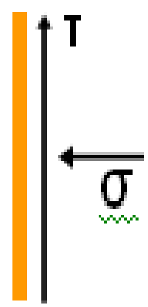
Stress, Strain and Deformation

1. Stress is a property at a point. It is a tensor.

$$\sigma = \lim_{\Delta A \to 0} (\Delta F / \Delta A)$$

2. There are normal stresses and there are shear stresses.



Normal stress σ

$$\sigma = \lim_{\Delta A \to 0} (\Delta N / \Delta A)$$

Shear stress τ

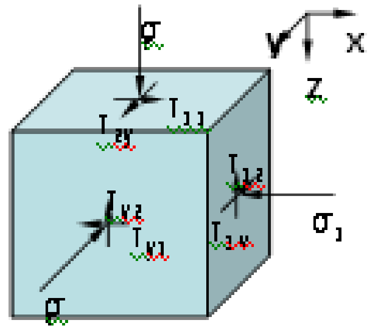
$$\tau = \lim_{\Delta A \to 0} (\Delta S / \Delta A)$$

Stress, Strain and Deformation

3. There are nine stress components on a small cube.

Three normal stresses σ_{xx} σ_{yy} σ_{zz}

Six shear stresses τ_{xy} τ_{yx} τ_{xz} τ_{zx} τ_{yz} τ_{zy}



4. These stress components can be listed out in matrix form.

σ_{xx}	τ_{yx}	τ_{zx}
τ [illegible]	[illegible]	[illegible]
τ_{xz}	τ_{yz}	σ_{zz}

Stress, Strain and Deformation

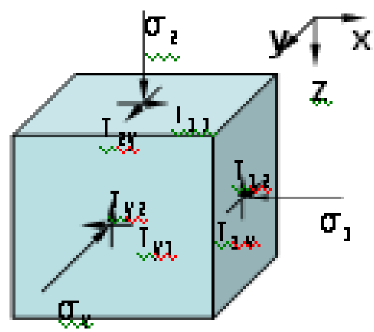
5. Corresponding shear stresses are equal. Hence matrix can be reduced to symmetrical.

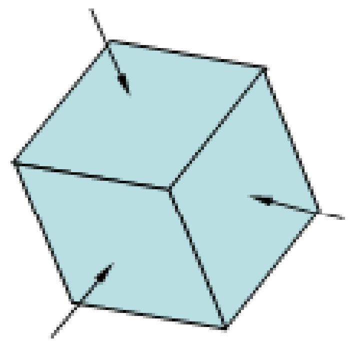
$$\tau_{xy} = \tau_{yx}, \quad \tau_{xz} = \tau_{zx}, \quad \tau_{yz} = \tau_{zy}$$

$$\begin{vmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_{yy} & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_{zz} \end{vmatrix}$$

Stress, Strain and Deformation

6. There is an inclination of the axes at which all shear stresses disappear (stress transformation). The remaining stresses are principal stresses.

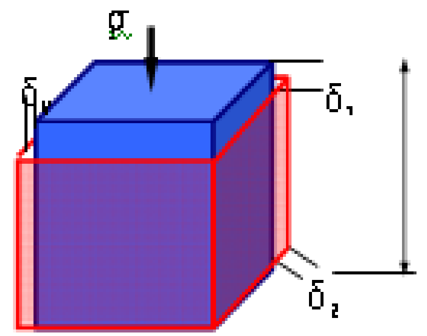




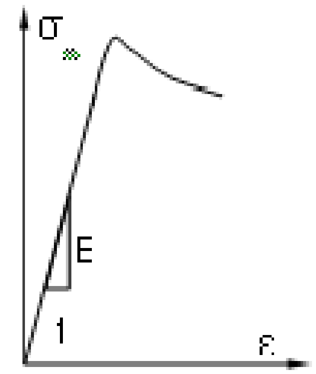
$$\begin{vmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{vmatrix}$$

Stress, Strain and Deformation

7. **Strains are deformations per lengths caused by stresses. In elastic region, they can be related by the Young's Modulus.**



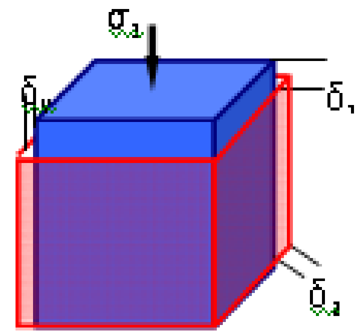
$\varepsilon = \delta_x / l$



$E = d\sigma_x / d\varepsilon_x$

Stress, Strain and Deformation

8. Strain in stress direction always causes strains in other directions. The ratio of strains is the Poisson's ratio.



$\nu = \varepsilon_y / \varepsilon_x$, $\nu = \varepsilon_z / \varepsilon_x$

Stress, Strain and Deformation

9. Stresses and strains are related by constitutive laws.

$$\begin{vmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{vmatrix} = \begin{vmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16} \\ S_{21} & S_{22} & S_{23} & S_{24} & S_{25} & S_{26} \\ S_{31} & S_{32} & S_{33} & S_{34} & S_{35} & S_{36} \\ S_{41} & S_{42} & S_{43} & S_{44} & S_{45} & S_{46} \\ S_{51} & S_{52} & S_{53} & S_{54} & S_{55} & S_{56} \\ S_{61} & S_{62} & S_{63} & S_{64} & S_{65} & S_{66} \end{vmatrix} \begin{vmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{vmatrix}$$

$$[\varepsilon] = [S]\,[\sigma]$$

$$\begin{vmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{vmatrix} = 1/E \begin{vmatrix} 1 & -v & -v & 0 & 0 & 0 \\ -v & 1 & -v & 0 & 0 & 0 \\ -v & -v & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2(1+v) & 0 & 0 \\ 0 & 0 & 0 & 0 & 2(1+v) & 0 \\ 0 & 0 & 0 & 0 & 0 & 2(1+v) \end{vmatrix} \begin{vmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{vmatrix}$$

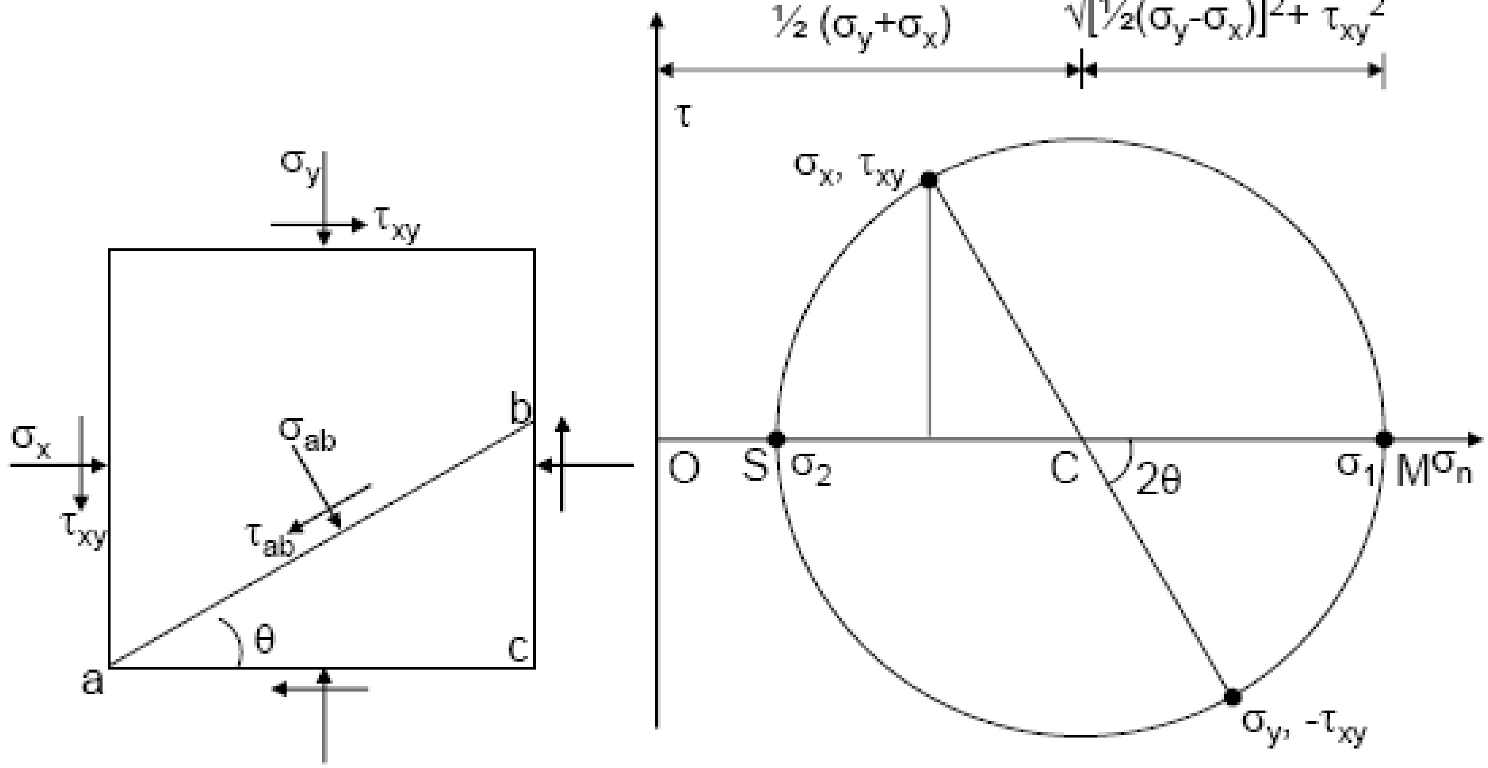
$\varepsilon_{xx} = [\sigma_{xx} - v\,(\sigma_{yy} + \sigma_{zz})] / E$

$\gamma_{xy} = \tau_{xy} / G$ where $G = E / [2\,(1+v)]$

E = Young's modulus
v = Poisson's ratio
G = shear modulus

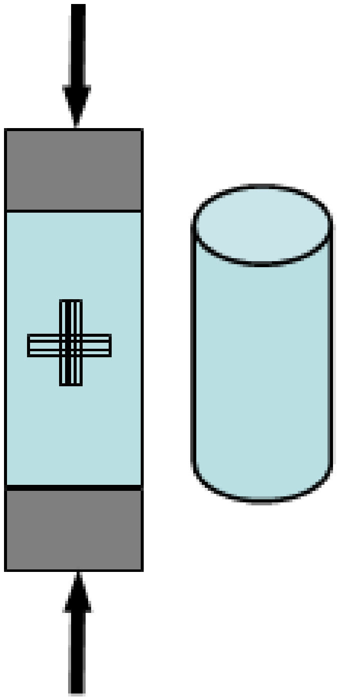
Stress, Strain and Deformation

10. Plane stresses and strains can be represented by Mohr circles.



Strength and Deformation

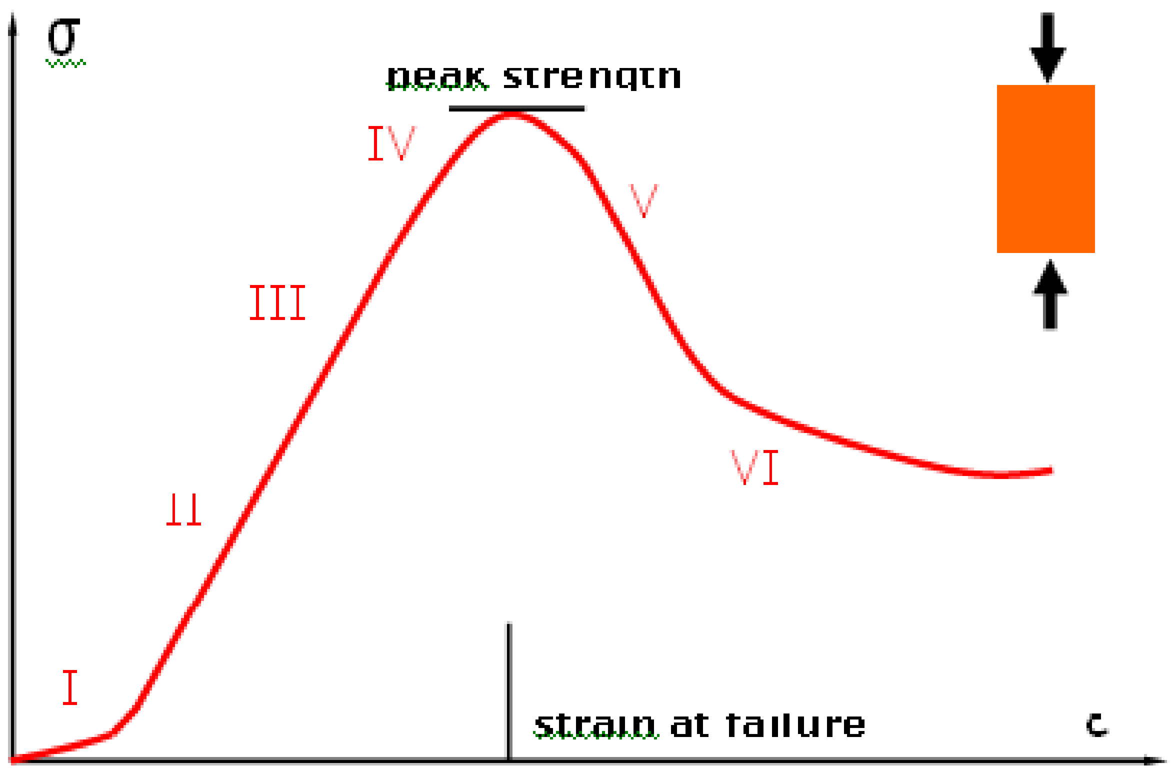
Uniaxial Compression



Uniaxial compressive strength is the ultimate stress a cylindrical rock specimen under axial load. It is the most important mechanical properties of rock material, used in design, analysis and modelling.

Along with measurements of load, axial and lateral deformations of the specimen are also measured.

Strength and Deformation



Strength and Deformation

Stage I – The rock is initially stressed, in addition to deformation; existing micro-cracks are closing, causing an initial non-linearity of the curve.

Stage II – The rock basically has a linearly elastic behaviour with linear stress-strain curves, both axially and laterally.

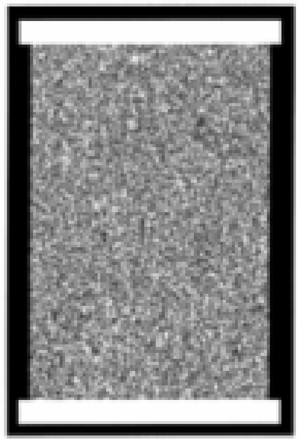
Stage III – The rock behaves near-linear elastic. The axial stress-strain curve is near-linear and is nearly recoverable.

