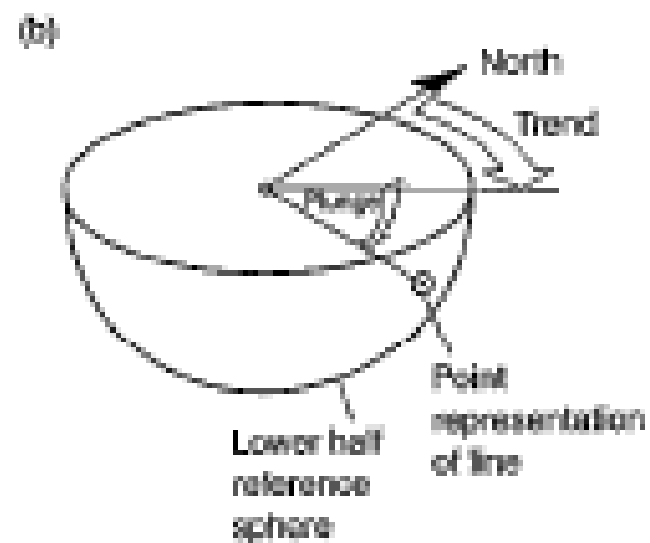
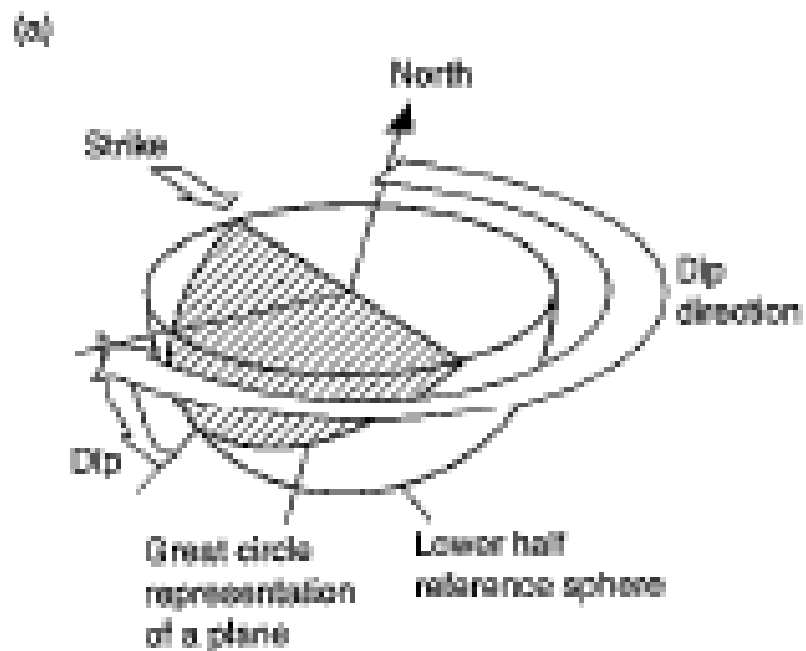


Figure 2.6 Equal area projections of plane and line: (a) plane projected as great circle and corresponding pole; (b) line projected as pole.

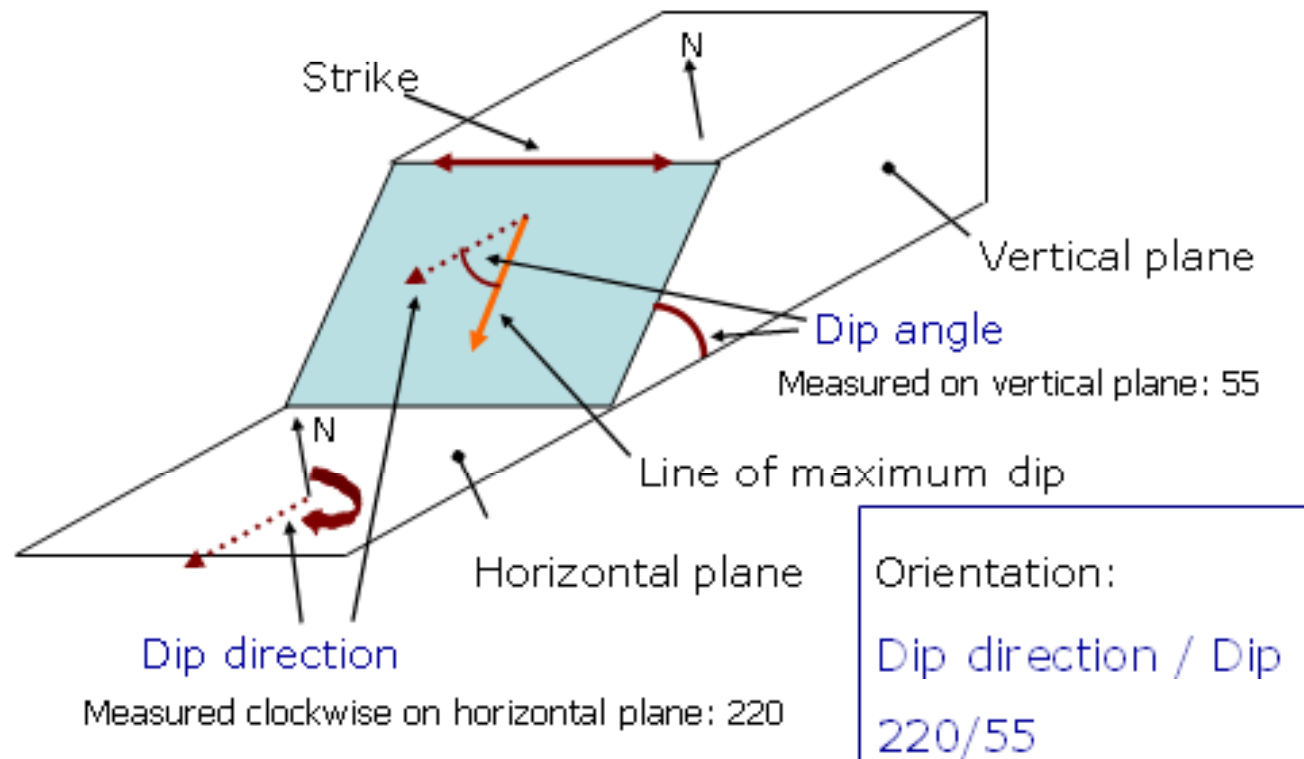
## Orientation

Attitude of discontinuity in space. Described by the dip direction  $\alpha$  (azimuth) and dip  $\psi$  of steepest declination in the plane of discontinuity.

Example: dip direction/dip  
(015°/35°)



# Geometrical Properties of Rock Joints



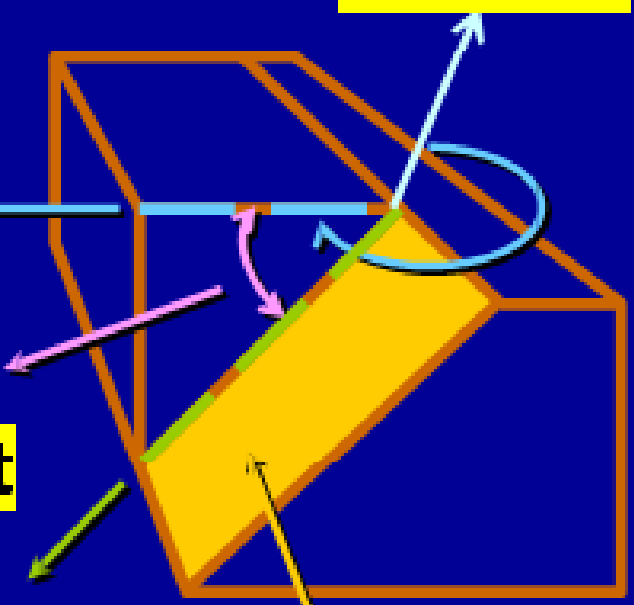
**Dip Direction  $\alpha$**

**Dip  $\psi$**

**Line of steepest  
declination**

**Mean Plane of the  
discontinuity**

**NORTH**



# Geometrical Properties of Rock Joints

---



**A geological compass to measure dip and direction of joint plane**



**An electronic geological compass**

**Geological compass  
used for measuring the  
orientation of a  
discontinuity plane**



**It allows reading directly  
in terms of dip and dip direction**

# Geometrical Properties of Rock Joints

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## Joint Plane Orientation

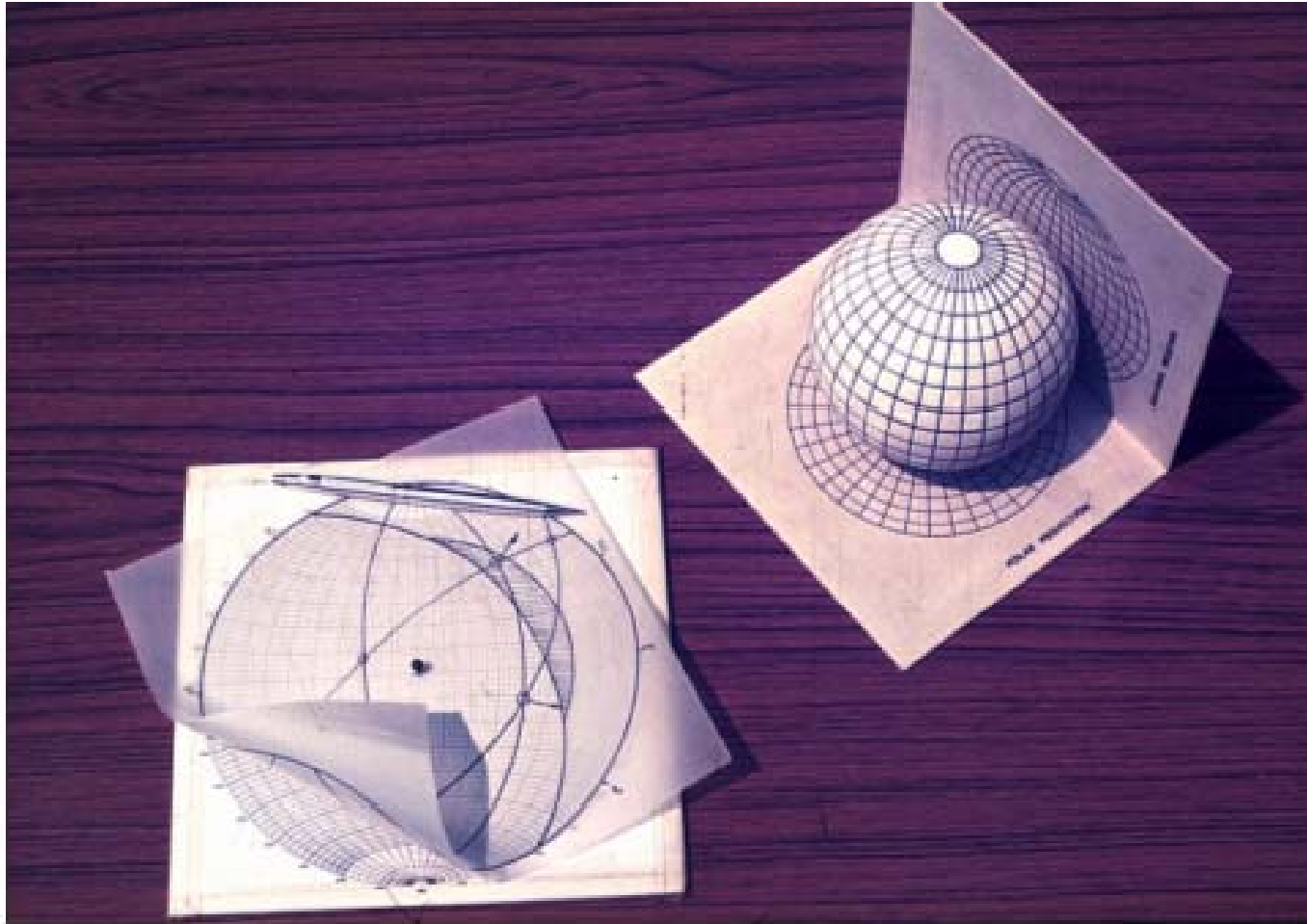
**Dip direction and strike direction are always perpendicular. Dip direction/dip format is generally used, e.g., 210/35, or 030/35. Sometimes, dip/strike format is used, e.g., 120/35SW (=dip direction/dip 210/35), or 120/35NE (=dip direction/dip 030/35). Normal (pole) to the plane is perpendicular to the plane. Orientation of the normal is given by:  
trend of normal = dip direction of the plane  $\pm$  180,  
plunge of normal = 90 – dip.**

# Geometrical Properties of Rock Joints

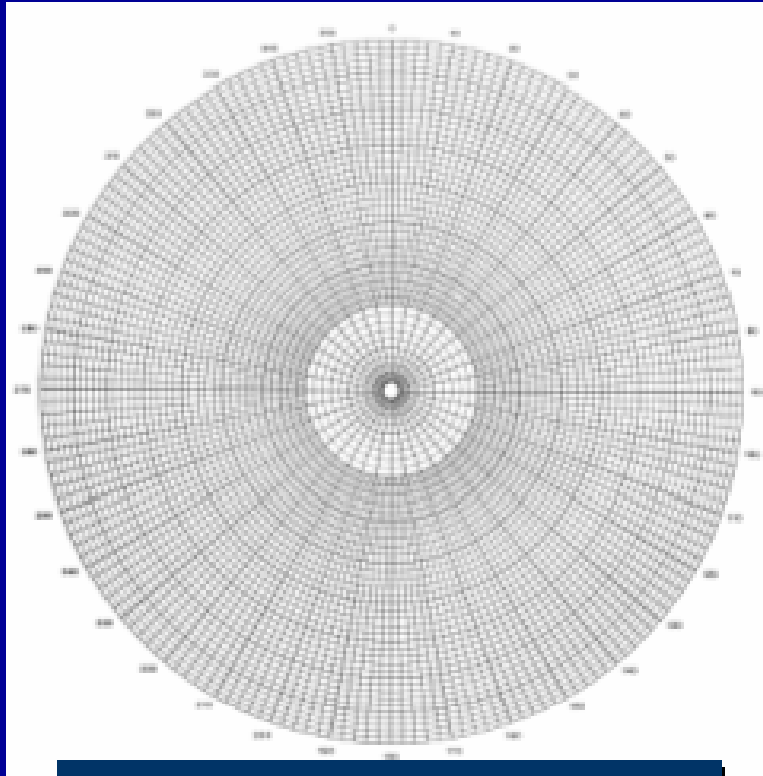
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## Joint Plane Orientation