Strength and Deformation

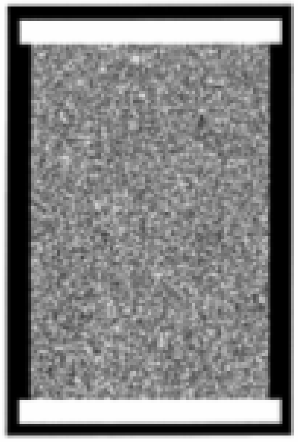
Stage IV – The rock is undergone a rapid acceleration of microcracking events and volume increase.

Stage V – The rock has passed peak stress, but is still intact, even though the internal structure is highly disrupt. The specimen is undergone strain softening (failure) deformation.

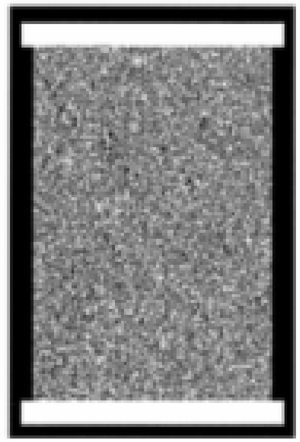
Stage VI – The rock has essentially parted to form a series of blocks rather than an intact structure.



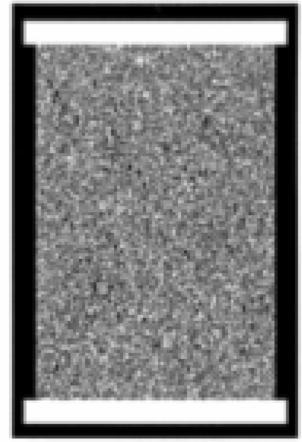
a. 56% peak stress



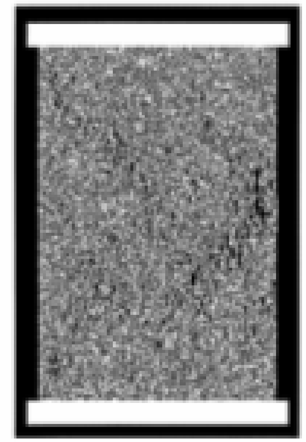
b. 65% peak stress



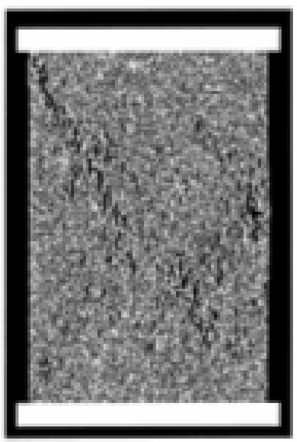
c. 92% peak stress



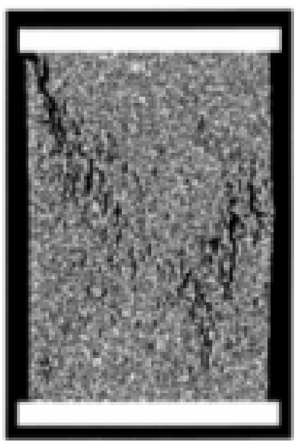
d. 98% peak stress



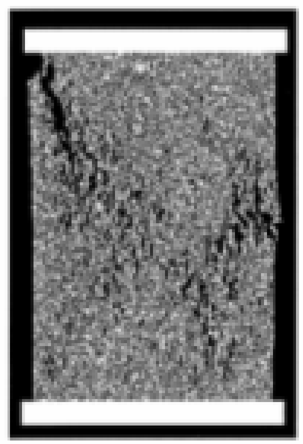
e. 100% peak stress



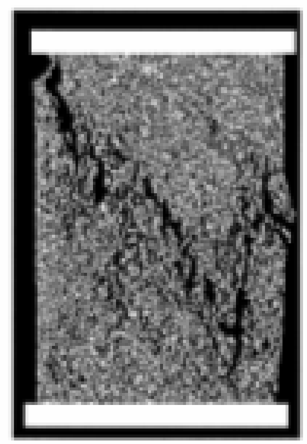
f. 96% peak stress



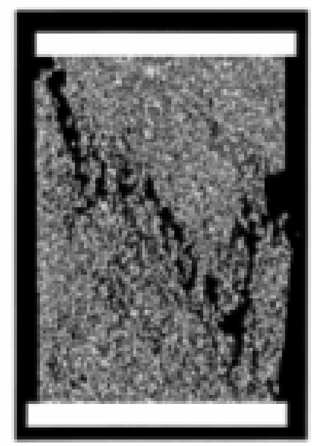
g. 92% peak stress



h. 78% peak stress



i. 75% peak stress



j. 37% peak stress

Strength and Deformation

Around peak stress (IV-V)

Post peak stress (VI)

Strength and Deformation

Uniaxial Stress-Strain at and after Peak

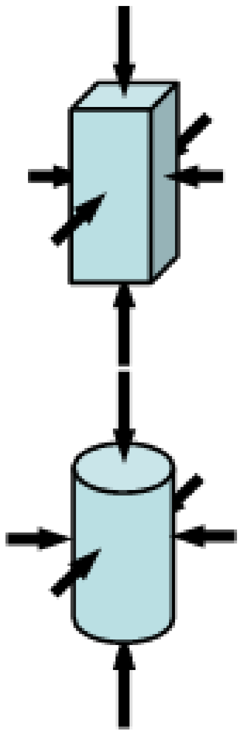
Rocks generally fail at a small strain, typically around 0.2 to 0.4%. Brittle rocks, typically crystalline rocks, have low strain at failure, while soft rock, such as shale and mudstone, tend to have relatively high strain at failure.

Most rocks, including all crystalline igneous, metamorphic and sedimentary rocks, behave brittle under uniaxial compression. A few soft rocks, mainly of sedimentary origin, behave ductile.

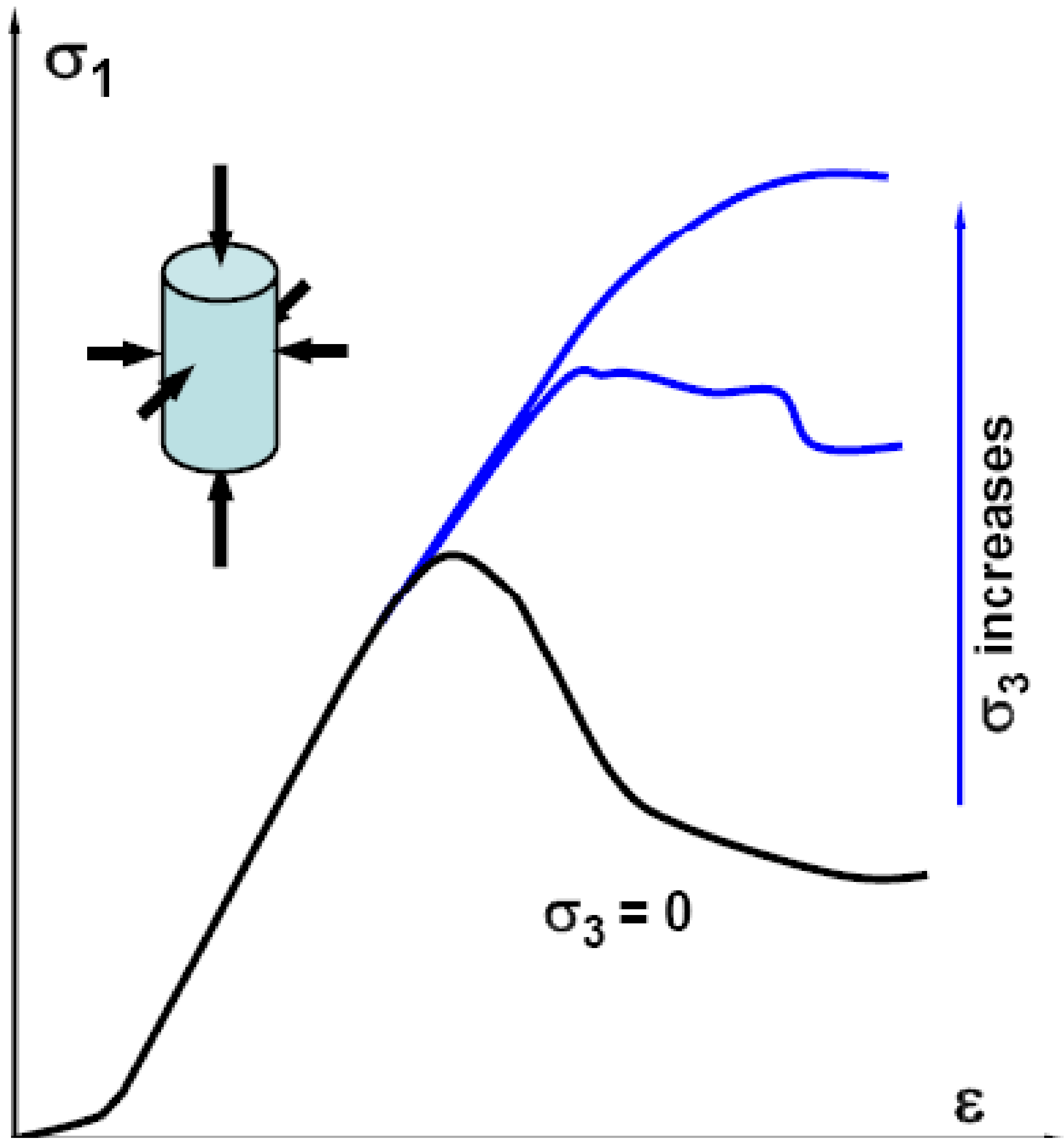
Strength and Deformation

Triaxial Compression

At depth, rock is subjected to axial and lateral stresses (triaxial), and compressive strength is higher in triaxial condition.



True triaxial compression means 3 different principal stresses. It is often simplified by making 2 lateral stresses equal to minor principal stress (axisymmetric triaxial test).



σ1
σ3 increases
σ3 = 0
ε

Strength and Deformation

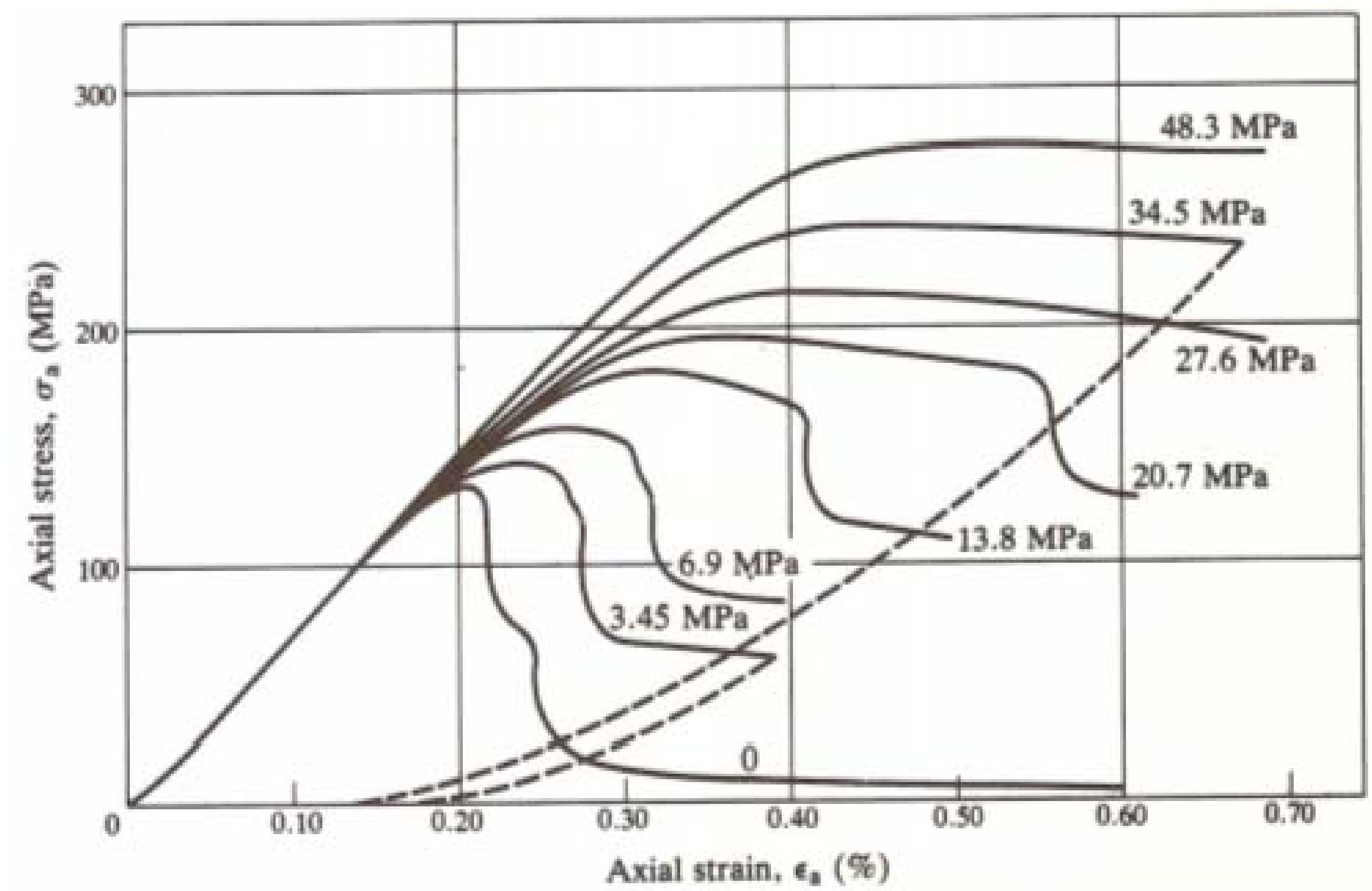
The behaviour of rock in triaxial compression changes with increasing confining pressure:

(a) Peak strength increases;

(b) Post peak behaviour from brittle gradually changes to ductile.

Stress-strain behaviour at elastic region appears the same as uniaxial compression.

Strength and Deformation



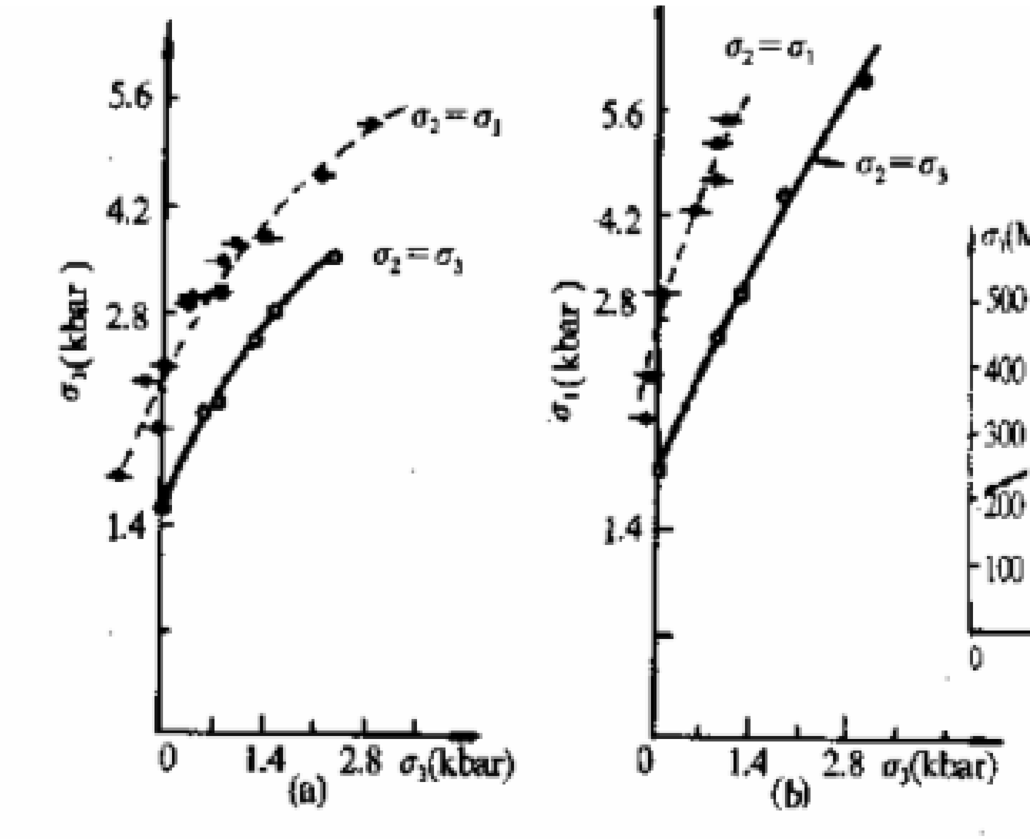
Strength and Deformation

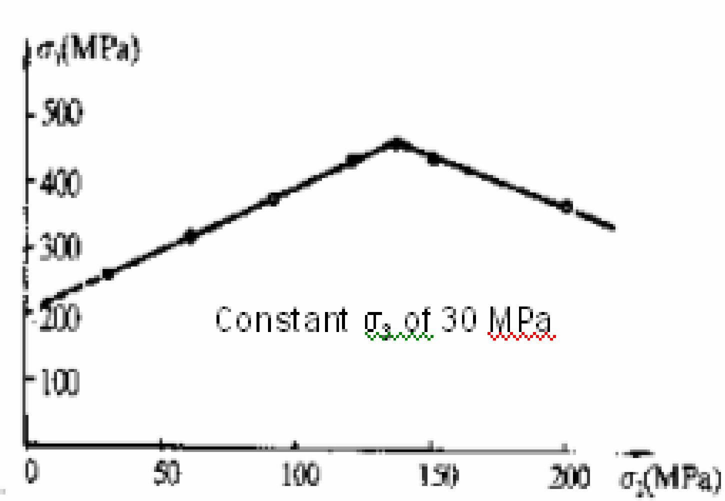
Effect of Intermediate Principal Stress

Axisymmetric triaxial test gives strength without considering the effect of intermediate principal stress (σ_2), and generally under-estimate the strength.

Rock triaxial compressive strength generally increases with σ_2 with a fixed σ_3. When σ_2 is excessively greater than σ_3, the strength may start to decrease.

Strength and Deformation





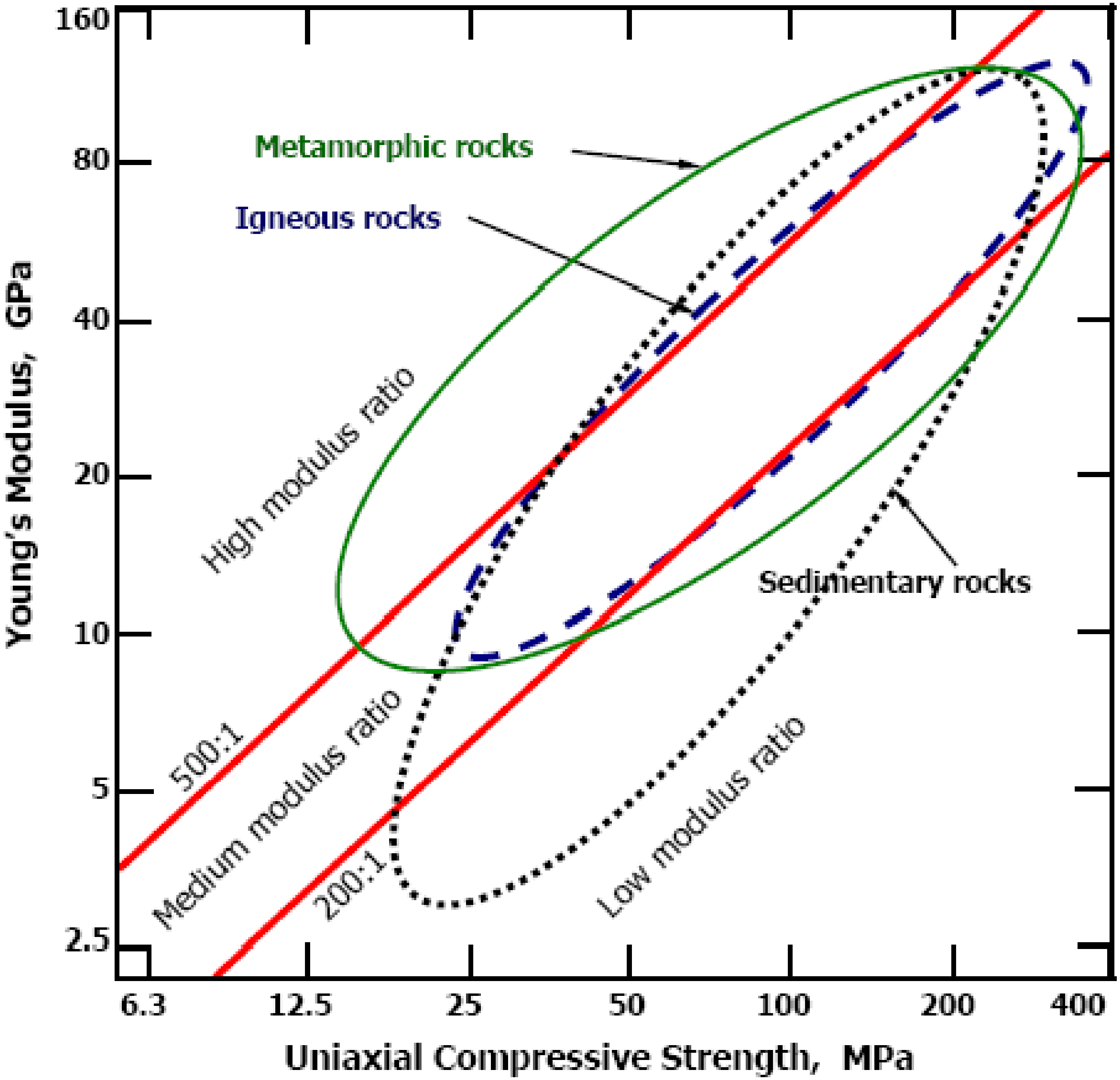
Strength and Deformation

Young's Modulus and Poisson's Ratio

Young's modulus and Poisson's Ratio can be experimentally determined from the stress-strain curve. They seem to be unaffected by change of confining pressure.

High strength rocks also tend to have high Young's modulus, depending on rock type and other factors.

For most rocks, the Poisson's ratio is between 0.15 and 0.4.

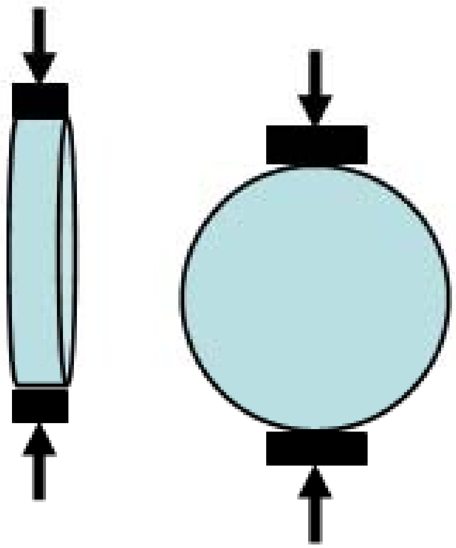

Metamorphic rocks
Igneous rocks
Sedimentary rocks
High modulus ratio
Medium modulus ratio
Low modulus ratio
500:1
200:1
Young's Modulus, GPa
160
80
40
20
10
5
2.5
6.3
12.5
25
50
100
200
400
Uniaxial Compressive Strength, MPa

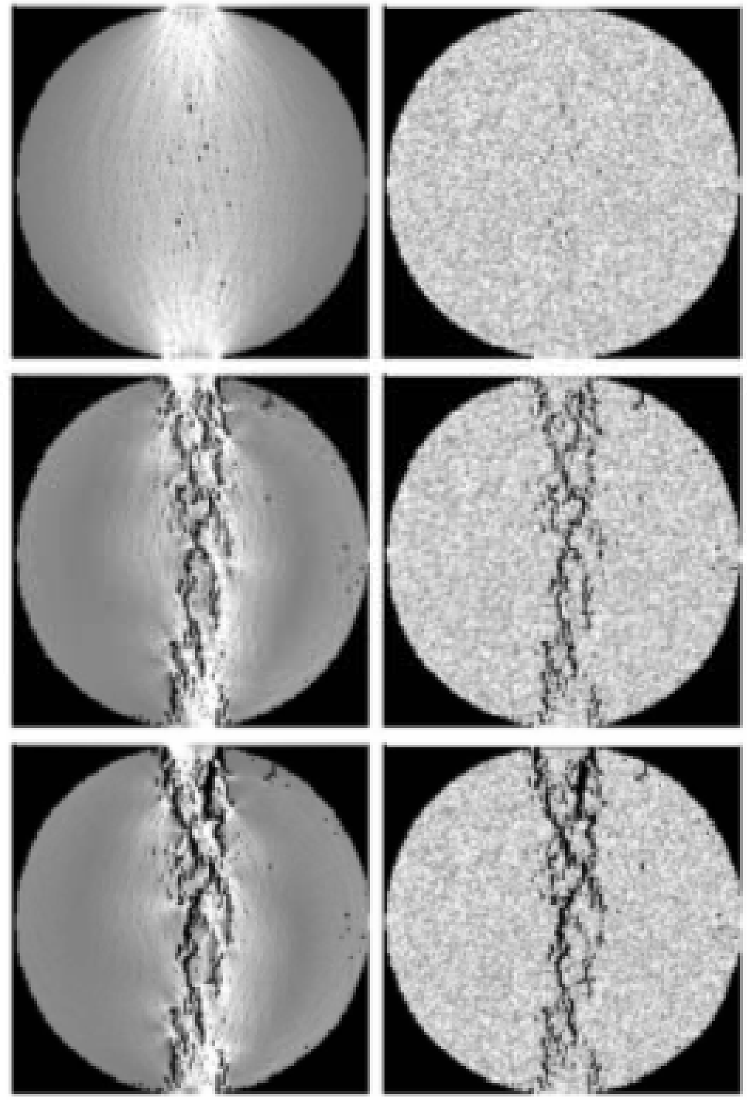
Strength and Deformation

Tensile Strength

Rock material generally has a low tensile strength, due to the pre-exiting microcracks in the rock material. The existence of microcracks may also be the cause of rock failing suddenly in tension with a small strain.

Rock material tensile strength can be obtained from several types of tests. The most common tensile test is the Brazilian tests.





Strength and Deformation

Shear Strength

Rock resists shear stress by two internal mechanisms, cohesion and internal friction. Cohesion is a measure of internal bonding of the rock material. Internal friction is caused by contact between particles, and is defined by the internal friction angle.

Shear strength of rock material ca be determined by direct shear test and by triaxial compression tests.

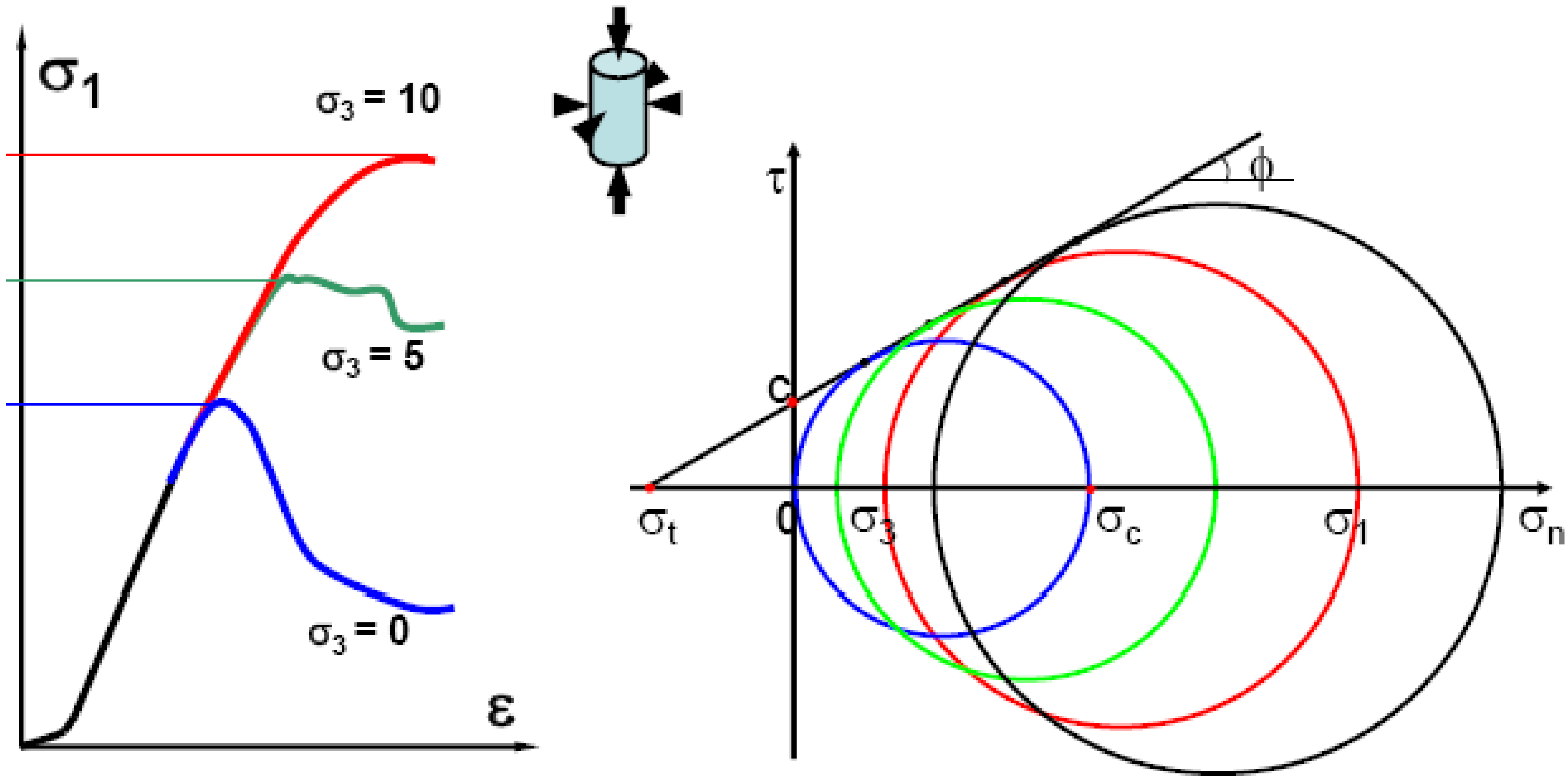
Strength and Deformation

Shear Strength from Triaxial Tests

From a series of triaxial tests, peak stresses (σ_1) are obtained at various lateral stresses (σ_3). By plotting Mohr circles, the shear envelope is defined and gives the cohesion and internal friction angle.