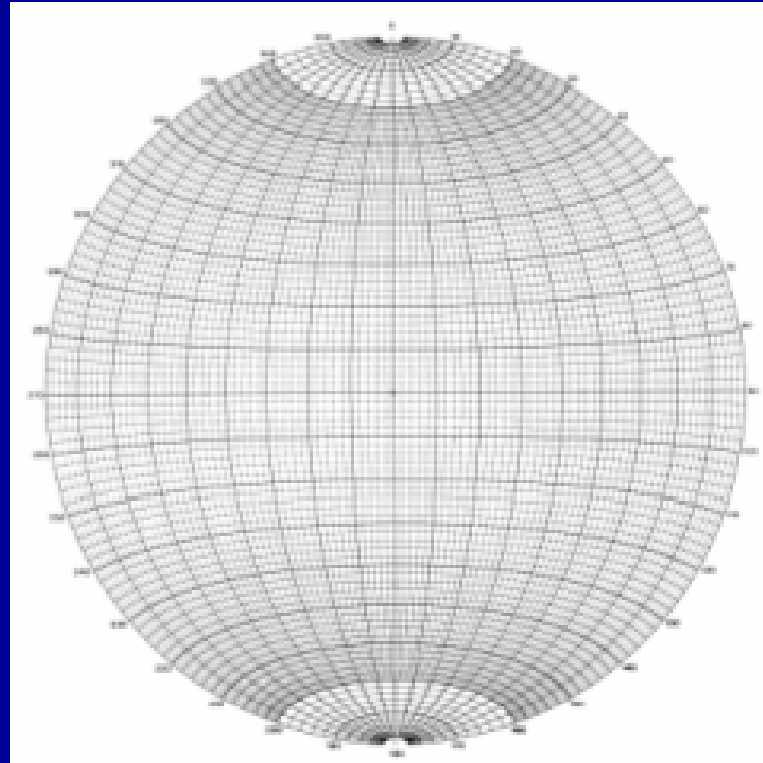
Orientation of a joint plane can be represented graphically using hemispherical projection method. The projection method is to represent a 3D plane by a 2D presentation. Use the projection, joint orientation data can be assessed in 2D form. It can be used to analyse large number of joint data and examine the rock slope stability and slide of rock block in underground excavation.