Test	σ_3 (MPa)	σ_1 (MPa)
1	0	41.2
2	1	52.6
3	3	74.1
4	5	90.3
5	10	122
6	15	151
7	20	172

Strength and Deformation



Strength and Deformation

Rock	UC Strength (MPa)	Tensile Strength (MPa)
Granite	100 – 300	7 – 25
Dolerite	100 – 350	7 – 30
Gabbro	150 – 250	7 – 30
Basalt	100 – 350	10 – 30
Sandstone	20 – 170	4 – 25
Shale	5 – 100	2 – 10
Dolomite	20 – 120	6 – 15
Limestone	30 – 250	6 – 25
Gneiss	100 – 250	7 – 20
Slate	50 – 180	7 – 20
Marble	50 – 200	7 – 20
Quartzite	150 – 300	5 – 20

Strength and Deformation

Compressive, Tensile and Shear Strengths

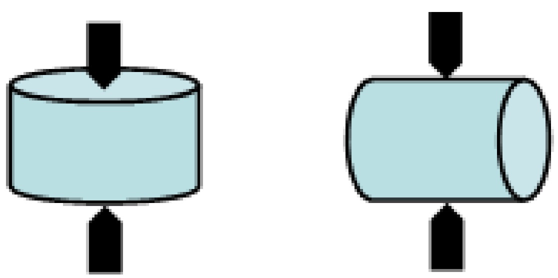
Tensile and shear strengths are important as rock fails mostly in tensile and in shear, even the loading may appears to be compression. Rocks generally have high compressive strength so failure in pure compression is not common.

Theoretically, the three strengths are related. This will be discussed in strength criteria.

Strength and Deformation

Point Load Index

Point load test is a simple index test for rock material. It gives the standard point load index, $I_{s(50)}$.



Granite	5 – 15
Gabbro	6 – 15
Andesite	10 – 15
Basalt	9 – 15
Sandstone	1 – 8
Mudstone	0.1 – 6
Limestone	3 – 7
Gneiss	5 – 15
Schist	5 – 10
Slate	1 – 9
Marble	4 – 12
Quartzite	5 – 15

Strength and Deformation

Correlation between Point Load Index and Strengths

$\sigma_c \approx 22\ I_{s(50)}$

Correction factor can vary between 10 and 30.

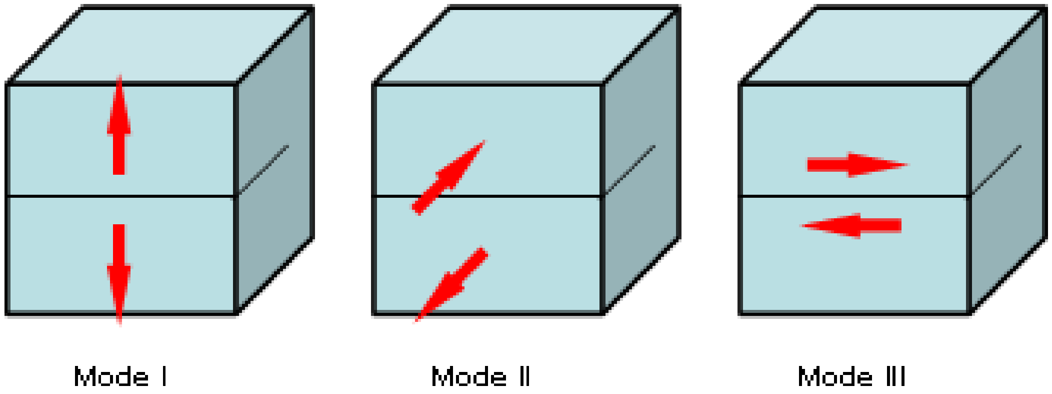
$\sigma_t \approx 1.25\ I_{s(50)}$

$I_{s(50)}$ should be used as an independent strength index.

Strength and Deformation

Fracture Toughness

Fracture toughness of rock materials measures the effectiveness of rock fracturing. It is typically measured by a toughness test. There are three fracture mode: (Mode I), (Mode II) (Mode III).



Strength and Deformation

Fracture Toughness
Correspondingly, there are three fracture toughness, K_{IC}, K_{IIC} and K_{IIIC}.

In rock mechanics, Mode I is associated with the crack initiation and propagation in a rock material.

Granite	0.11 – 0.417
Dolerite	>0.41
Gabbro	>0.41
Basalt	>0.41
Sandstone	0.027 – 0.041
Shale	0.027 – 0.041
Limestone	0.027 – 0.041
Gneiss	0.11 – 0.41
Schist	0.005 – 0.027
Slate	0.027 – 0.041
Marble	0.11 – 0.41
Quartzite	>0.41

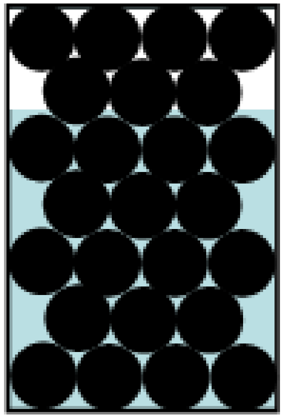
Physical and Engineering Properties

Density, Porosity and Water Content

They are standard physical properties.

Density = Bulk mass / Bulk volume
Porosity = Non-solid volume / Bulk volume

Water content = Volume of water / Bulk volume



Physical and Engineering Properties

Dry density of rock material is generally between 2.5 and 2.8 g/cm^3. High density generally mean low porosity.

Porosity is generally low for crystalline rocks, e.g., granite (<5%) and can be high for clastic sedimentary rocks, e.g., sandstone (up to 50%). Porosity effects permeability.

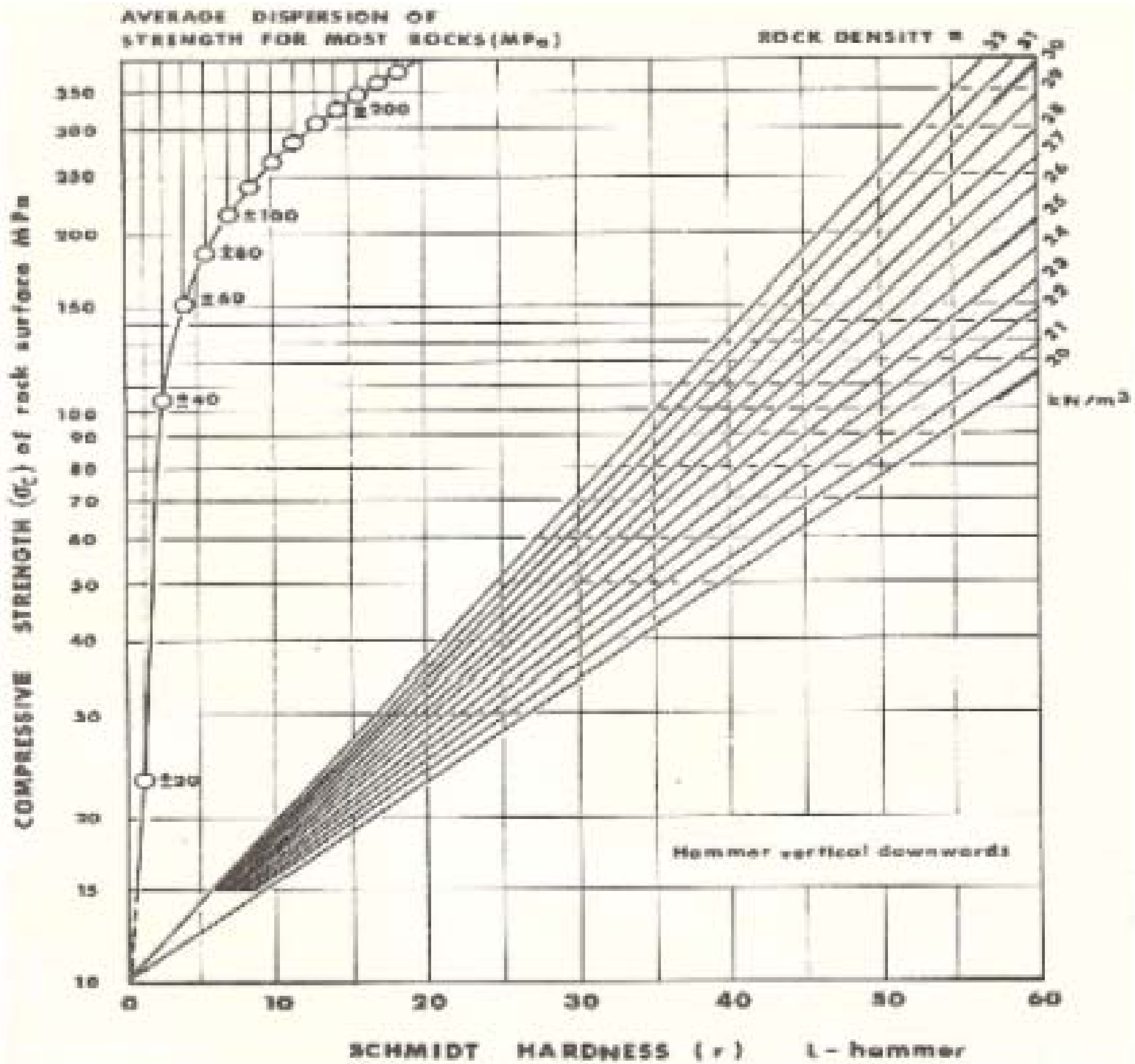
Water content depends saturation. Wet rock tends to have slightly lower strength.

Physical and Engineering Properties

Hardness

Hardness is the characteristic of a solid material to resist permanent deformation. Rock material hardness depends on several factors, including mineral composition and density. A typical measure is the Schmidt rebound hardness number.

Schmidt hardness can be correlated to rock strength.



AVERAGE DISPERSION OF
STRENGTH FOR MOST ROCKS (MPa)
ROCK DENSITY =
kN/m3
COMPRESSIVE STRENGTH (σ_c) of rock surface MPa
SCHMIDT HARDNESS (r) L-hammer
Hammer vertical downwards
±20
±40
±60
±80
±100
±200

Physical and Engineering Properties

Abrasivity

Abrasivity measures the abrasiveness of a rock materials against other materials, e.g., steel.

Abrasivity is highly influenced by the amount of quartz mineral in the rock material. The higher quartz content gives higher abrasivity.

Abrasivity are measured by tests, e.g., Cerchar test gives Cerchar Abrasivitiy Index (CAI).

Physical and Engineering Properties

Cerchar Abrasivitiy Index (CAI)

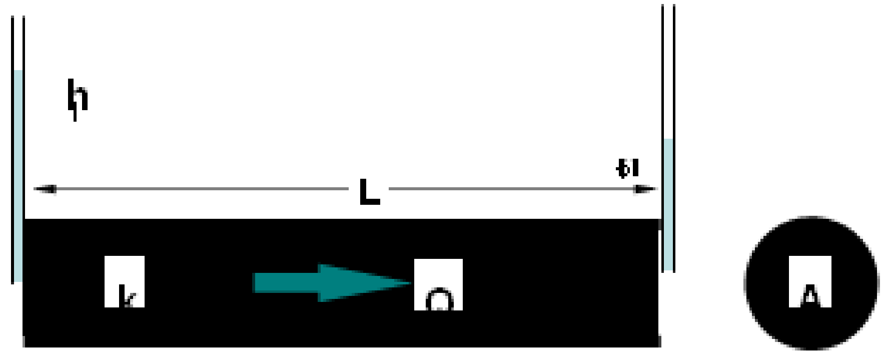
Granite	4.5 – 5.3
Diorite	4.2 – 5.0
Andesite	2.7 – 3.8
Basalt	2.0 – 3.5
Sandstone	1.5 – 3.5, 2.8 – 4.2
Shale	0.6 – 1.8
Limestone	1.0 – 2.5
Gneiss	3.5 – 5.3
Slate	2.3 – 4.2
Quartzite	4.3 – 5.9

Physical and Engineering Properties

Permeability

Permeability is a measure of the ability of a material to transmit fluids. It is given by the Darcy's law,

$$Q = A\,k\,(h_1 - h_2)/L$$



Q = Flow rate
k = Coefficient of permeability
A = cross section area
L = length
h_1, h_2 = hydraulic head

Physical and Engineering Properties

Permeability

Most rocks have very low permeability. Permeability of rock material is governed by porosity. Porous rocks such as sandstones usually have high permeability while granites have low permeability. Permeability of rock materials, except for those porous one, has limited interests. In the rock mass, flow is concentrated in fractures.

Physical and Engineering Properties

Wave Velocity

Two types of wave are often used in velocity measurements: longitudinal (P) wave and shear (S) wave. P wave is the fastest travelled wave and therefore is the most commonly used one in wave velocity measurements.

http://www.matter.org.uk/Schools/Content/Seismology/pandswaves.html

Physical and Engineering Properties

Wave Velocity

Wave velocity is related to the compaction degree (density and porosity) of the rock material. A well compacted rock has generally high velocity as the grains are in good contact and wave travels through solid grains.

P-wave velocity of igneous rocks, gneiss and quartzite is 5000-7000 m/s, and of shale, sandstone and conglomerate 3000-5000 m/s.

Physical and Engineering Properties

Wave Velocities and Deformation Modulus

Wave velocity can be used to estimate the modulus of the rock material. The modulus estimated is generally slightly higher than the modulus determined from static tests.