**Spherical projections plot and analyze joint orientation data**



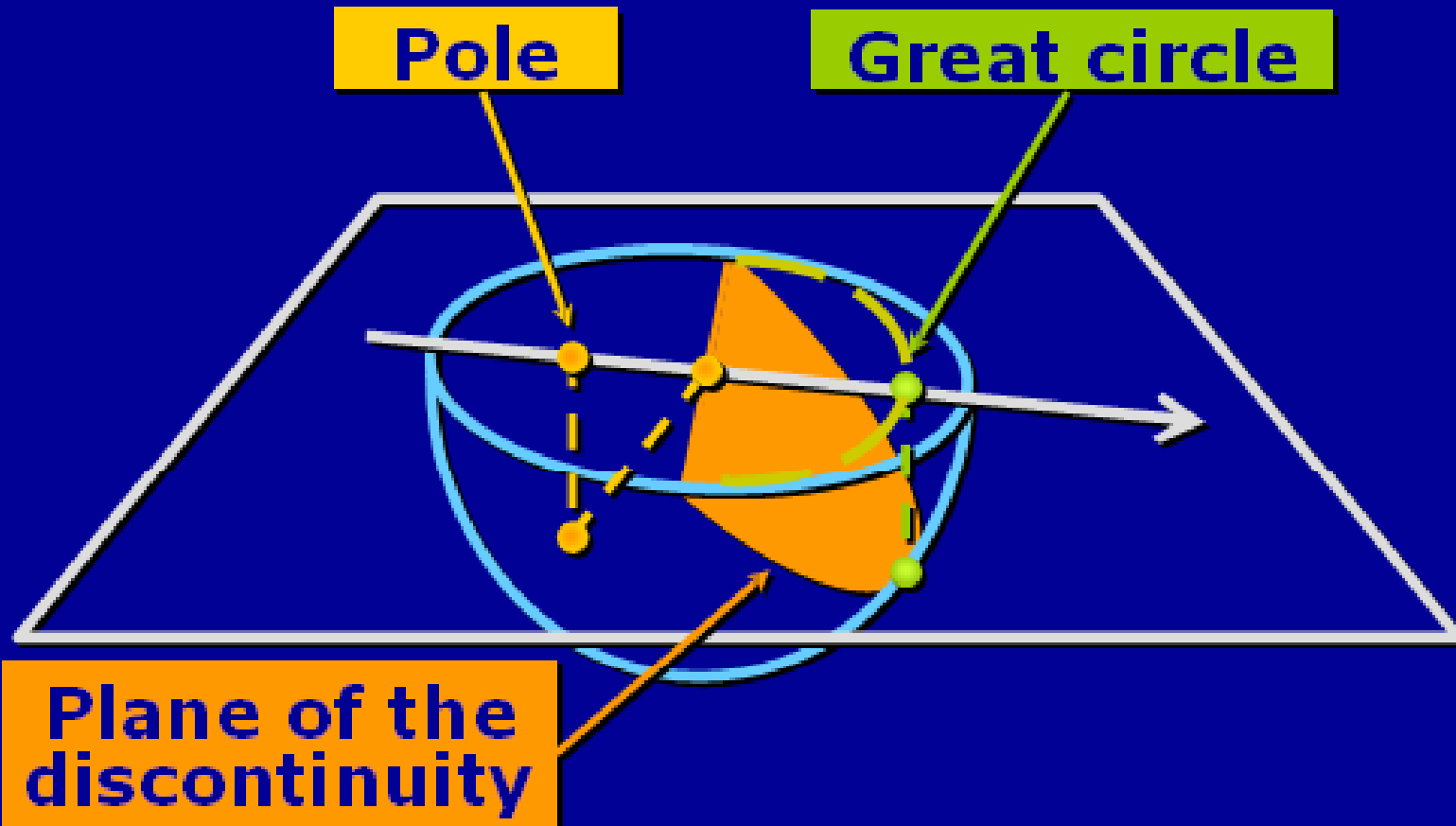
**Polar equal-area  
stereonet**

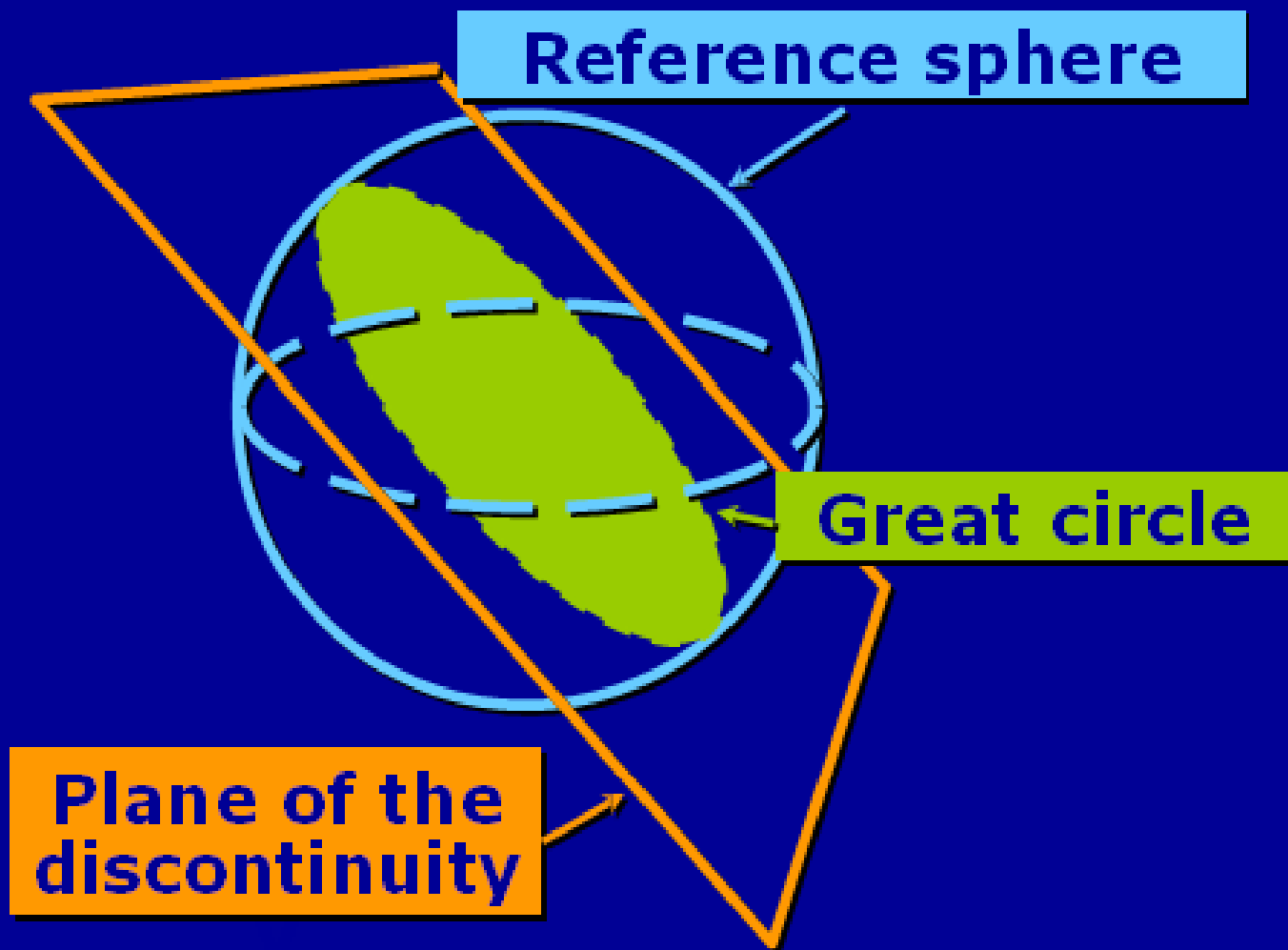


**Equatorial equal-area  
stereonet**







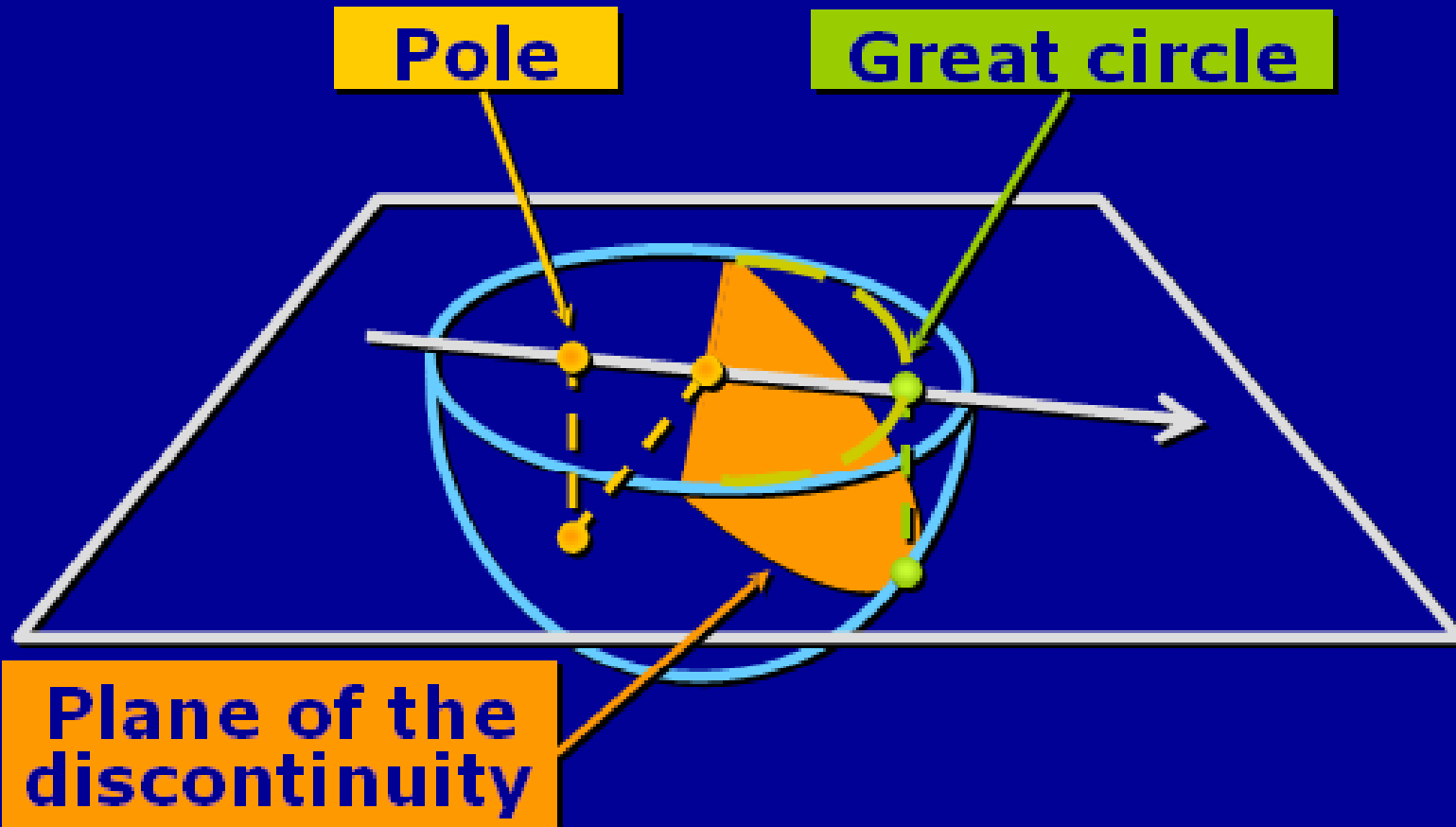


**Reference sphere**

**Great circle**

**Plane of the discontinuity**





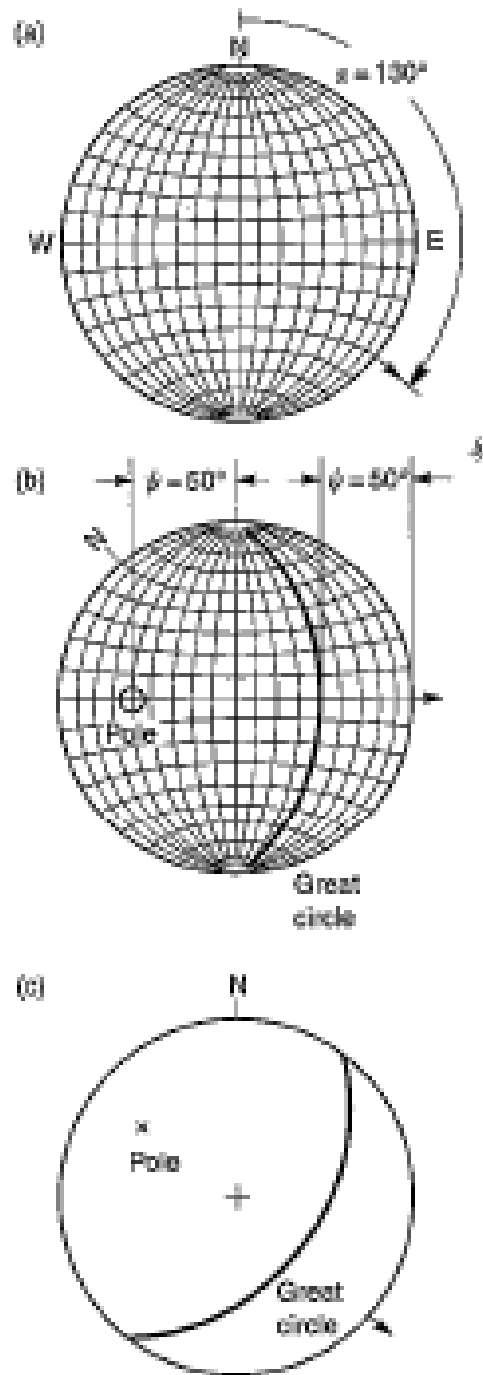


Figure 2.13 Construction of great circles and a pole representing a plane with orientation 50 (dip)/130 (dip direction) on an equal area net: (a) with the tracing paper located over the stereonet, by means of the center pin, trace the circumference of the net and mark the north point. Measure off the dip direction of 130° clockwise from north and mark this position on the circumference of the net; (b) rotate the net about the center pin until the dip direction mark lies on the W-E axis of the net, that is, the net is rotated through 40° counterclockwise. Measure 50° from the outer circle of the net and trace the great circle that corresponds to a plane dipping at this angle. The position of the pole, which has a dip of (90-50), is found by measuring 50° from the center of the net as shown, or alternatively 40° from the outside of the net. The pole lies on the projection of the dip direction line which, at this stage of the construction, is coincident with the W-E axis of the net; (c) the tracing is now rotated back to its original position so that the north mark on the tracing coincides with the north mark of the net. The final appearance of the great circle and the pole representing a plane dipping at 50° in a dip direction of 130° is as illustrated.

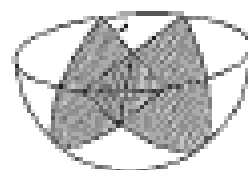
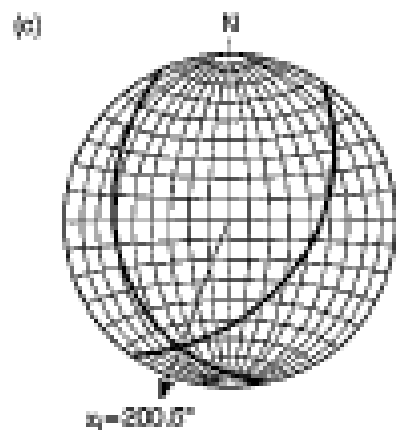
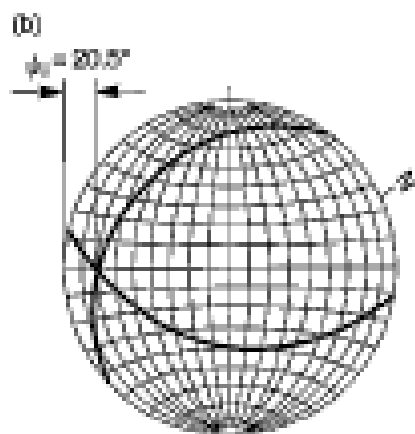
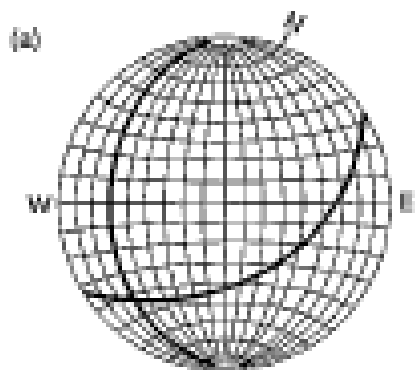


Figure 2.14 Determination of orientation (plunge and trend) of line intersection between two planes with orientations  $50/130$  and  $30/250$ : (a) the first of these planes has already been drawn in Figure 2.13. The great circle defining the second plane is obtained by marking the  $250^\circ$  dip direction on the circumference of the net, rotating the tracing until the mark lies on the W-E axis and tracing the great circle corresponding to a dip of  $30^\circ$ ; (b) the tracing is rotated until the intersection of the two great circles lies along the W-E axis of the stereonet, and the plunge of the line of intersection is measured as  $20.5^\circ$ ; (c) the tracing is now rotated until the north mark coincides the north point on the stereonet and the trend of the line of intersection is found to be  $200.5^\circ$ .

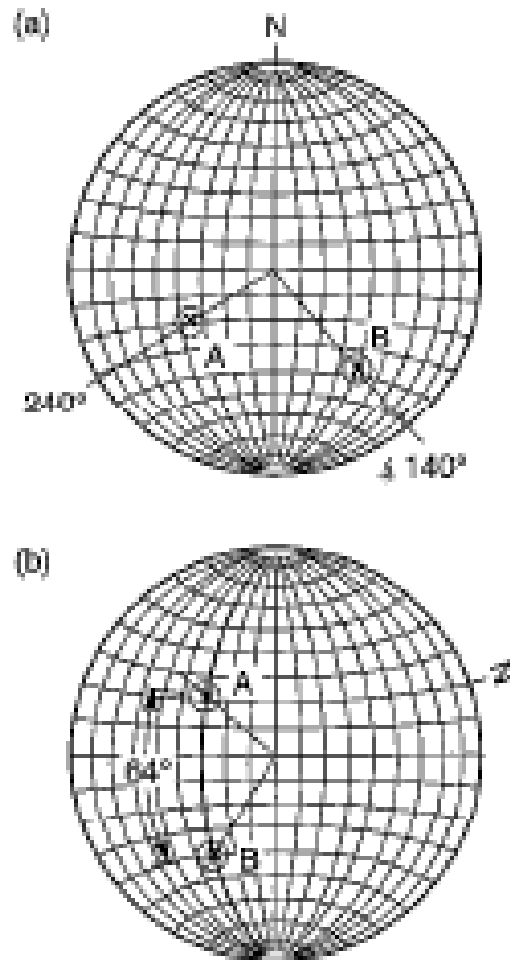
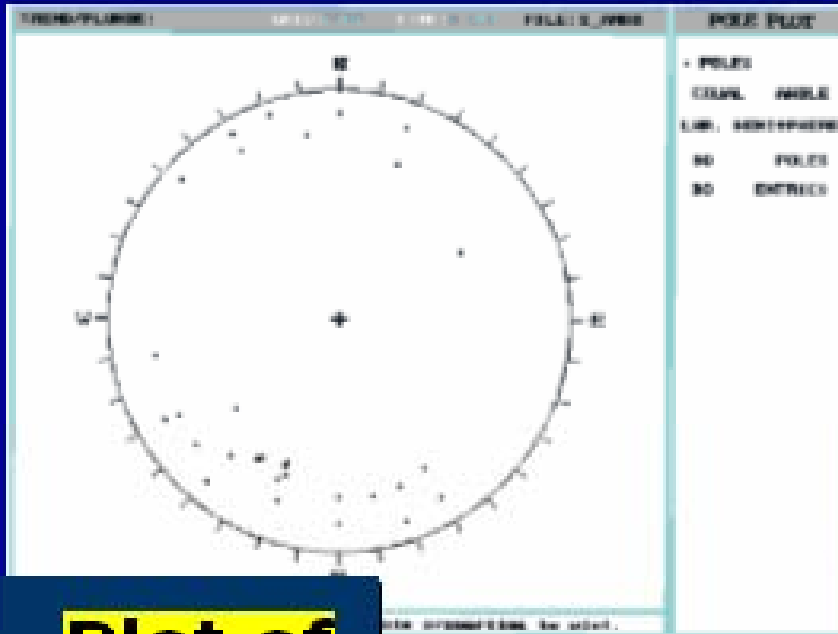


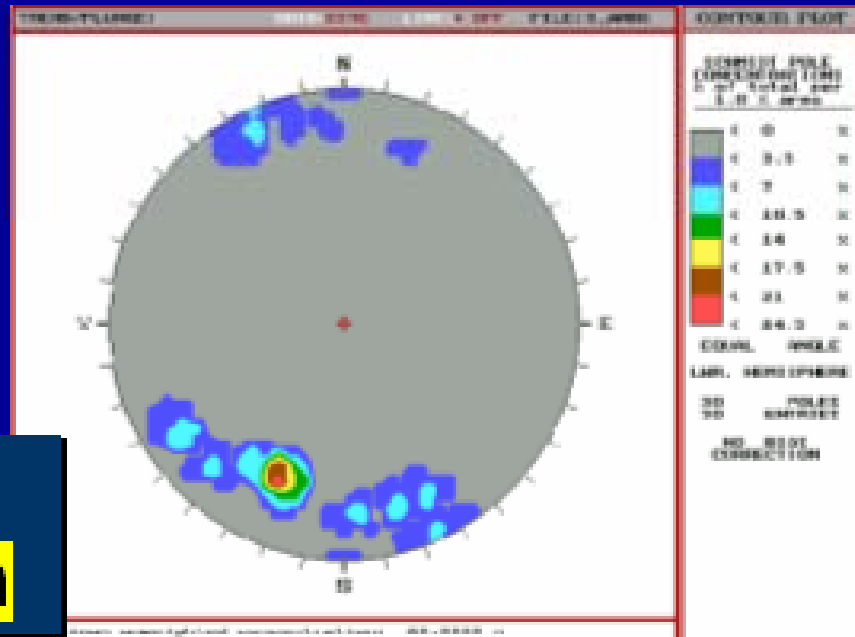
Figure 2.15 Determination of angle between lines with orientations 54/240 and 40/140: (a) the points A and B that define the poles of these two lines are marked on the stereonet as described in Figure 2.13 for locating the pole; (b) the tracing is rotated until the two poles lie on the same great circle on the stereonet. The angle between the lines is determined by counting the small circle divisions between A and B, along the great circle; this angle is found to be 64°. The great circle on which A and B lie defines the plane that contains these two lines. The dip and direction of this plane are 60° and 200° respectively.