Elastic modulus $E_s = \rho v_s^2$ (GPa), (g/cm^3), (km/s)

Shear modulus $G_s = \rho v_s^2$ (GPa), (g/cm^3), (km/s)

Poisson's ration $\nu_s = [1–2(v_s/v_p)^2] / \{2[1–(v_s/v_p)^2]\}$

Failure Criteria of Rock Material

Strength and Strength Criterion

Yield strength is defined as the stress at which a material begins to plastically deform. It generally represents an upper limit to the load that can be applied.

A yield strength criterion is a hypothesis concerning the limit of stress under any combination of stresses. They are generally described in term of three principal stresses.

Failure Criteria of Rock Material

Common Strength Criteria

Tresca-Guest criterion

$$Max\ (|\sigma_1 - \sigma_2|, |\sigma_2 - \sigma_3|, |\sigma_3 - \sigma_1|) = \sigma_0$$

von Mises criterion

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2{\sigma_0}^2$$

Mohr-Coulomb criterion

$$\max\left(\frac{|\sigma_1 - \sigma_2|}{2} - c + K\frac{\sigma_1 + \sigma_2}{2}, \frac{|\sigma_2 - \sigma_3|}{2} - c + K\frac{\sigma_2 + \sigma_3}{2}, \frac{|\sigma_3 - \sigma_1|}{2} - c + K\frac{\sigma_3 + \sigma_1}{2}\right) = 0$$

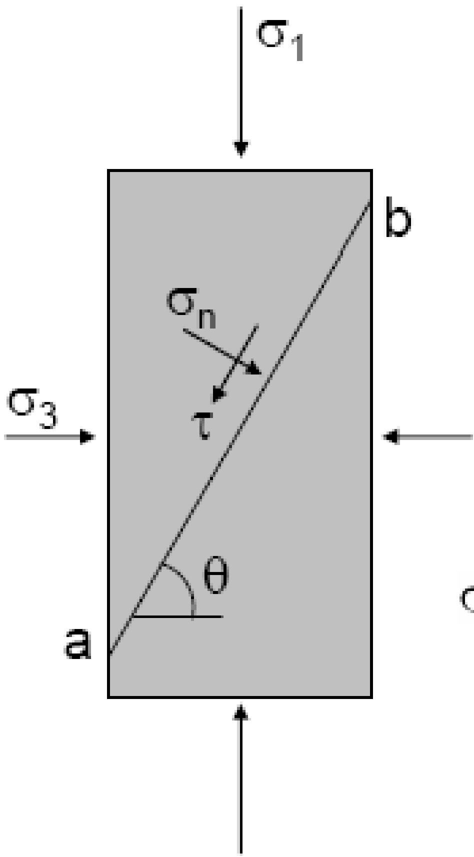
Failure Criteria of Rock Material

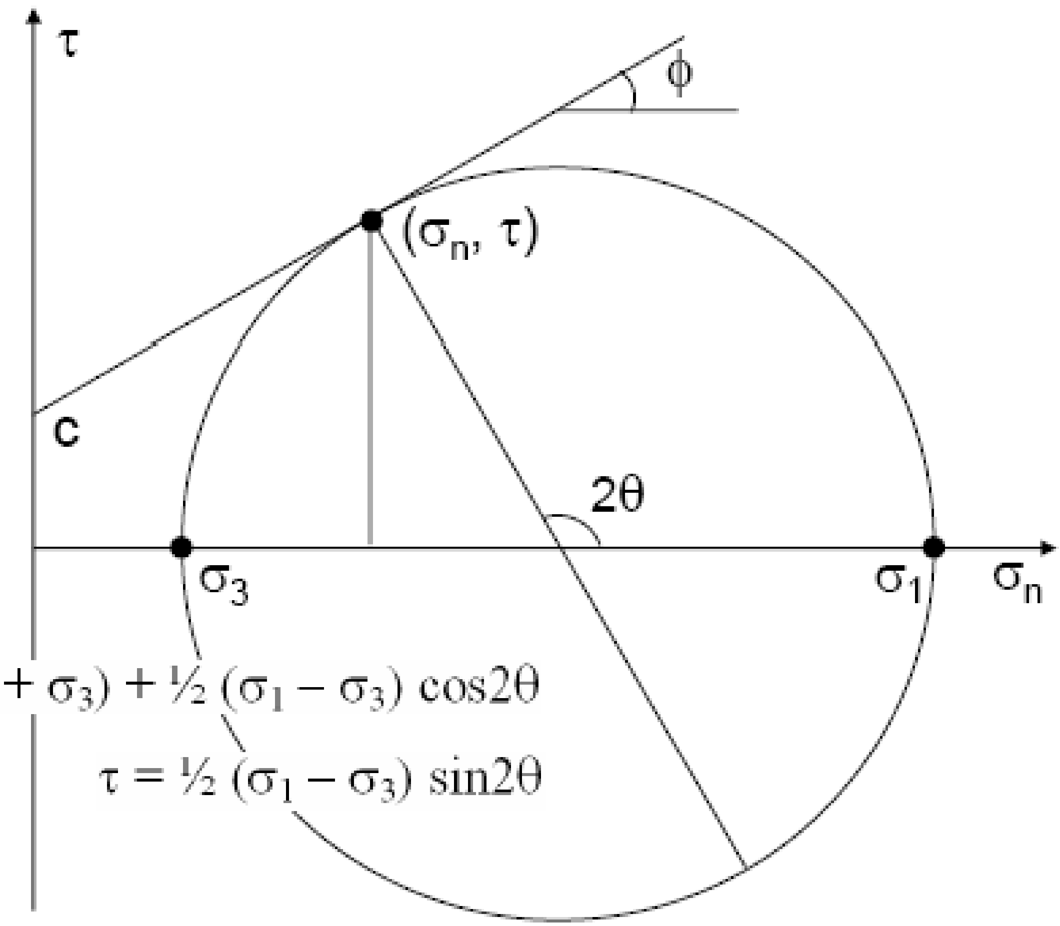
Mohr-Coulomb Criteria of Rock Material

Mohr-Coulomb criterion is a two-parametric criterion. It takes shearing into account. It considers the major and minor principal stresses only (the two principal stresses making the largest difference).

The criterion assumes that a shear failure plane is developed in the rock material. When failure occurs, the stresses developed on the failure plane are on the yield surface (envelope in 2D).

Failure Criteria of Rock Material





$$\sigma_n = \tfrac{1}{2}(\sigma_1 + \sigma_3) + \tfrac{1}{2}(\sigma_1 - \sigma_3)\cos 2\theta$$

$$\tau = \tfrac{1}{2}(\sigma_1 - \sigma_3)\sin 2\theta$$

Failure Criteria of Rock Material

Coulomb's shear strength is made up of two parts, a constant cohesion (c) and a normal stress (σ_n) dependent frictional component, angle of internal friction (φ),

$\tau = c + \sigma_n \tan\varphi$

It is a straight line, with an intercept c on the τ-axis and an angle of φ with the σ_n-axis.

Failure Criteria of Rock Material

From the Mohr's circle diagram,

$$\sigma_n = \tfrac{1}{2}(\sigma_1 + \sigma_3) + \tfrac{1}{2}(\sigma_1 - \sigma_3)\cos 2\theta$$

$$\tau = \tfrac{1}{2}(\sigma_1 - \sigma_3)\sin 2\theta$$

By combining the above two equations with
$\tau = c + \sigma_n \tan\varphi$,

$$\tfrac{1}{2}(\sigma_1 - \sigma_3)\sin 2\theta = c + [\tfrac{1}{2}(\sigma_1 + \sigma_3) + \tfrac{1}{2}(\sigma_1 - \sigma_3)\cos 2\theta]\tan\phi$$

$$\sigma_1 = \frac{2c + \sigma_3\,[\sin 2\theta + \tan\phi\,(1 - \cos 2\theta)]}{\sin 2\theta - \tan\phi\,(1 + \cos 2\theta)}$$

Failure Criteria of Rock Material

Rock fails with the formation of a shear failure plane a-b, i.e., the stress condition on the a-b plane satisfies the shear strength condition. In the diagram, when the Mohr circle touches the Mohr-Coulomb strength envelope, the stress condition on the a-b plane meets the strength criterion.

From the Mohr circle, the failure plane is defined by θ, and

$$\theta = \frac{1}{4}\pi + \frac{1}{2}\varphi$$

Failure Criteria of Rock Material

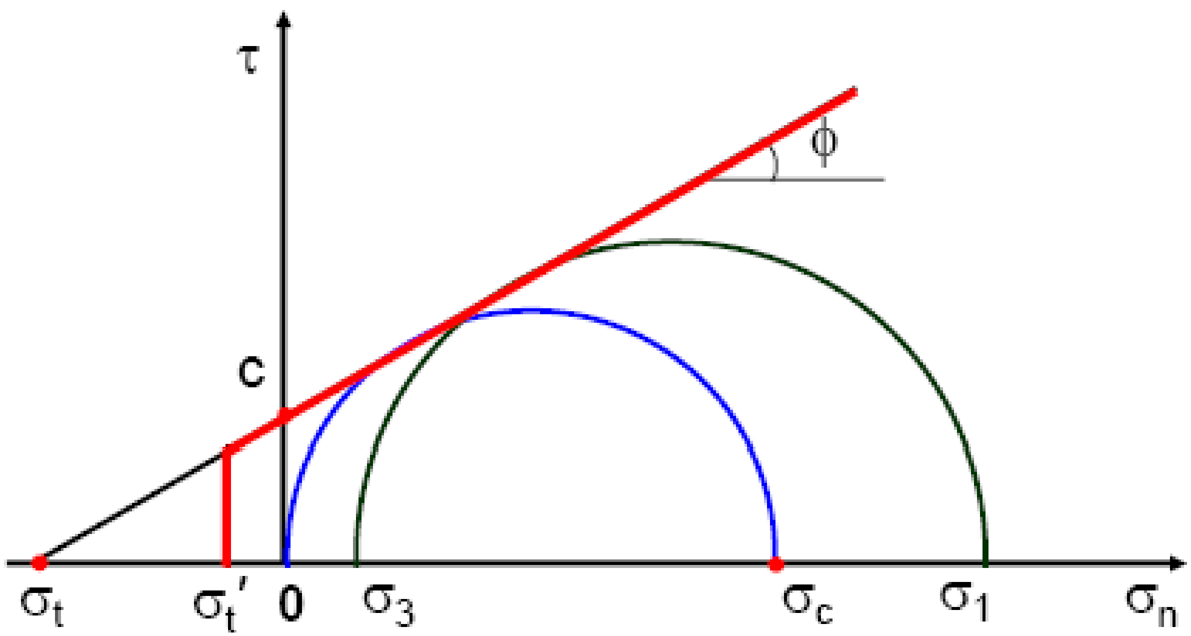
$$\sigma_1 = \frac{2c\cos\phi + \sigma_3 (1 + \sin\phi)]}{1 - \sin\phi}$$

$$\sigma_c = \frac{2\,c\,\cos\phi}{1 - \sin\phi}$$

$$\sigma_t = \frac{2\,c\,\cos\phi}{1 + \sin\phi}$$

Failure Criteria of Rock Material

Actual rock tensile strengths are lower than the criterion. A tensile cut-off is usually applied at a selected value of uniaxial tensile stress, σ_t', at about 1/10 σ_c.



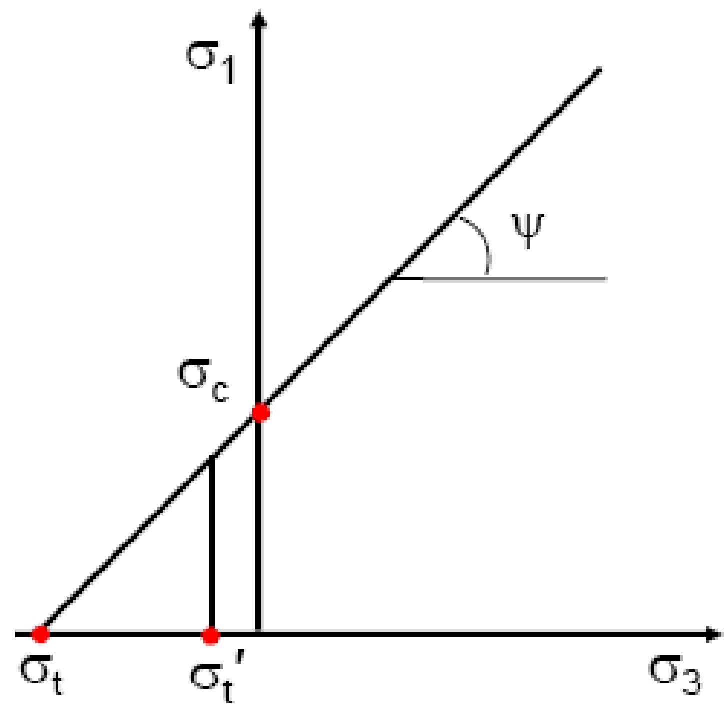
Failure Criteria of Rock Material

Mohr-Coulomb criterion can also be shown in σ_1–σ_3 plots.

$$\sigma_1 = \sigma_c + \sigma_3 \tan\psi$$

$$\tan\psi = \frac{1 + \sin\phi}{1 - \sin\phi}$$

$$\sigma_1 = \sigma_c + \sigma_3 \frac{1 + \sin\phi}{1 - \sin\phi}$$



Failure Criteria of Rock Material

Comment on the Mohr-Coulomb Criterion

The Mohr-Coulomb criterion is only suitable for the low range of confining stress. At high confining stress, it overestimates the strength. It also overestimates tensile strength. In most cases, rock engineering deals with shallow problems and low confining stress, so the criterion is widely used, due to its simplicity and popularity.

Failure Criteria of Rock Material

Griffith Strength Criterion

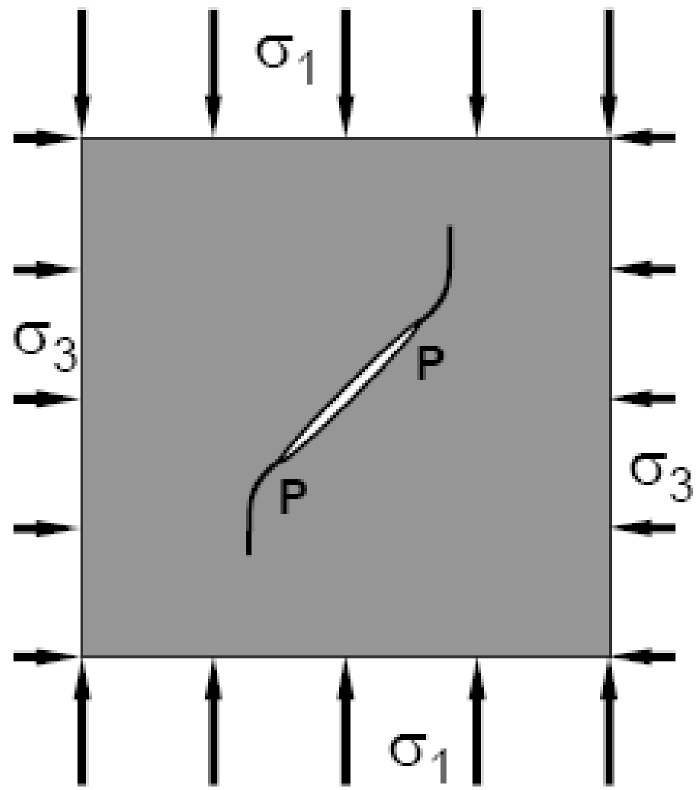
Griffith criterion is based on mechanics of brittle fracture, using elastic strain energy concepts. It describes the behaviour of crack propagation of an elliptical nature by considering the energy involved. The equation basically states that when a crack is able to propagate enough to fracture a material, that the gain in the surface energy is equal to the loss of strain energy.

Failure Criteria of Rock Material

Under compression, elliptical crack will propagate from the points of maximum tensile stress concentration. it gives the following criterion for crack extension:

$(\sigma_1 - \sigma_3)^2 - 8\,\sigma_t\,(\sigma_1 + \sigma_3) = 0$
if $\sigma_1 + 3\,\sigma_3 > 0$

$\sigma_1 + \sigma_t = 0$
if $\sigma_1 + 3\,\sigma_3 < 0$



Failure Criteria of Rock Material

When $\sigma_3 = 0$, $\sigma_1 - 8\sigma_t = 0$ or $\sigma_1 = 8\sigma_t$

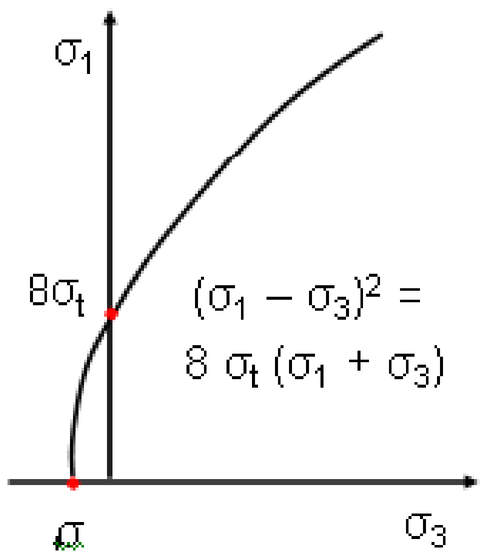
Uniaxial compressive stress at crack extension is always 8 times the uniaxial tensile strength.

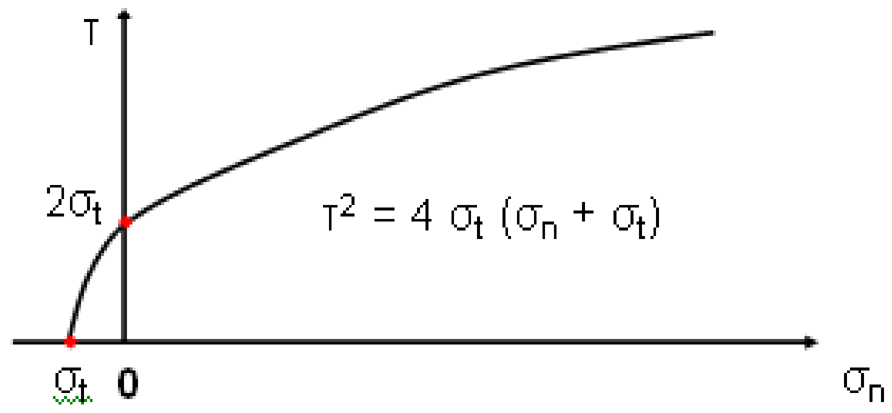
It can also be expressed in terms of the shear stress (τ) and normal stress (σ_n) acting on the plane containing the major axis of the crack,

$\tau^2 = 4\sigma_t(\sigma_n + \sigma_t)$

When $\sigma_n = 0$, $\tau = 2\sigma_t$, it represents the cohesion.

Failure Criteria of Rock Material





Failure Criteria of Rock Material

Comment on the Griffith Strength Criterion

The plane compression Griffith theory did not provide a very good model for the peak strength of rock under multiaxial compression. It gives only good estimate of tensile strength, and under-estimate compressive strength, particularly at high lateral stresses. A number of modifications to Griffith's solution were introduced, but they are not in practical use today.

Failure Criteria of Rock Material

Hoek-Brown Strength Criterion

The classic strength theories have been found not to apply to rock materials over a wide range of applied compressive stress conditions, a number of empirical strength criteria have been introduced for practical use. One of the most widely used criteria is the Hoek-Brown criterion for isotropic rock materials and rock masses.

Failure Criteria of Rock Material

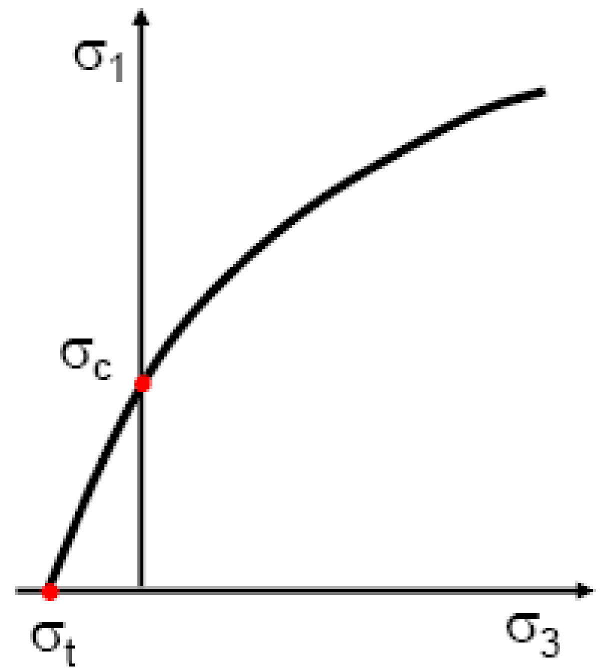
Hoek and Brown criterion is expressed by the equation:

$$\sigma_1/\sigma_c = \sigma_3/\sigma_c + (m\ \sigma_3/\sigma_c + 1.0)^{1/2}$$

or

$$\sigma_1 = \sigma_3 + (m\ \sigma_3\ \sigma_c + \sigma_c^2)^{1/2}$$

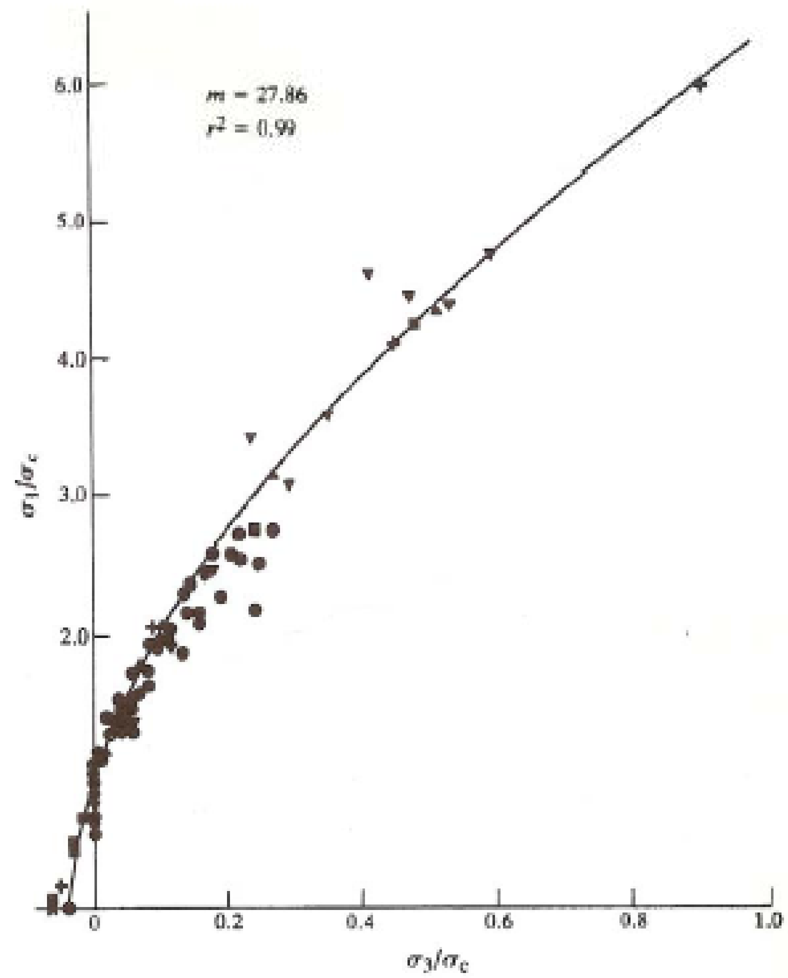
m is a parameter that changes with rock type.

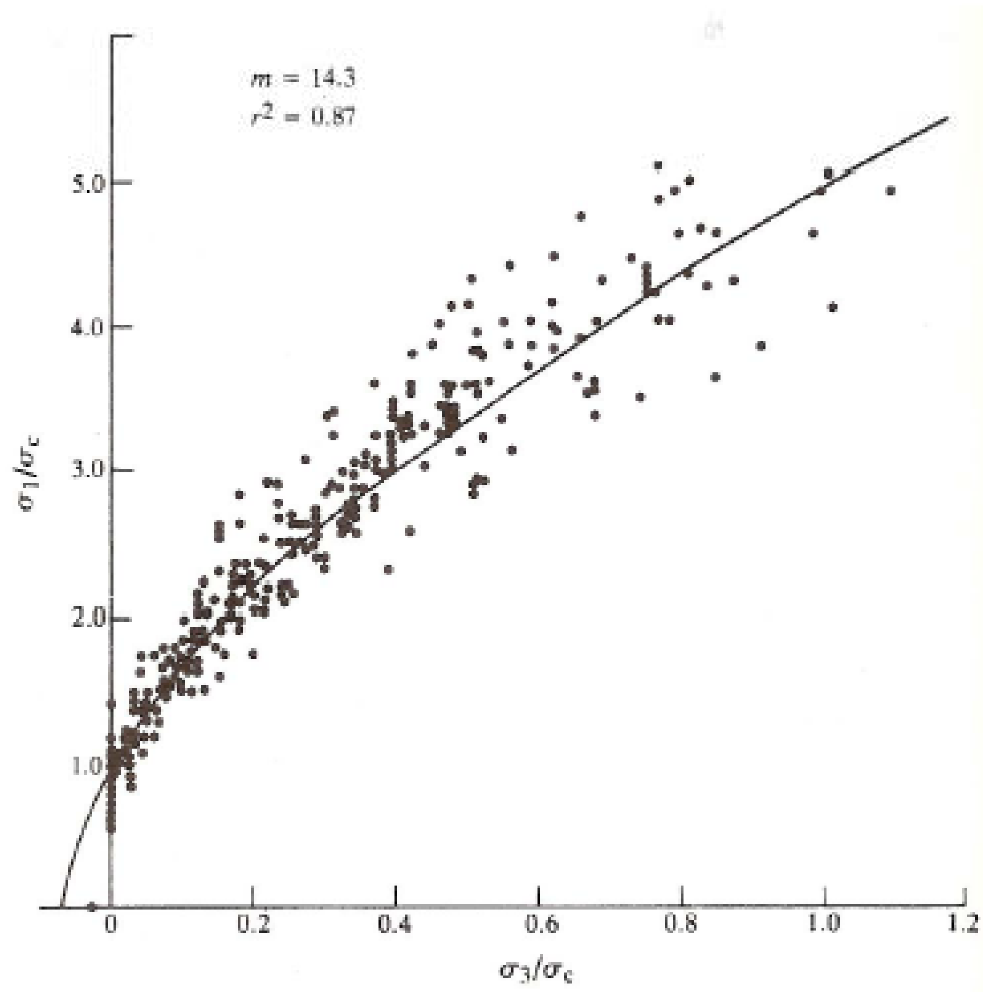


Failure Criteria of Rock Material

(a) $m \approx 7$ for carbonate rocks with well developed crystal cleavage (dolomite, limestone, marble);
(b) $m \approx 10$ for lithified fine grain sedimentary and low grade matemorphic rocks (mudstone, siltstone, shale, slate);
(c) $m \approx 15$ for coarse sedimentary rocks of strong crystals and poorly developed crystal cleavage (sandstone, quartzite);
(d) $m \approx 17$ for fine-grained igneous crystalline rocks (andesite, dolerite, diabase, rhyolite, basalt);
(e) $m \approx 25$ for coarse-grained igneous and metamorphic rocks (gabbro, diorite, granite, gneiss).

Failure Criteria of Rock Material





Failure Criteria of Rock Material

Comment on the Hoek-Brown Strength Criterion

The Hoek-Brown strength envelope is not a straight line. It is a curve. At high stress level, the envelope curves down, so it gives low strength estimate than the Mohr-Coulomb envelope.