# Example

## Computer analysis of structural data



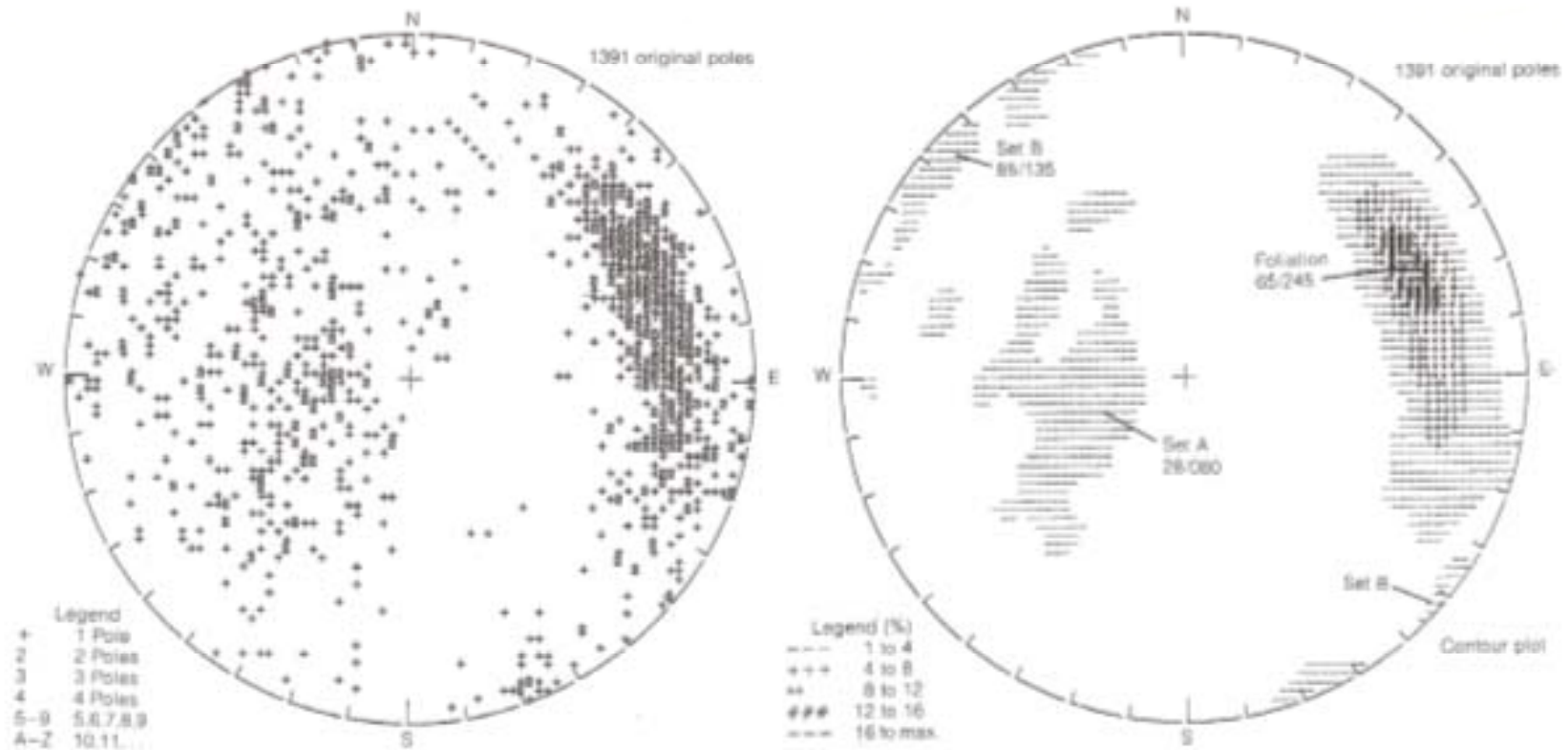
Plot of poles



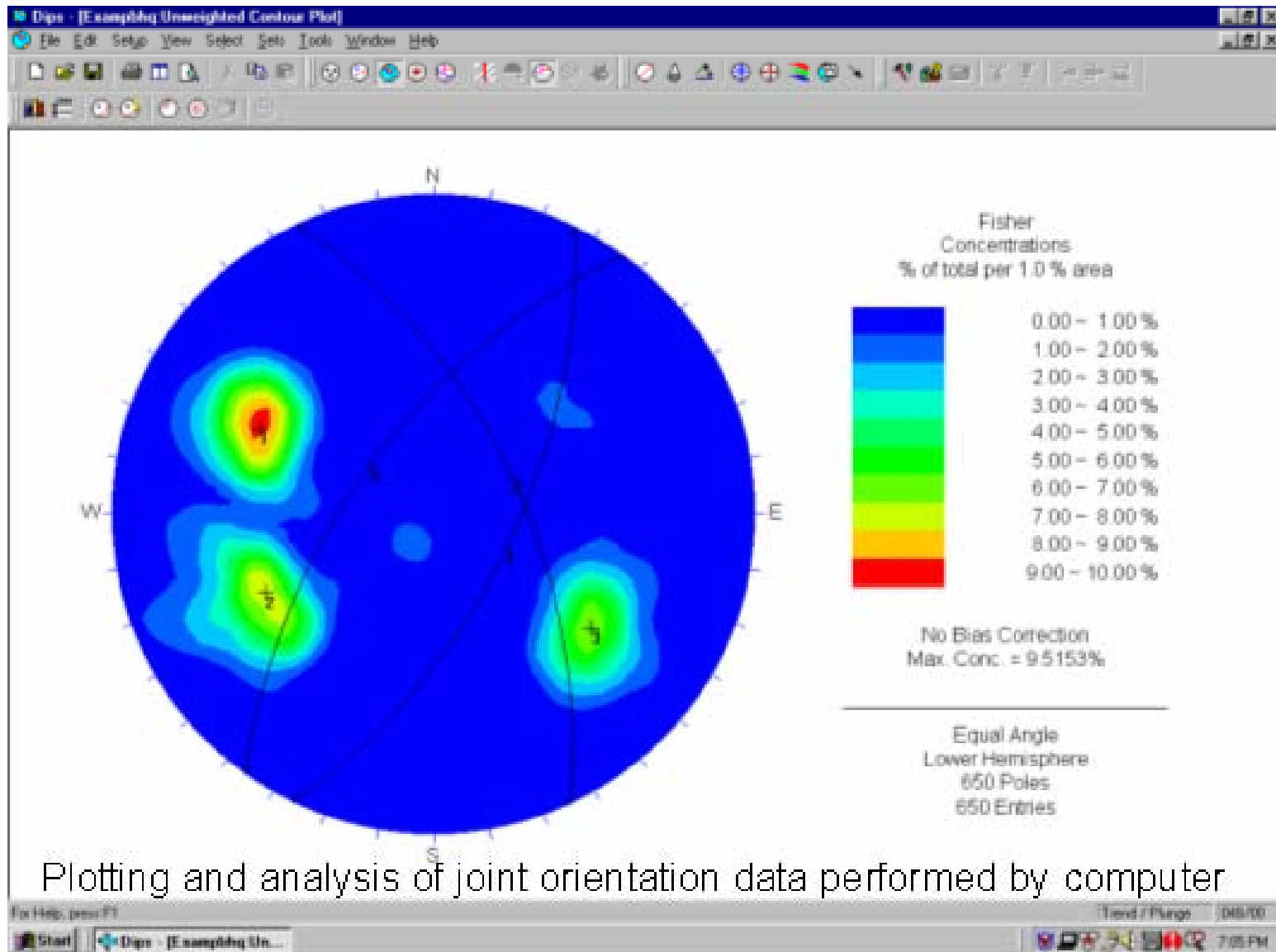
Contour diagram



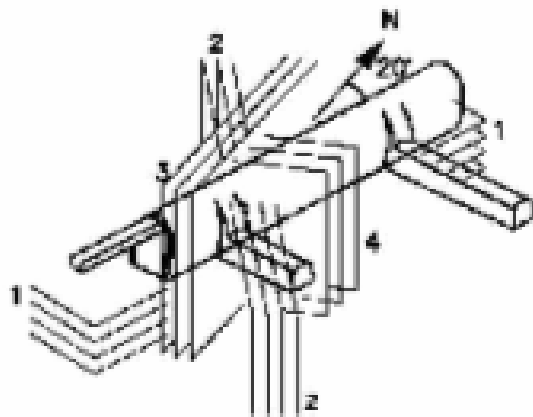
# Geometrical Properties of Rock Joints



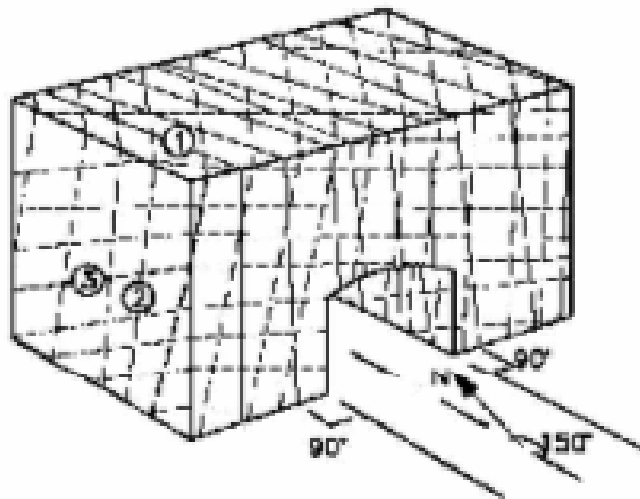




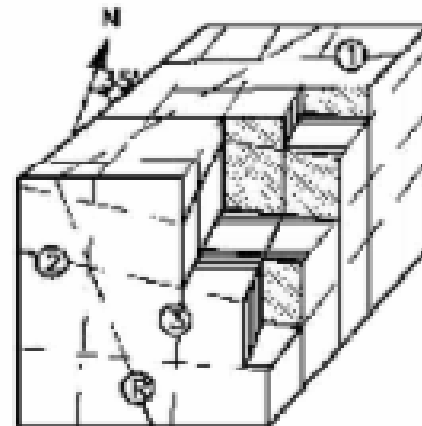
# Perspective views and block diagrams for engineering structures



1.  $200^\circ/10^\circ$
2.  $230^\circ/05^\circ$
3.  $095^\circ/90^\circ$
4.  $180^\circ/88^\circ$



1.  $055^\circ/85^\circ$
2.  $285^\circ/70^\circ$
3.  $030^\circ/32^\circ$



1.  $200^\circ/88^\circ$
2.  $130^\circ/15^\circ$
3.  $085^\circ/85^\circ$

# Geometrical Properties of Rock Joints

---

## **Joint Spacing, Frequency, Block Size, and RQD**

**Fracturing degree of a rock mass is controlled by the number of joint in the rock mass. More joints mean that less average spacing between joints. Joint spacing controls the size of individual rock blocks. It controls the mode of failure and flow. For example, a close spacing gives low mass cohesion and circular or even flow failure.**

# Geometrical Properties of Rock Joints

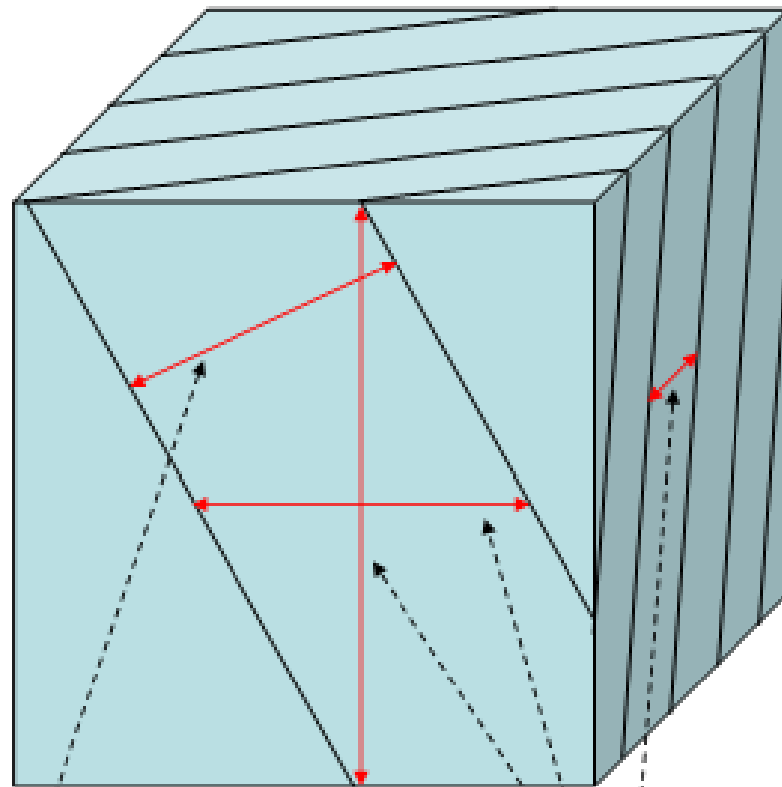
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## Joint Spacing

Joint spacing is the perpendicular distance between joints. For a joint set, is usually expressed as the mean spacing of that joint set. Often the apparent spacing is measured.

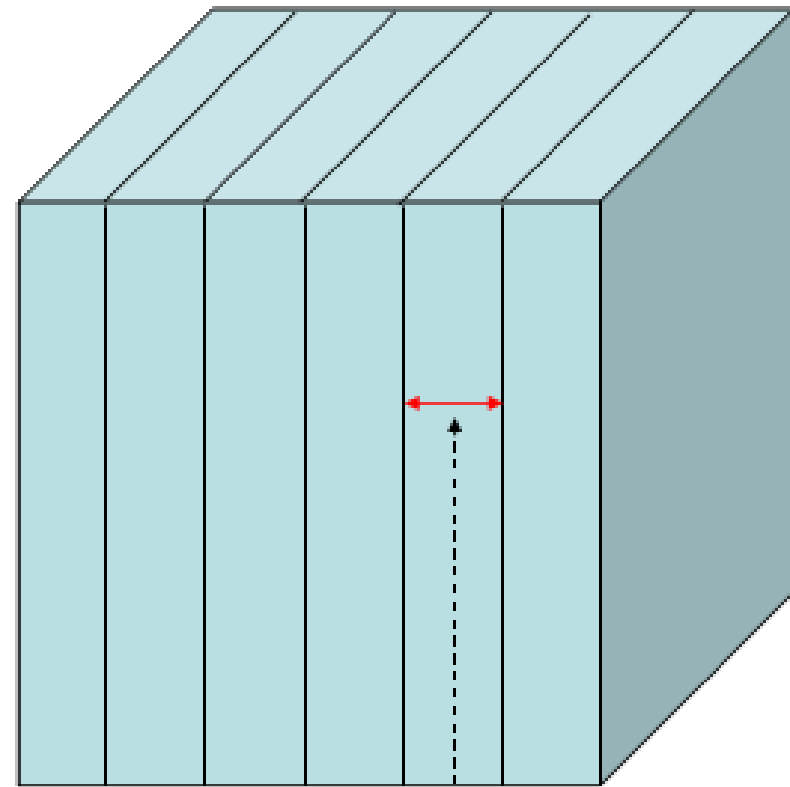
Measurements of joint spacing are different on different measuring faces and directions. For example, in a rock mass with mainly vertical joints, measurements in vertical direction have far greater spacing than that in horizontal direction.

# Geometrical Properties of Rock Joints



Apparent spacing  
on the plane

Apparent spacing in  
x, y and z directions



True spacing

# Geometrical Properties of Rock Joints

---

## Classification of joint spacing

| Description             | Joint Spacing (m) |
|-------------------------|-------------------|
| Extremely close spacing | < 0.02            |
| Very close spacing      | 0.02 – 0.06       |
| Close spacing           | 0.06 – 0.2        |
| Moderate spacing        | 0.2 – 0.6         |
| Wide spacing            | 0.6 – 2           |
| Very wide spacing       | 2 – 6             |
| Extremely wide spacing  | > 6               |

# Geometrical Properties of Rock Joints

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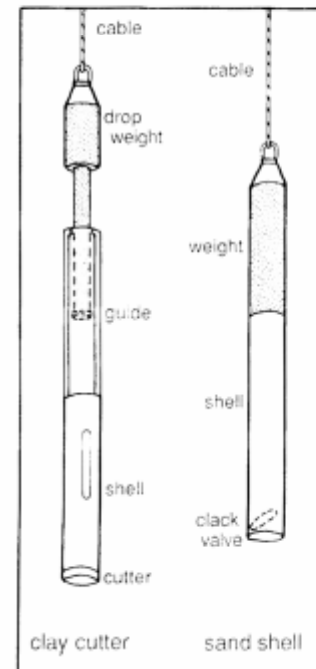
## Joint Frequency

Joint frequency ( $\lambda$ ), is defined as number of joint per metre length. It is therefore simply the inverse of joint spacing ( $s_j$ ), i.e.,

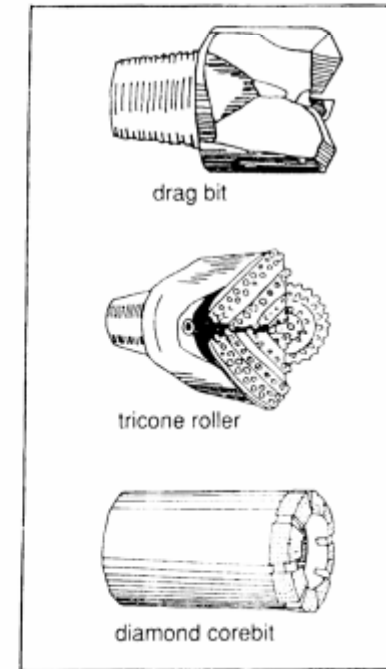
$$\lambda = 1 / s_j$$

# Site investigation boreholes

- Percussion drilling
  - soils/soft clay rocks
  - core recovery
- Rotary coring
  - soil or rock >100m deep
  - core recovery
- Rock probing
  - rotary percussion rig
  - soil or rock
  - no core recovery



*Alternative shells for light percussion drills*



*Alternative drill bits for rock penetration*



# Rotary rig



# Core bit



# Core drilling



# Rock core



1529 GEOLOGICAL AND ROCK INVESTIGATION AT JARONG ISLAND HDD2-2 BOX-15

| CR | LENGTH(M)     | TCR(%) | SCR(%) | RDD(%) | DATE     |
|----|---------------|--------|--------|--------|----------|
| 90 | 188.90-191.90 | 100    | 100    | 97     | 03-04-04 |
| 91 | 191.90-194.10 | 98     | 98     | 98     | 07-04-04 |
| 92 | 194.10-195.90 | 97     | 97     | 94     | 07-04-04 |
| 93 | 195.90-197.05 | 98     | 98     | 98     | 07-04-04 |



# Total Core Recovery (R)

$$R = \frac{\text{Summed length of core recovered}}{\text{Length drilled}}$$

Depends upon:

- quality of the rock mass?
- stability of/lack of vibration in, the drill rig
- choice of core barrel/skill of the operator

# Rock Quality Designation (RQD) or Modified Core Recovery

$$\text{RQD} = \frac{\sum x_i}{L}$$

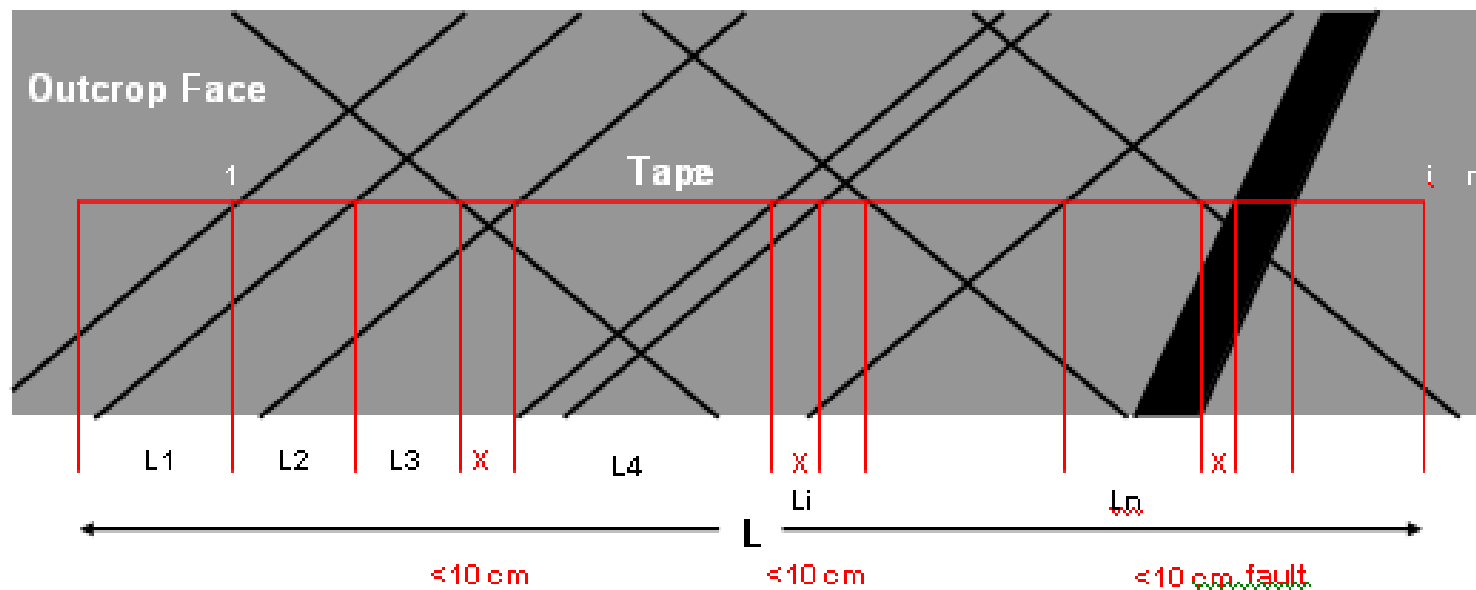
$x_i$  = lengths of individual pieces of core  $\geq 10$  cm

L is the total length of the drill run

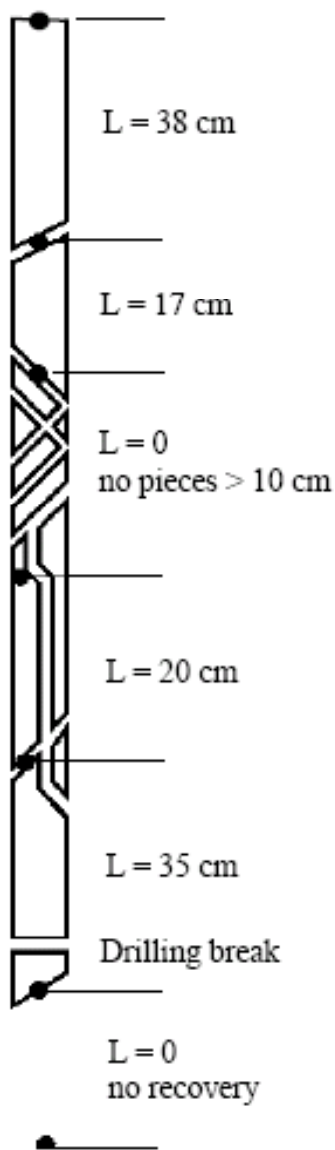
# Geometrical Properties of Rock Joints

$$\text{RQD} = (L_1 + L_2 + \dots + L_n) / L \times 100\%$$

$$\lambda = \text{number of joints} / \text{length} = n / L$$







Total length of core run = 200 cms

$$RQD = \frac{\sum \text{Length of core pieces } > 10 \text{ cm length}}{\text{Total length of core run}} \times 100$$

$$RQD = \frac{38 + 17 + 20 + 35}{200} \times 100 = 55 \%$$

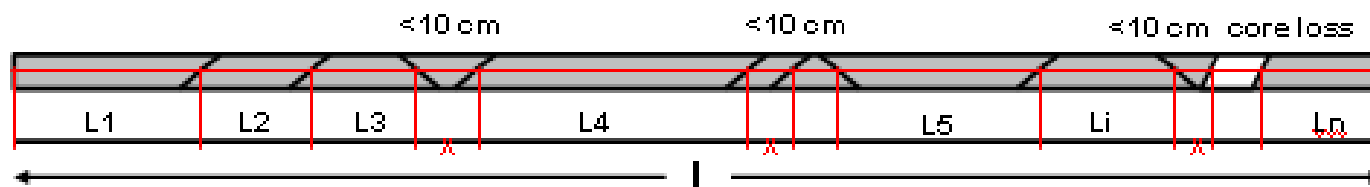
Figure 4.1: Procedure for measurement and calculation of *RQD* (After Deere, 1989).

# Geometrical Properties of Rock Joints

## RQD

**Rock Quality Designation (RQD)** is defined as the percentage of rock cores that have length equal or greater than 10 cm over the total drill length.

$$\text{RQD} = \frac{\sum L_i}{L} \times 100\%, \quad L_i > 10 \text{ cm}$$



$$\text{RQD} = \frac{(L_1 + L_2 + \dots + L_n)}{L} \times 100\%$$

# Geometrical Properties of Rock Joints

---

**RQD can be correlated to joint frequency ( $\lambda$ ):**

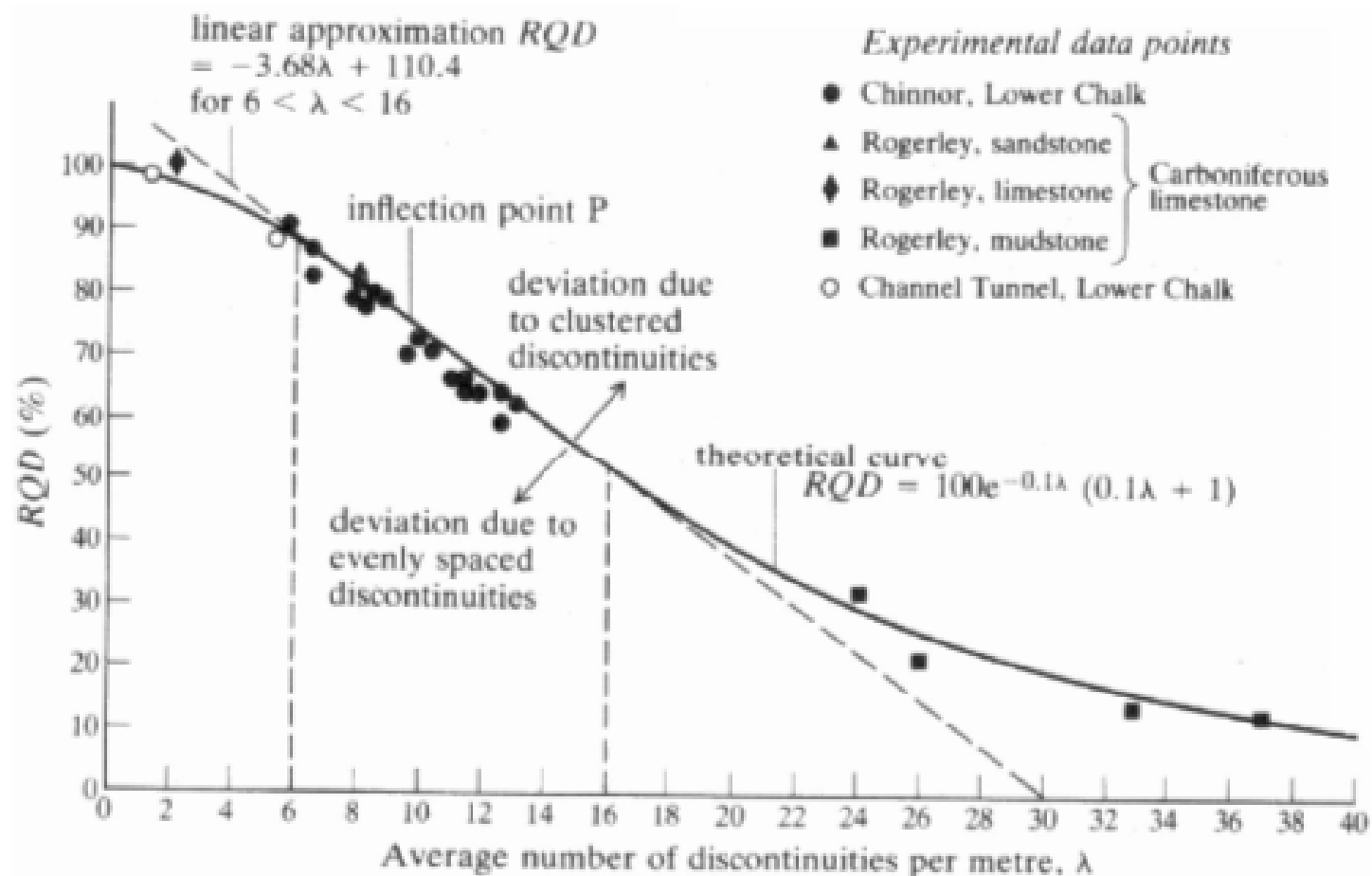
$$\mathbf{RQD = 100 (0.1\lambda + 1) e^{-0.1\lambda}}$$

**For  $\lambda = 6$  and  $16/m$ , it can be approximated by:**

$$\mathbf{RQD = 110.4 - 3.68\lambda}$$

**RQD was initially proposed as an attempt to describe rock quality, in reality, it only describes fracturing degree, but not other properties such as joint alteration, groundwater and rock strength.**

# Geometrical Properties of Rock Joints



# Geometrical Properties of Rock Joints

---

## Block Size and Volumetric Joint Count

Joint space also defines the size of rock blocks. When a rock mass contains more joints numbers, the joints have lower average spacing and smaller block size.

RQD can be related to volumetric joint count  $J_v$  by:

$RQD = 115 - 3.3 J_v$ , for  $J_v$  between 4.5 and 30.

$J_v < 4.5$ ,  $RQD = 100\%$ ,  $J_v > 30$ ,  $RQD = 0\%$ .