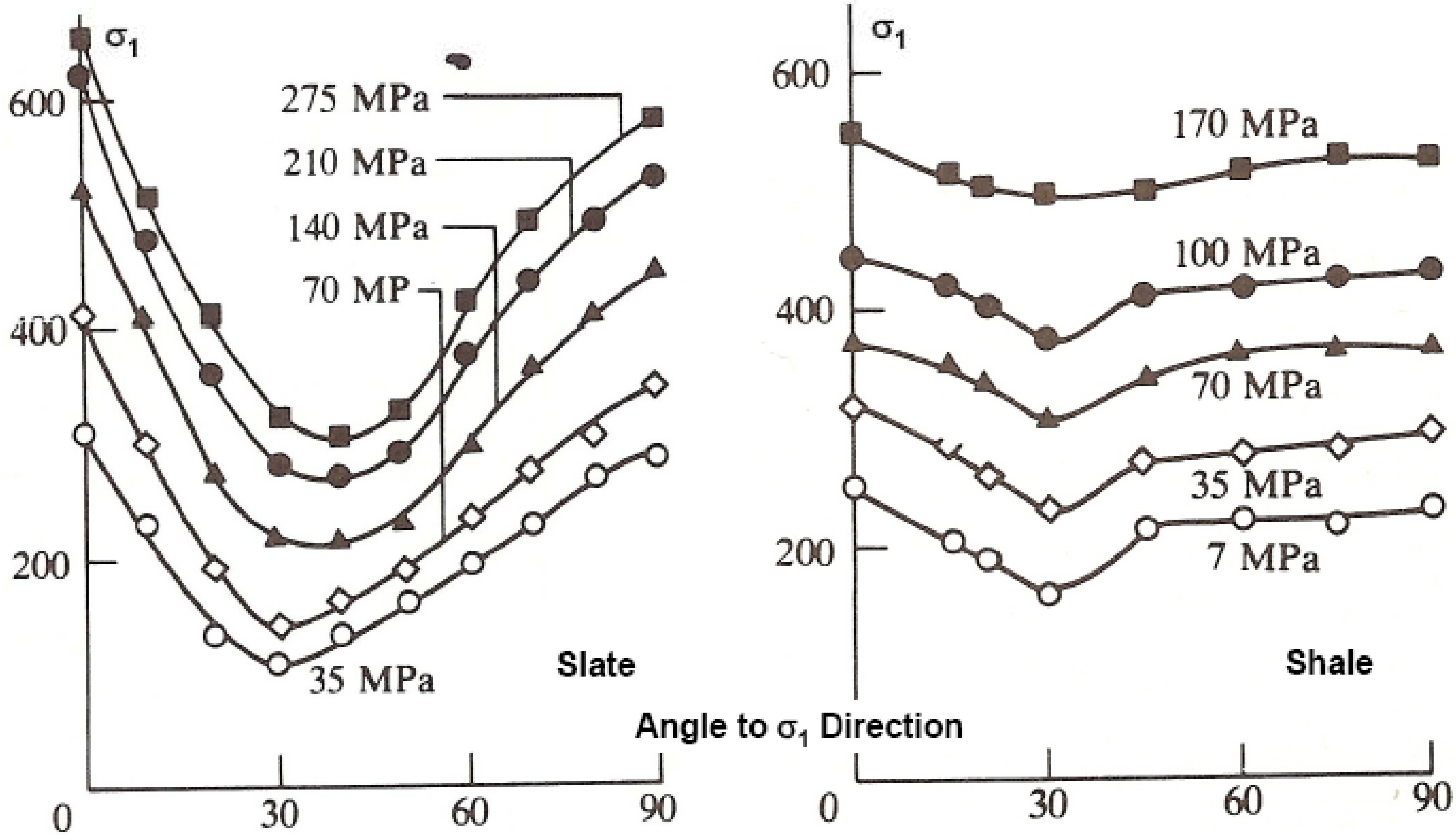
It is an empirical criterion based on substantial test results on various rocks. It is very easy to use and select parameters. It is also extended to rock masses. It is widely used in rock mechanics and engineering design.

Effects of Rock Microstructures

Strength of Anisotropic Rock Material

Rocks, such as shale and slate, is transverse isotropy.

Peak strengths developed by transversely isotropic rocks in compression vary with the orientation of the plane of isotropy, with respect to the principal stress directions



σ1
600
400
200
0
30
60
90
275 MPa
210 MPa
140 MPa
70 MP
35 MPa
Slate
σ1
600
400
200
0
30
60
90
170 MPa
100 MPa
70 MPa
35 MPa
7 MPa
Shale
Angle to σ1 Direction

Effects of Rock Microstructures

Analytical solutions

$$(\sigma_1 - \sigma_3) = \frac{2\,(c_w + \sigma_3 \tan\phi_w)}{(1 - \tan\phi_w \cot\beta)\sin 2\beta}$$

c_w = cohesion of the plane of weakness; φ_w = angle of friction of the plane; β= inclination of the plane.

Mminimum strength occurs when

$\tan 2\beta = -\cot\varphi_w$, or $\beta = 45 + \frac{1}{2}\varphi_w$, and it gives,

$(\sigma_1 - \sigma_3)_{min} = 2(c_w + \sigma_3 \tan\varphi_w)\,[(1 + \tan^2\varphi_w)^{1/2} + \tan\varphi_w]$

Effects of Rock Microstructures

Strength of Anisotropic Rock Material

When the weakness plane is at an angle of $(45+\frac{1}{2}\varphi_w)$, the strength is the lowest. For rocks, φ_w is about 30 to 50°, hence β is about 60 to 70°. In triaxial compression tests, intact rock specimens generally fails to form a shear plane at an angle about 60 to 70°. When the rock containing an existing weakness plane that is about to become a failure plane, the rock has the lowest strength.

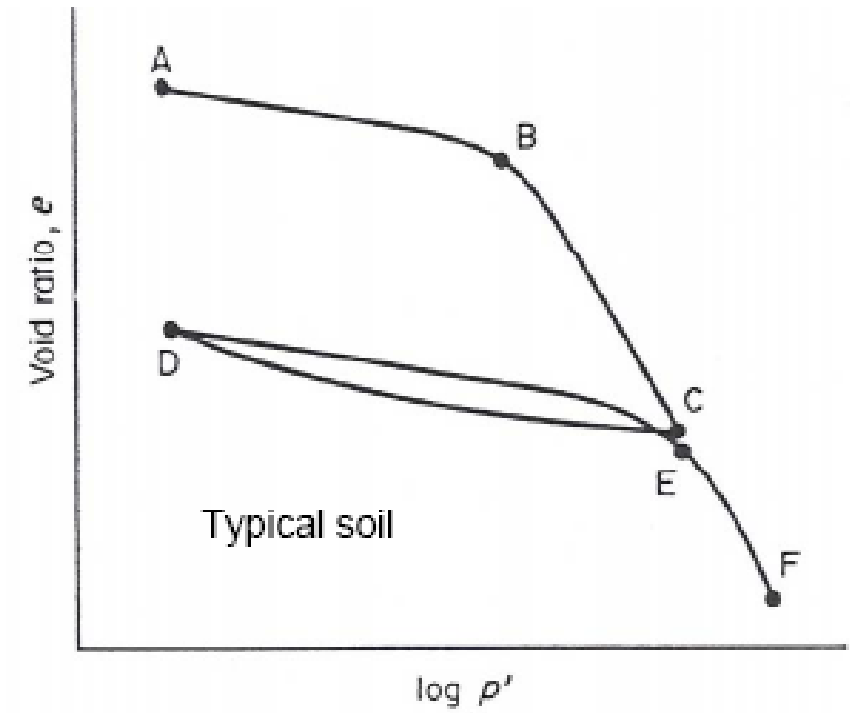
Effects of Rock Microstructures

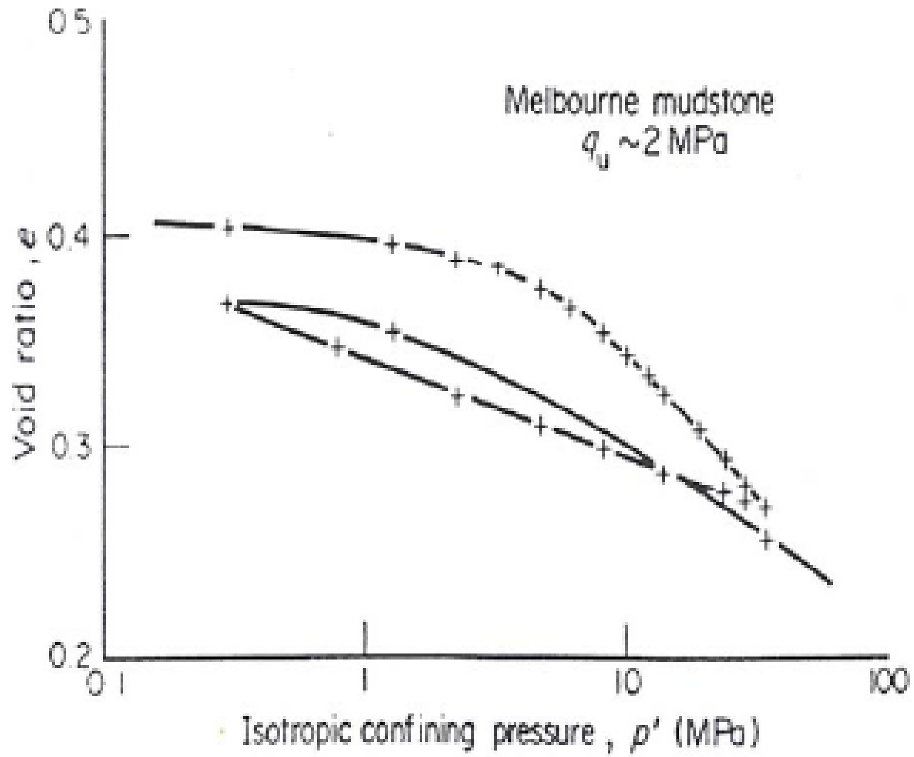
Mechanical Behaviour of Soft and Weathered Rocks

Rocks have uniaxial compressive strength below 25 MPa is considered as soft rocks or weak rocks. Soft rocks are usually sediments in the process of consolidation and solidification. Weathered rocks can also be soft or weak rocks.

Soft rocks may have the isotropic compression response similar in detail to the response of consolidated soils

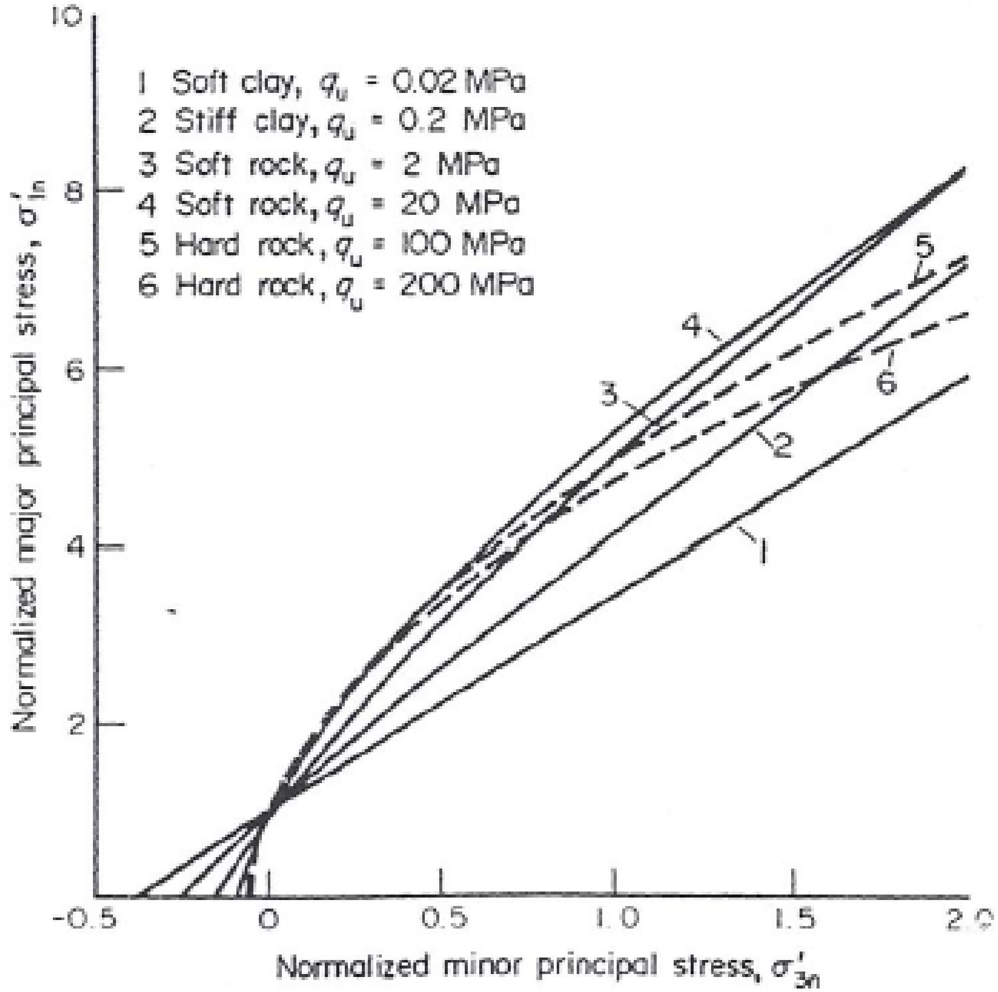
Effects of Rock Microstructures





Effects of Rock Microstructures

Strength character of soft rocks is between soil and hard rock. Soil strength is generally represented by the Mohr-Coulomb criterion (straight line), while for hard rocks, it is by the Hoek-Brown criterion (curve).



Effects of Rock Microstructures

With increasing rock strength, characteristics progressively changes from linear M-C criterion for soil to curved H-B criterion for hard rocks,

$$\sigma_1/\sigma_c = \{[\sigma_3 (1 + \sin\phi)] / [B \sigma_c (1 - \sin\phi)] + 1\}^B$$

$1.0 \geq B \geq 0.5$. B approaches to 1.0 for soil, the equation becomes the M-C criterion.

B approaches to 0.5 for rock, the equation becomes a parabolic envelope similar to the H-B criterion.

Laboratory Tests of Rock Material

Density, Porosity and Water Content

Uniaxial Compression Test

Triaxial Compression Test

Brazilian Tensile Test

Point Load Test

Ultrasonic Wave Velocity Measurement

Schmidt Hammer Rebound Hardness

Cerchar Abrasivity Test

Slake Durability Test

Unconfined Swelling Test

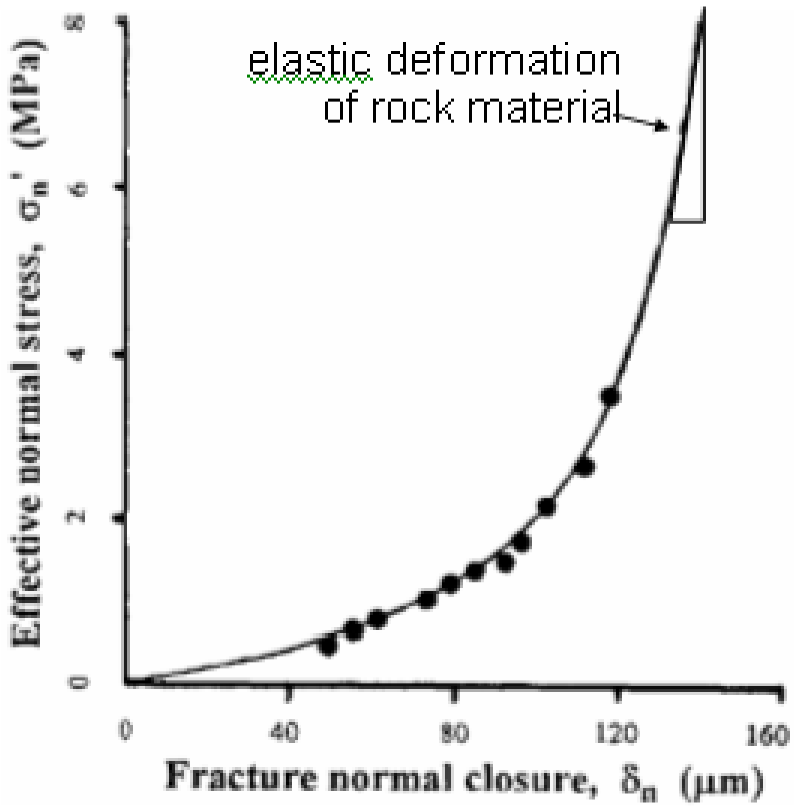
Mechanical and Hydraulic Properties

Normal Stiffness and Displacement

A natural joint always has opening aperture of less than 1 mm to a few mm. With increasing normal stresses, the opening closes, and contact areas of the joint surfaces increase. The normal stress – normal displacement curve is non-linear. The normal stiffness, slope of the curve, is therefore not a constant.