# Indirect Methods of determination of RQF

Seismic Method -

$$RQD = (V_f / V_l)^2 * 100$$

Ratio of velocity in the field to that in the lab

Volumetric Count -

$$RQD = 115 - 3.3 * J_v$$

where  $J_v$  is a measure of number of joints  
within a unit volume of rock mass

# Geometrical Properties of Rock Joints

## ISRM suggested block size designations

| Designation         | Volumetric Joint Count, joints/m <sup>3</sup> |
|---------------------|-----------------------------------------------|
| Very large blocks   | < 1                                           |
| Large blocks        | 1 – 3                                         |
| Medium-sized blocks | 3 – 10                                        |
| Small blocks        | 10 – 30                                       |
| Very small blocks   | > 30                                          |
| Crushed rock        | > 60                                          |

# Geometrical Properties of Rock Joints

---

## Joint Surface Roughness and Matching

**A joint is an interface of two contacting surfaces. The surfaces can be smooth or rough; they can be in good contact and matched, or they can be poorly contacted and mismatched.**

**The condition of contact also governs the aperture of the interface. The interface can be filled with intrusive or weathered materials.**





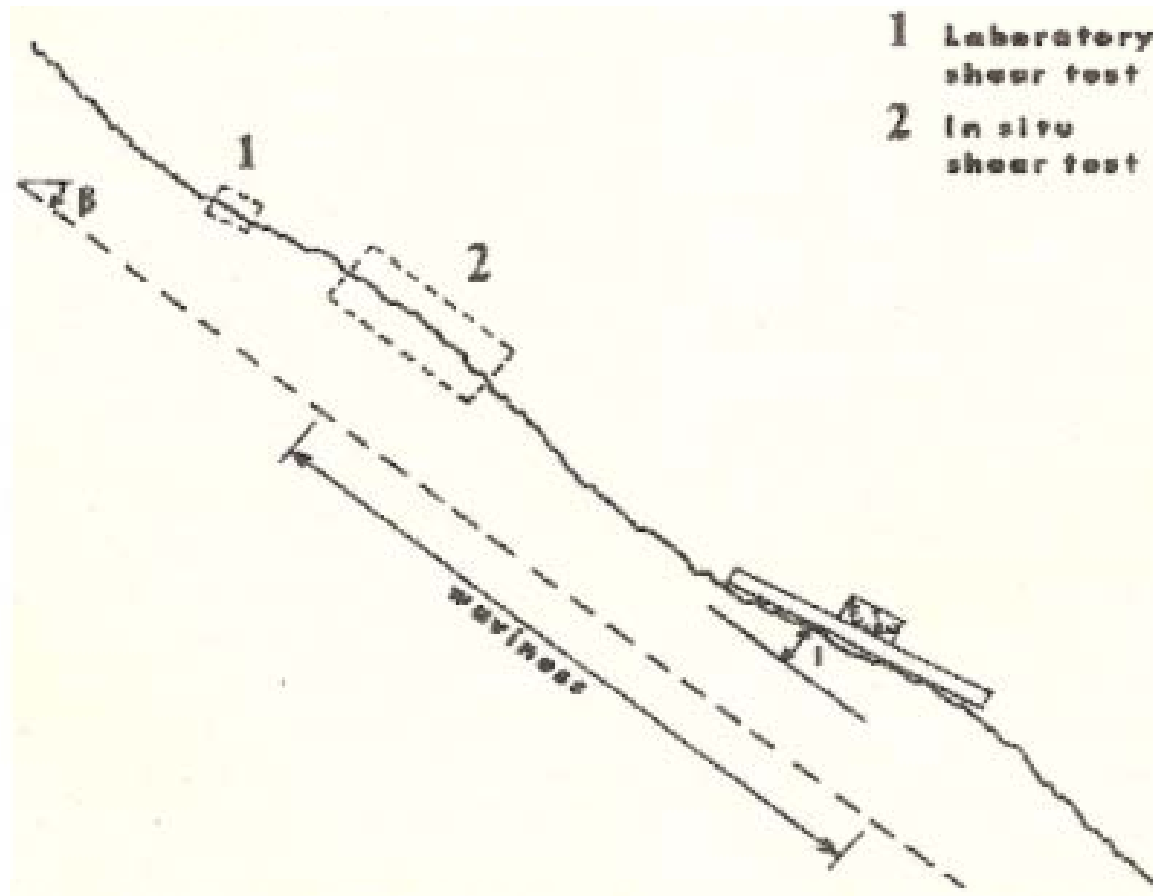
# Geometrical Properties of Rock Joints

---

## Joint Roughness

Joint surface roughness is a measure of surface unevenness and waviness relative to its mean plane. The roughness is characterised by large scale waviness (undulation) and small scale unevenness (irregularity) of a joint surface. It is the principal governing factor the direction of shear displacement and shear strength, and in turn, the stability of potentially sliding blocks.

# Geometrical Properties of Rock Joints



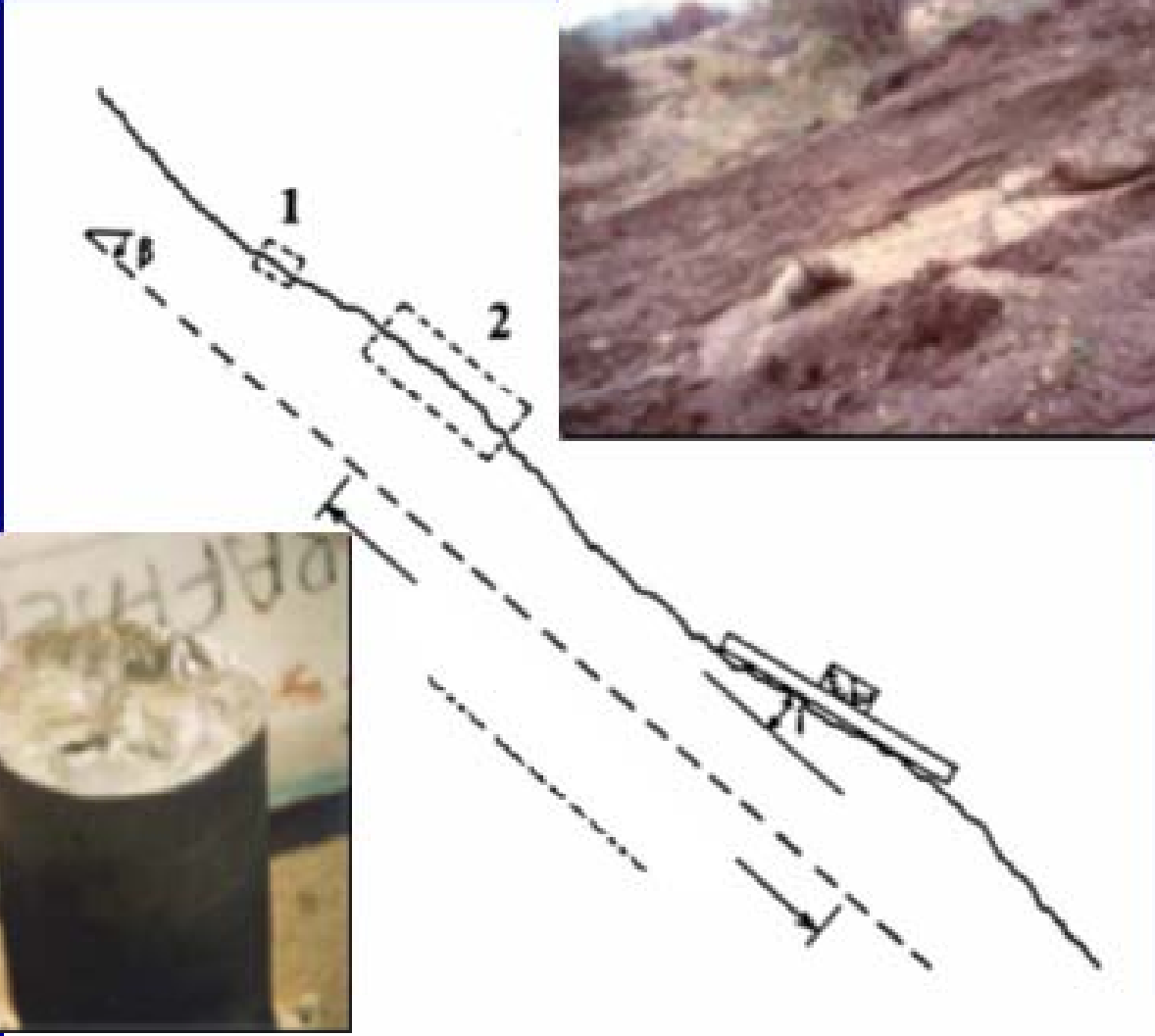
# Geometrical Properties of Rock Joints

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## Joint Roughness

Roughness should first be described in metre scale (step, undulating, and planar) and then in centimetre scale (rough, smooth, and slickensided), as suggested by ISRM. It is not a quantitative measure.

Joint Roughness Coefficient (JRC) is a quantitative measure of roughness, varying from 0 for the smooth flat surface to 20 for the very rough surface. Joint roughness is affected by geometrical scale.



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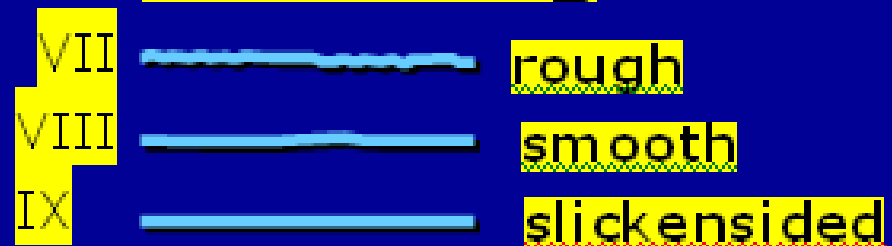
**Typical roughness profiles and suggested nomenclature**



**Stepped**






**Undulating**




**Planar**






The length of  
each profile is in  
the range  
1 to 10 m

I		rough
II		smooth
III		slickensided

### Stepped

IV		rough
V		smooth
VI		slickensided

### Undulating

VII		rough
VIII		smooth
IX		slickensided

### Planar



Description of joint types		JRC <sub>20</sub>	JRC <sub>100</sub>
I	rough	20	11
	smooth		
	stepped		
II	smooth	14	9
	rough		
	stepped		
III	slickensided	11	8
	rough		
	stepped		
IV	rough	14	9
	smooth		
	undulating		
V	smooth	11	8
	rough		
	undulating		
VI	slickensided	7	6
	rough		
	undulating		
VII	rough	2.5	2.3
	smooth		
	planar		
VIII	smooth	1.5	0.9
	rough		
	planar		
IX	slickensided	0.5	0.4
	rough		
	planar		

JRC number is obtained by directly comparing the actual joint surface profile with the typical profile in the chart.

JRC<sub>20</sub> is the profile for 20 cm and JRC<sub>100</sub> for 100 cm. The value of JRC decreases with increasing size.

# Geometrical Properties of Rock Joints

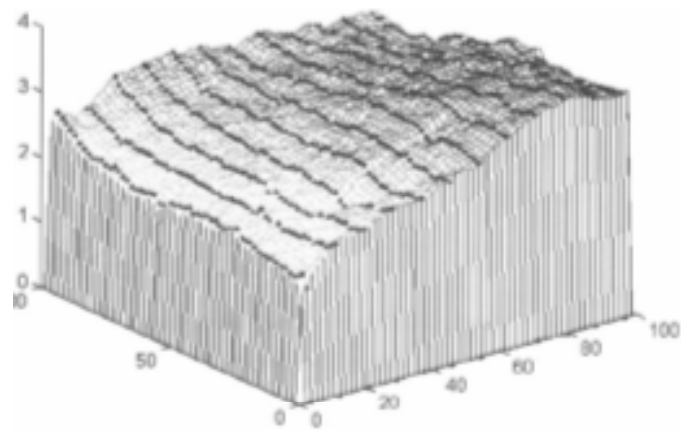
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## Joint Roughness in 3D

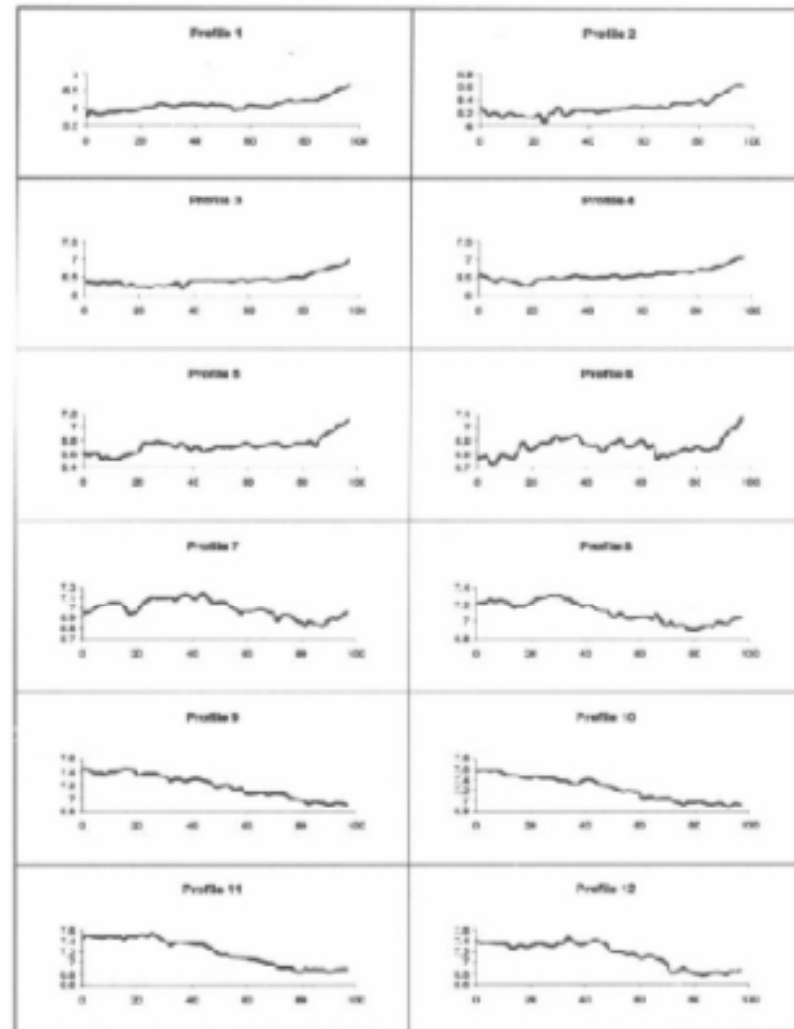
In reality, profiles of joint surfaces are 3D features. ISRM and JRC descriptions are 2D based. It is therefore suggested to take several linear profiles of a surface for the description and JRC indexing.

Joint surface is a rough profile that can be described by statistic method and fractal. Fractal method is applicable not only in 2D (linear profile), but also in 3D (surface plane profile). It is a useful tool to quantify the surface profile.

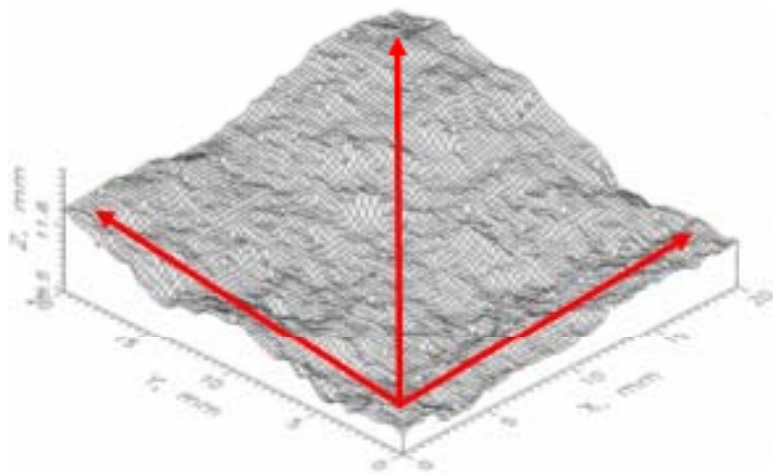




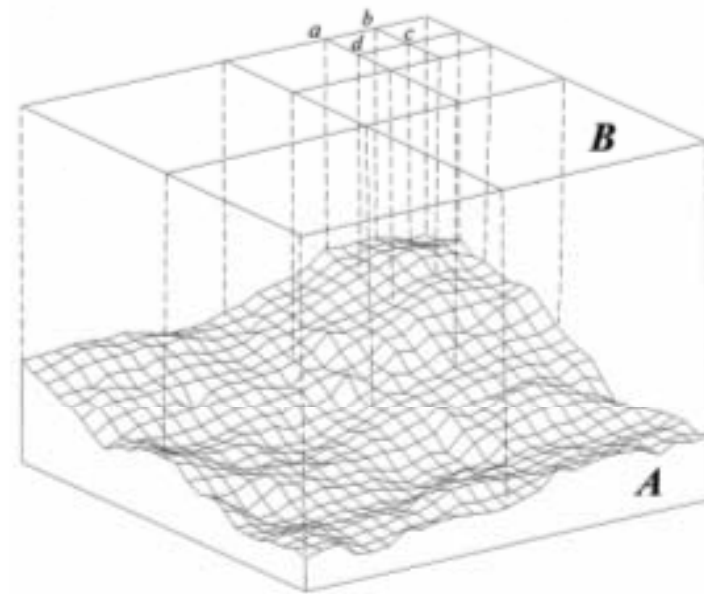
A joint surfaces is 3D. Each 2D measurement may give defferent linear profiles.



## Geometrical Properties of Rock Joints



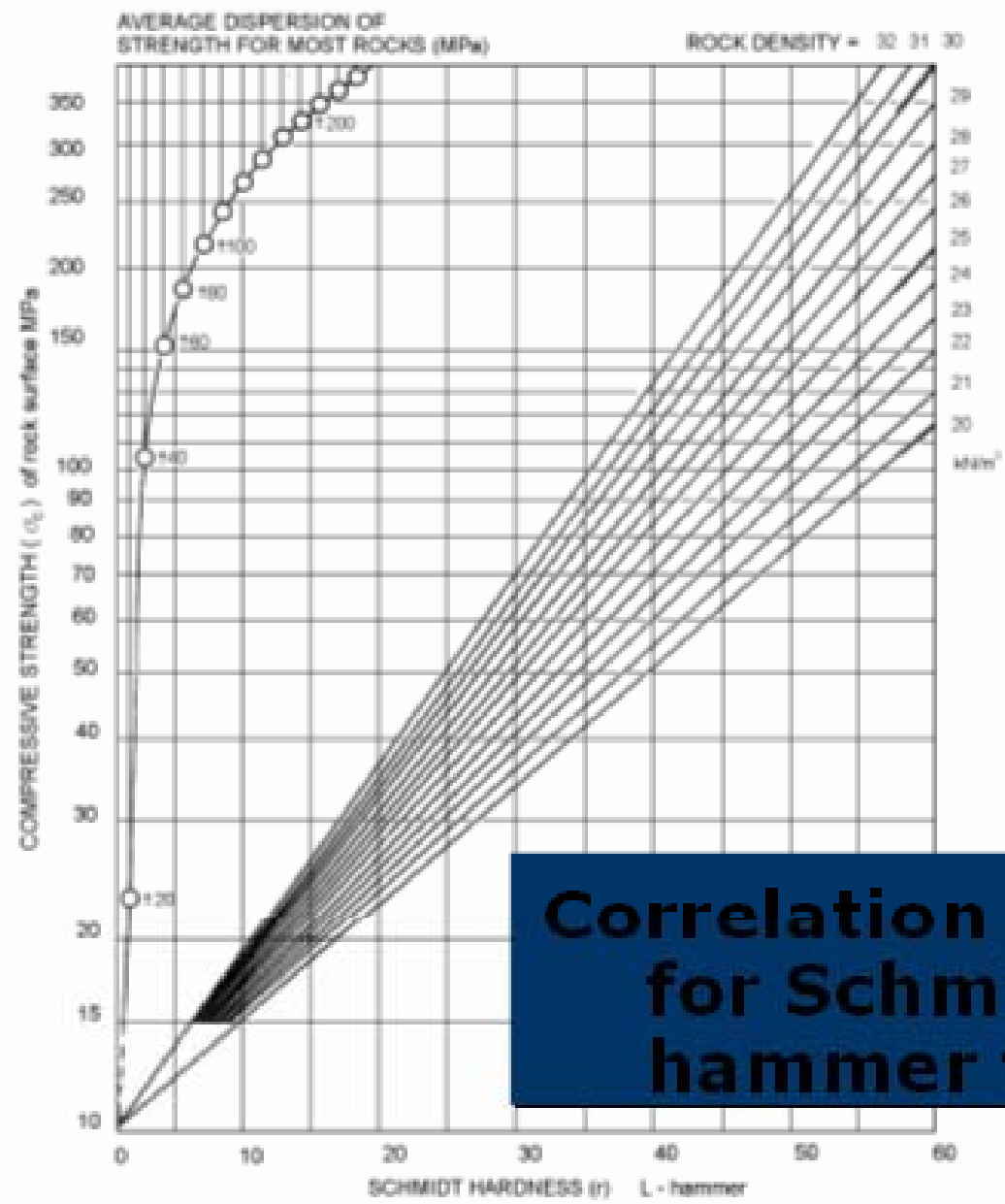
A joint surfaces in 3D, also noting change of linear profiles in directions.



Calculating fractal for a 3D surface profile.

## **Wall strength**

**It is the compressive strength of the rock comprising the walls of of a discontinuity. It is a very important component of shear strength and deformability, especially if the walls are in direct rock to rock contact as in the case of unfilled joints**



**Correlation chart  
for Schmidt  
hammer test**

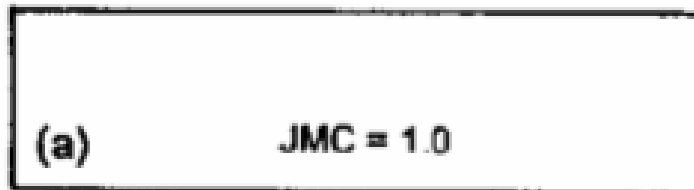
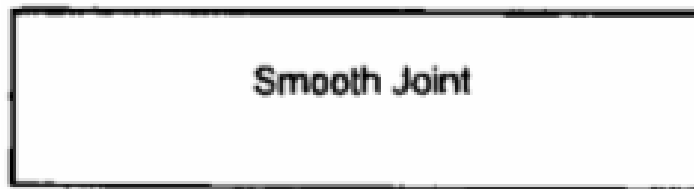
# Geometrical Properties of Rock Joints

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## Joint Matching

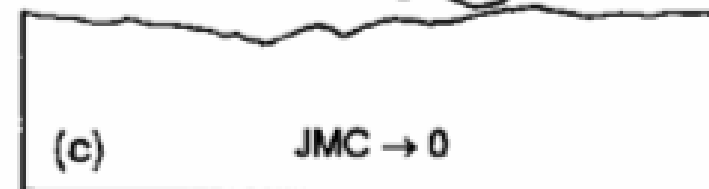
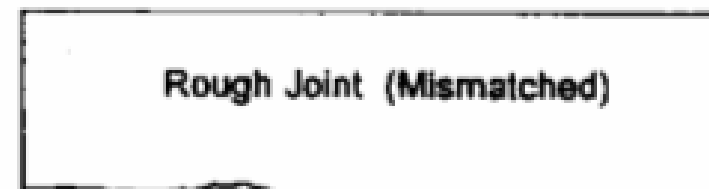
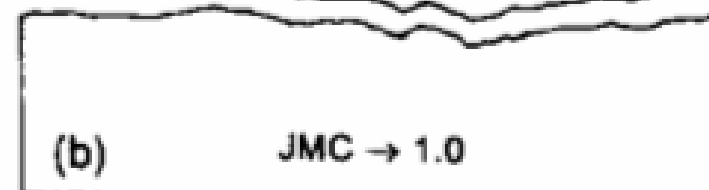
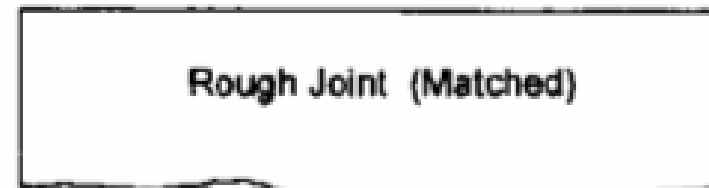
**A joint is an interface of two surfaces. Properties of a joint are also controlled by the relative positioning of the two surfaces, in addition to the profiles. For example, joints in fully contacted and interlocked positions has little possibility of movement and is also difficult to shear, as compared to the same rough joints in point contact where movement can easily occur. Often, joints are differentiated as matched and mismatched. A Joint Matching Coefficient (JMC) has been suggested.**

# Geometrical Properties of Rock Joints



JMC is 1 for completely matched joint and two surfaces fully in contact.

JMC is 0 for completely mismatched joint and two surfaces in contact at a few points only.



# Geometrical Properties of Rock Joints

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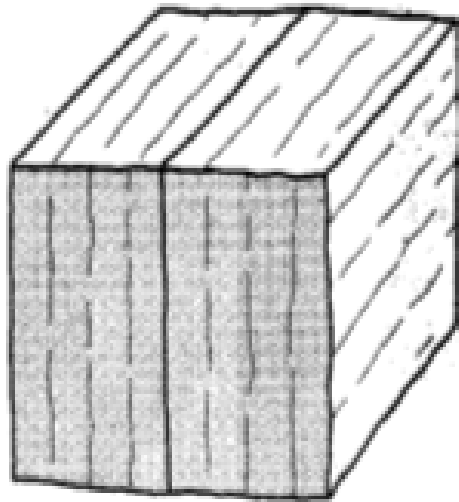
## Joint Aperture and Filling

In a natural joint, it is very seldom that the two surfaces are in complete contact. There usually exists an opening or a gap between the two surfaces. The perpendicular distance separating the adjacent rock walls is termed as aperture. Joint opening is either filled with air and water (open joint) or with infill materials (filled joint). Open or filled joints with large apertures have low shear strength. Aperture also associates with flow and permeability.

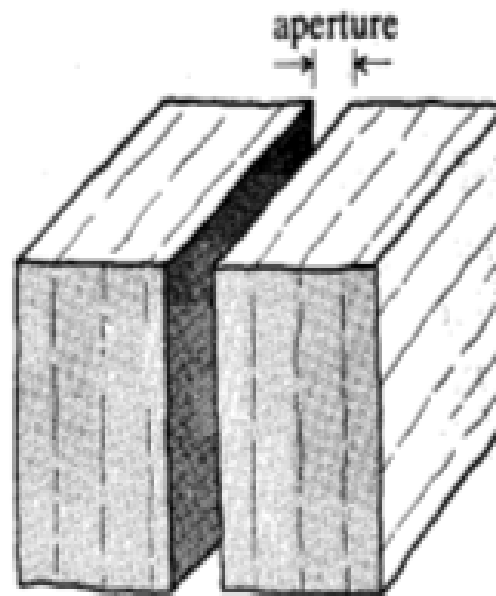
# Geometrical Properties of Rock Joints

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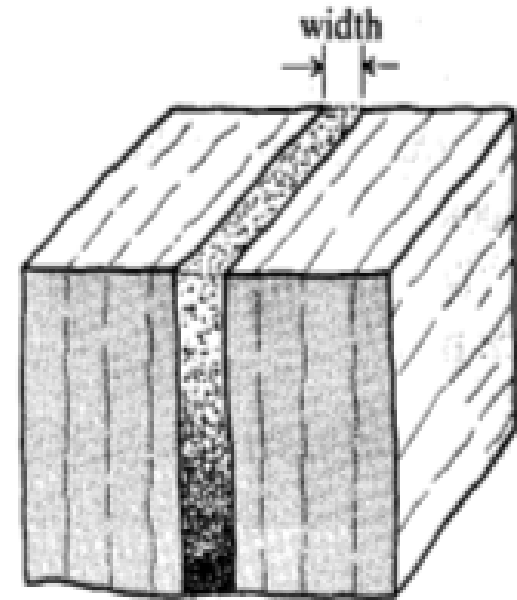
Closed discontinuity



Open discontinuity

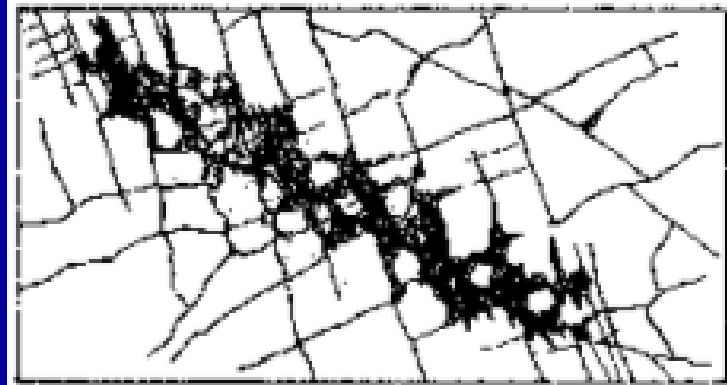
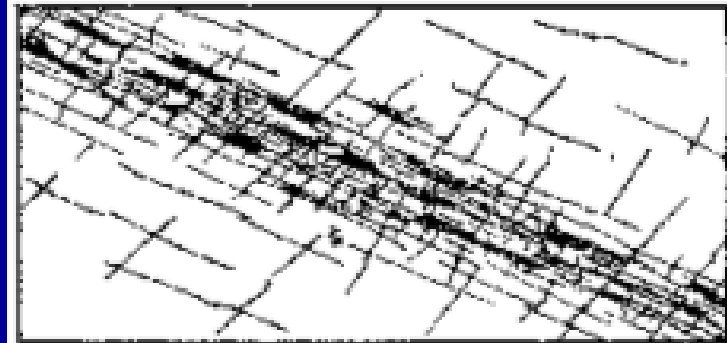
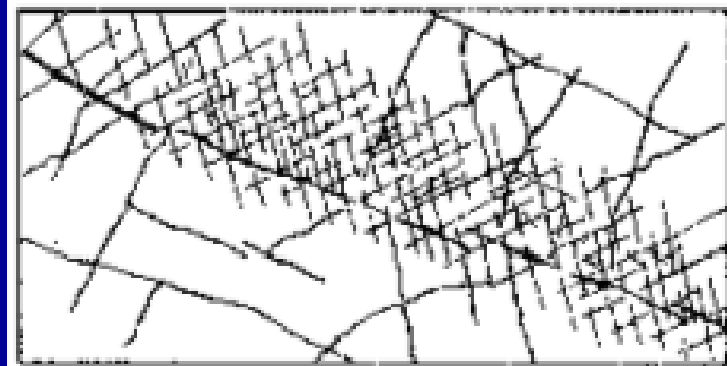
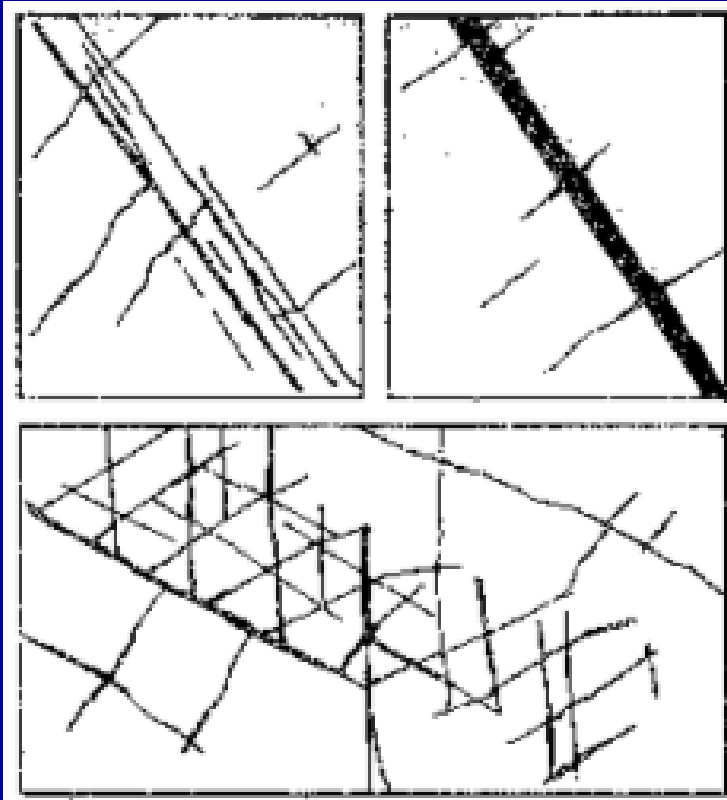


Filled discontinuity





# Examples of field sketches of filled discontinuities



# Geometrical Properties of Rock Joints

## Classification of discontinuity aperture

Aperture	Description	
< 0.1 mm	Very tight	
0.1 ~ 0.25 mm	Tight	"Closed feature"
0.25 ~ 0.5 mm	Partly open	
0.5 ~ 2.5 mm	Open	"Gapped Feature"
2.5 ~ 10 mm	Widely open	
1 ~ 10 cm	Very widely open	
10 ~ 100 cm	Extremely widely open	"Open feature"
> 1 m	Cavernous	

# Geometrical Properties of Rock Joints

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## **Joint Aperture and Filling**

**Aperture can be the real aperture and equivalent hydraulic aperture. The later is particularly important when permeability is concerned.**

**Filling is material in the rock discontinuities separating the adjacent rock surfaces. In general, properties of the filling material affect shear strength, deformability and permeability of the discontinuities.**

## **Seepage**

**Water seepage through rock masses results mainly from flow through water conducting discontinuities (secondary permeability)**



# Mechanical and Hydraulic Properties

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## Key Mechanical and Hydraulic Properties of Rock Joints

**Normal Stiffness** – deals with normal load and normal displacement

**Shear Strength** – deals with shear stress, shear displacement and dilation

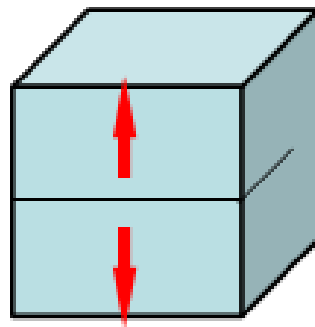
**Permeability** – deals with flow and hydraulic conductivity

# Strength and Deformation

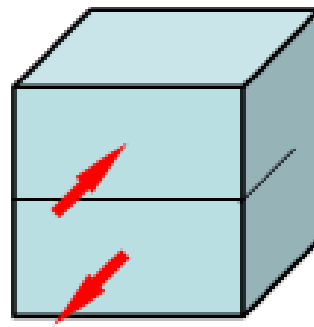
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## Fracture Toughness

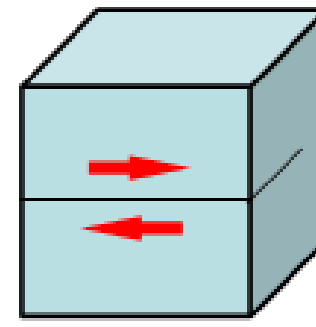
Fracture toughness of rock materials measures the effectiveness of rock fracturing. It is typically measured by a toughness test. There are three fracture mode: (Mode I), (Mode II) (Mode III).



Mode I



Mode II



Mode III

## Strength and Deformation

**Fracture Toughness**  
Correspondingly, there are three fracture toughness,  $K_{IC}$ ,  $K_{IIIC}$  and  $K_{IIIIC}$ .

**In rock mechanics, Mode I is associated with the crack initiation and propagation in a rock material.**

Granite	0.11 – 0.417
Dolerite	>0.41
Gabbro	>0.41
Basalt	>0.41
Sandstone	0.027 – 0.041
Shale	0.027 – 0.041
Limestone	0.027 – 0.041
Gneiss	0.11 – 0.41
Schist	0.005 – 0.027
Slate	0.027 – 0.041
Marble	0.11 – 0.41
Quartzite	>0.41



# Mechanical and Hydraulic Properties

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## Stress and Deformation at Discontinuity

**Stresses are often disturbed by a discontinuity. For a rock fracture with opening, normal stress on the fracture walls is zero and there are stress concentrations on the contact points. The stress field is no longer the same as in the continuous material.**

**For a closed joint, the stress field may be continuous although strain may not.**

# Mechanical and Hydraulic Properties

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## **Stress and Displacement at Discontinuity**

**Displacement at discontinuity is not continuous. For example, at a fracture plane, sliding or shear displacement may occur. There may be much greater normal displacement at fracture than those of the material.**

**Discontinuities can range from a fully-welded interface to an opening containing different material. The mechanics of are vary different.**

# Mechanical and Hydraulic Properties

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## **Stress and Displacement at Discontinuity**

**For a fully-welded interface between two different materials, it has the continuities both is stress and displacement. Discontinuity is the change of materials at the interface.**

**For a fully-contacted smooth interface, the interface representing a weak plane of shearing.**

**For a locally-contacted fracture with gaps, both stress and displacement are discontinuous.**

# Mechanical and Hydraulic Properties

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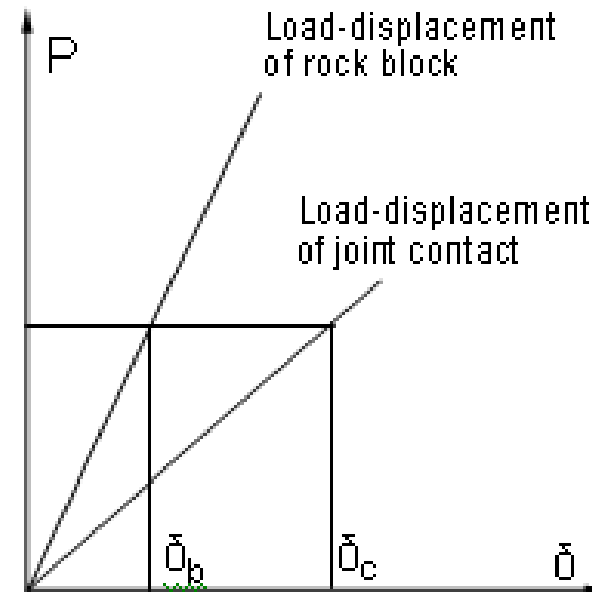
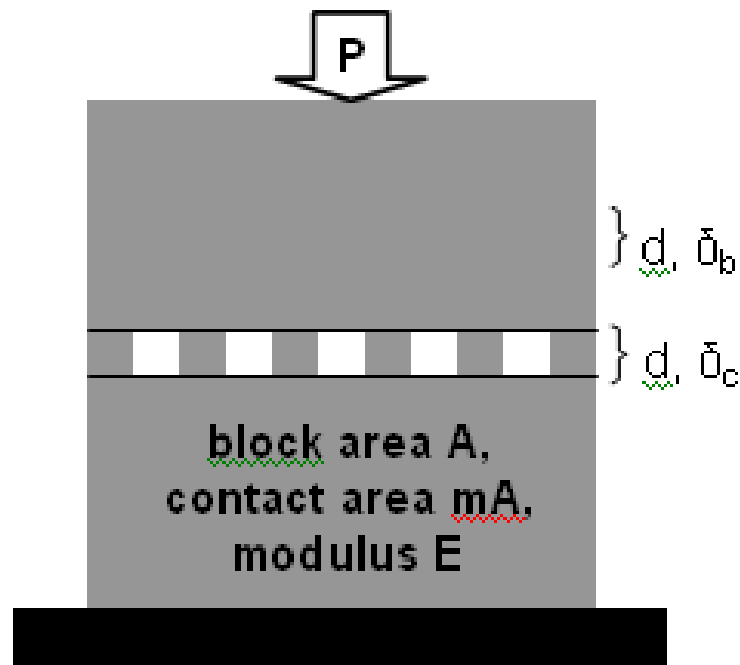
## Normal Stiffness and Displacement

Normal stress and displacement of fully-contact discontinuity is continuous and therefore can be dealt with continuum approach.

For locally-contacted fractures, there are voids between the two sides, stress-displacement function is discontinuous.

- (a) An idealised pillar-contacted fracture
- (b) An idealised prism-contacted fracture

# Mechanical and Hydraulic Properties



Assume block and contact have the same material,  $m$  is between 0 and 1. Contact area does not change and fail.

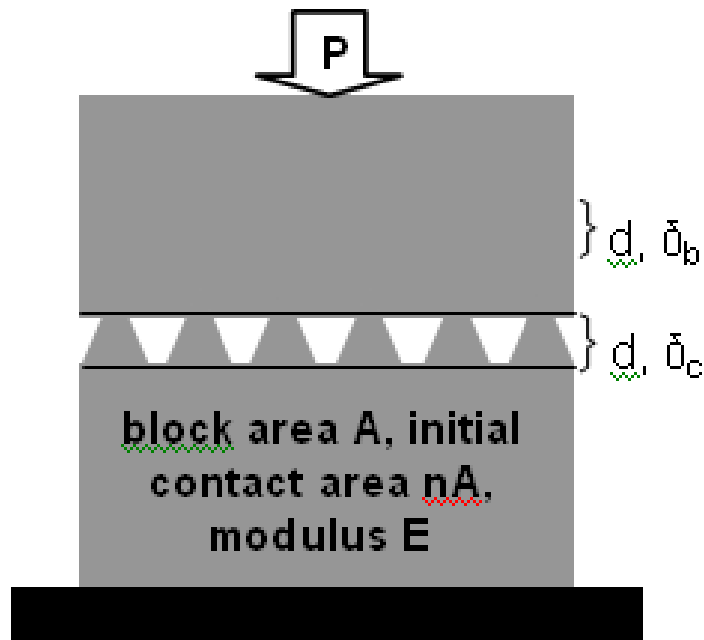
$$\delta_b = (P D) / (A E)$$

$$\delta_c = (P D) / (m A E)$$

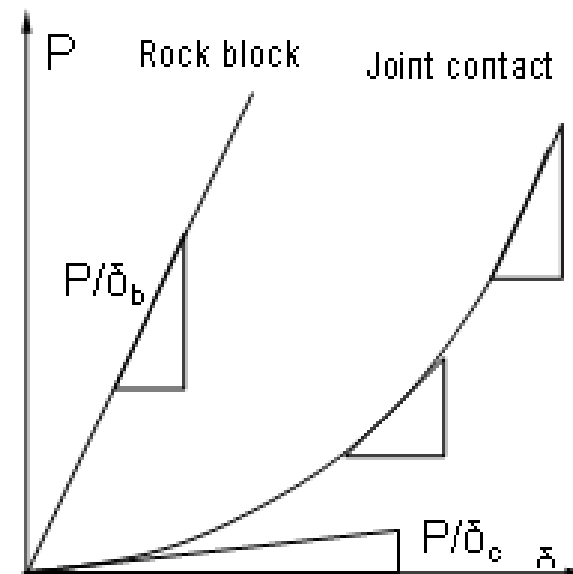
$$\delta = \frac{P D}{A E} \left( \frac{1}{m} \right)$$

$$P / \delta = \frac{A E}{D} \left( \frac{1}{m} \right)$$

# Mechanical and Hydraulic Properties



Assume block and contact have the same material,  $n$  is between 0 and 1. Contact area increases with contact closure, but does not fail.



At initial condition

$$P/\bar{\delta}_c = nP/\bar{\delta}_b$$

At complete closure

$$P/\bar{\delta}_c = P/\bar{\delta}_b$$