When the joint is completely closed, displacement is then only by the deformation of the rock material.

Mechanical and Hydraulic Properties



Stress-deformation curve of a natural joint in a granite, showing non-linear characteristics of joint stiffness.

At high normal stress, joint is closed, the normal stiffness approaches that of rock material. When the joint is completely closed, there is no further closure of the joint, the displacement is then only by the elastic deformation of the rock material.

Mechanical and Hydraulic Properties

Normal Stiffness and Displacement Equations

Hyperbolic Equation (Goodman)

$$\frac{\sigma_n - \sigma_{ni}}{\sigma_{ni}} = A\left(\frac{d_n}{d_{max} - d_n}\right)^t$$

σ_n = normal stress, d_n = normal displacement, d_{max} = maximum possible closure, σ_{ni} = an initial seating pressure, A, t = experimentally determined constants.

Mechanical and Hydraulic Properties

Normal Stiffness and Displacement Equations

Hyperbolic Equation for Matched Natural Joints (Barton-Bandis)

$$\sigma_n = \frac{k_{ni}\, d_n}{1 - (d_n / d_{max})} \qquad d_n = \frac{\sigma_n}{k_{ni} + (\sigma_n / d_{max})}$$

σ_n = normal stress, d_n = normal displacement, d_{max} = maximum possible closure, k_{ni} = normal stiffness of the fracture at initial seat stress.

Mechanical and Hydraulic Properties

Normal Stiffness and Displacement Equations

Semi-Logarithmic Equation for Mismatched Joints (Barton-Bandis)

$$\log \sigma_n = p + q\, d_n$$

σ_n = normal stress, d_n = normal displacement, p and q are material constants.

Mismatched rock fractures exhibit much lower normal stiffness than matched fractures.

Mechanical and Hydraulic Properties

Normal Stiffness and Displacement Equations

Logarithmic Equation for Natural Joints (Zhao-Brown)

$$\frac{d_{max} - d_n}{d_{max} - d_{ni}} = 1 - A\ \ln(\sigma_n/\sigma_{ni})$$

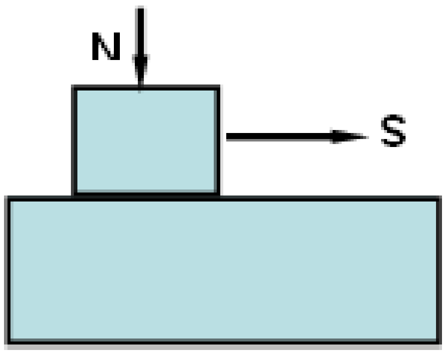
d_{ni} = displacement at a reference normal stress σ_{ni}, usually equal to the seating pressure, A = a constant varying from 0.16 to 0.21.

Mechanical and Hydraulic Properties

Shear and Friction between Contact Planes

The most common shear phenomenon of a discontinuity is the sliding between two contact surfaces. The friction theory gives the relationship between the friction angle φ,

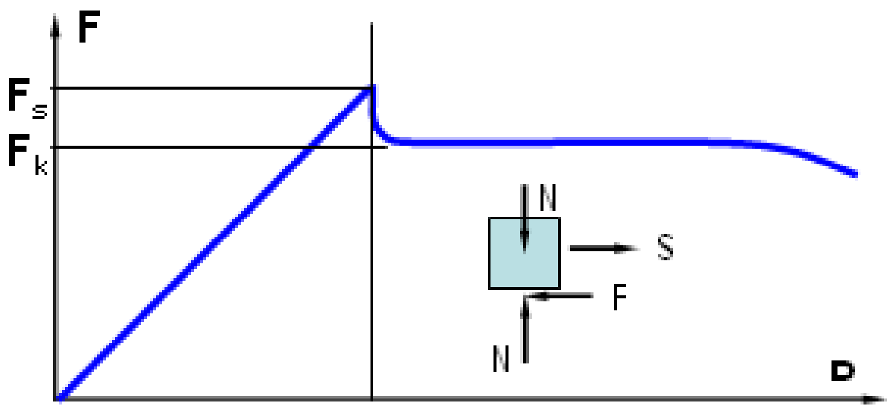
the normal force (N) and shear force (S), as



$$S = N \tan\varphi$$

Mechanical and Hydraulic Properties

When slipping at the surface of contact is about to occur, the maximum static frictional force is proportional to the normal force. When slipping is occurring, the kinetic frictional force is proportional to the normal force.



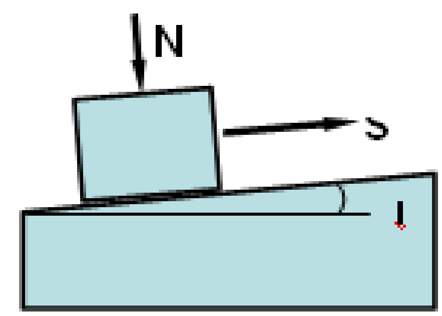
Mechanical and Hydraulic Properties

If the contact surface is at an inclined up angle (i), from the force diagram, along the sliding direction, the normal force is N cos(i) + S sin(i), the shear force is S cos(i) – N sin(i). By friction theory,

$$S\cos(i) - N\sin(i) = [\,N\cos(i) + S\sin(i)\,]\tan\varphi,$$

$$S - N\tan(i) = N\tan\varphi + S\tan\varphi\tan(i),$$

$$S = \frac{N\,[\,\tan\varphi + \tan(i)\,]}{1 - \tan\varphi.\tan(i)}$$

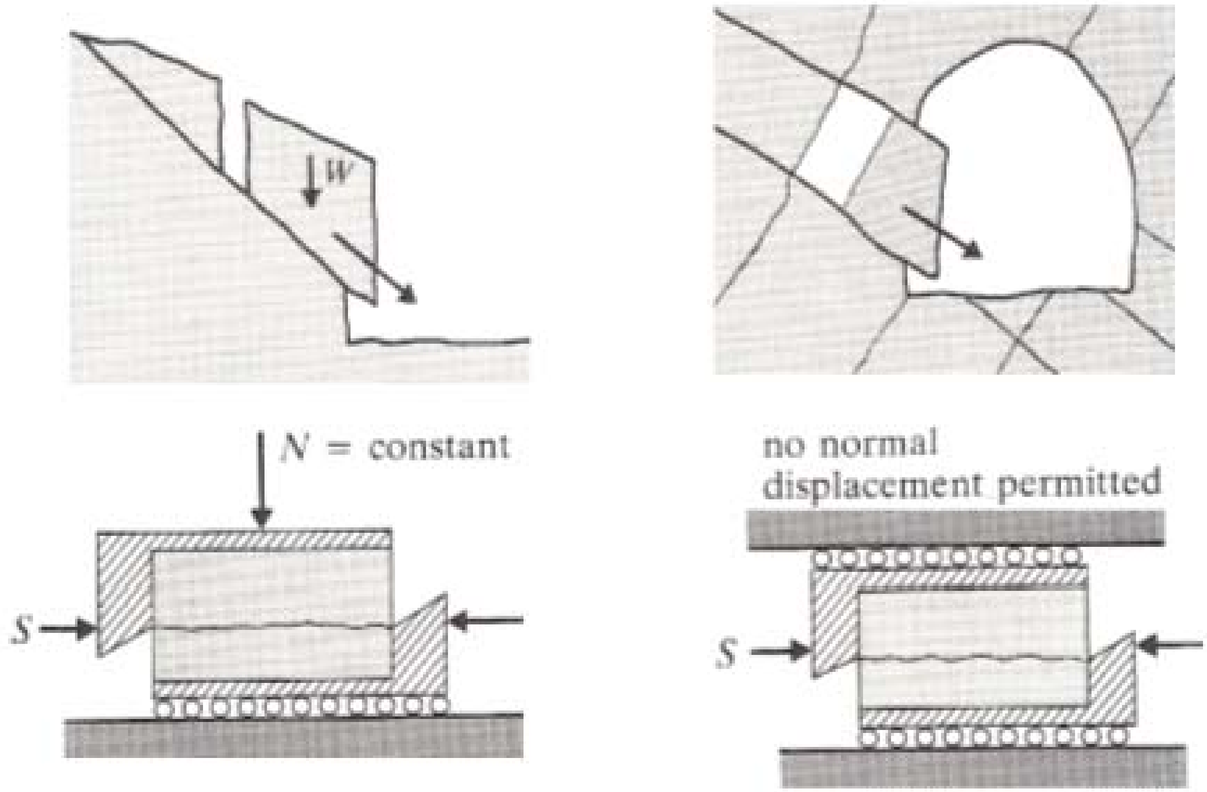


$$S = N\tan(\varphi + i)$$

Mechanical and Hydraulic Properties

Shear Strength of Rock Joints

Shear behaviour of rock joints is perhaps one of most important feature in civil engineering rock mechanics. Conditions for sliding of rock blocks along existing joints and faults at slope or excavation opening are governed by the shear strengths developed on the sliding rock discontinuities.



In slope, shear is subjected to a constant normal load generated by the weight of the blocks. In tunnel, shear is subjected to constant stiffness due to the constraints of lateral displacement.

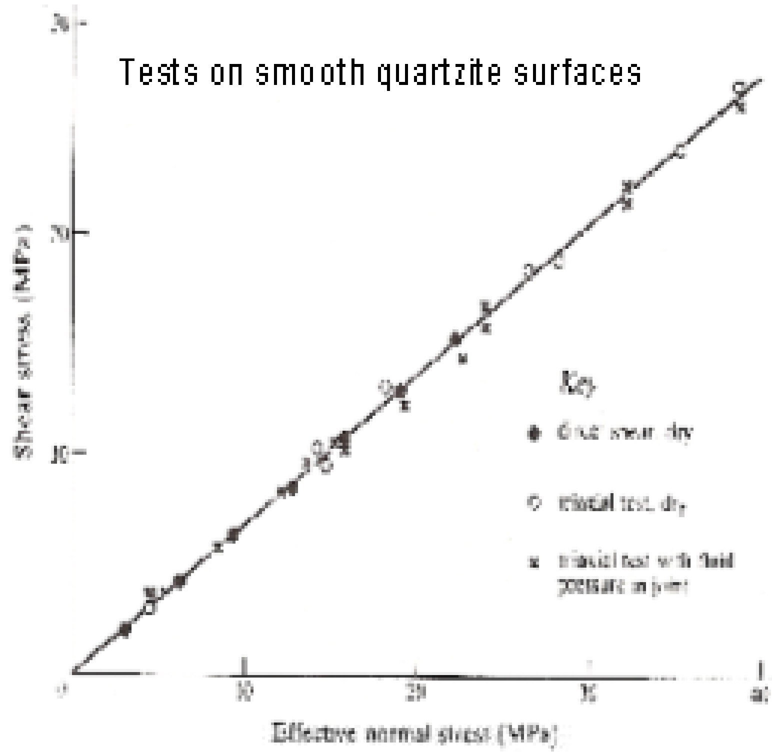
Mechanical and Hydraulic Properties

Shear Strength of Smooth Joints

Shear of smooth joint surface follows the friction theory, i.e.,

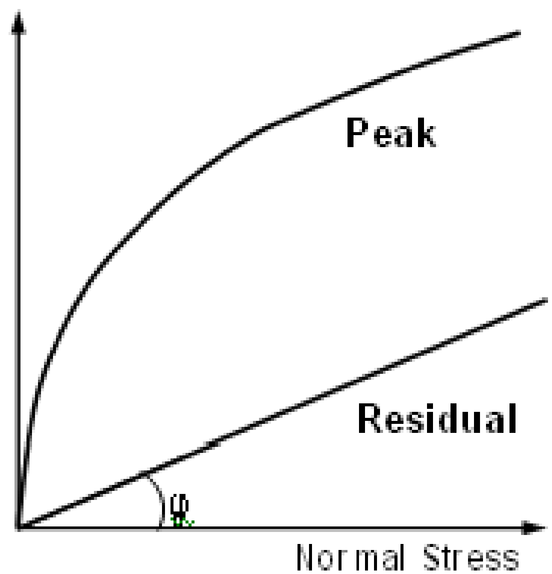
$$\tau = \sigma_n \tan \varphi_b$$

φ_b is often called as basic frictional angle. For most rocks, it is about 25~35° .



Mechanical and Hydraulic Properties

Shear Strength of Rough Joints



Mechanical and Hydraulic Properties

Shear Strength of Rough Joints

In tests, shear stress quickly reaches a peak (peak strength). With shearing progressed, shear stress stabilises to a residual level (residual strength). For rough joints, peak shears strength is significantly higher than the residual strength.

Residual strength follows the linear friction law, i.e., $\tau_r = \sigma_n \tan \varphi_r$. For most rocks, φ_r is about 25~35°, about the same as φ_b.

Mechanical and Hydraulic Properties

Shear Strength of Rough Joints

Peak shear strength does not follow the linear fiction law. Gradient of the peak shear strength – normal stress decreases with increasing normal stress.

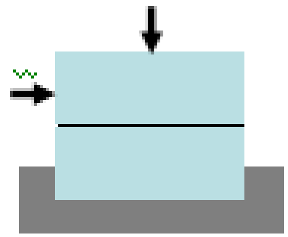
At low normal stress, friction angle for rough joints can be as high as 70°, the frictional angle decreases with increasing normal stress. At high normal stress, the frictional angle approaches to φ_b.

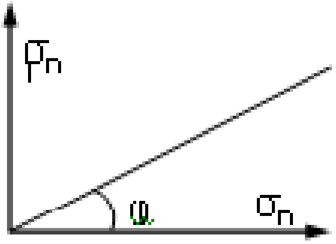
Mechanical and Hydraulic Properties

Peak Shear Strength Models

Linear Friction Model – shear on horizontal smooth plane

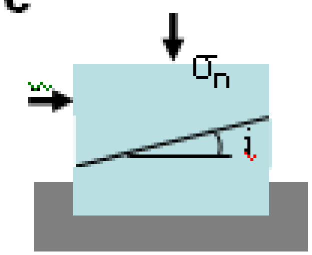
$$\tau = \sigma_n \tan \varphi$$

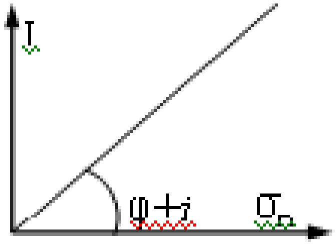




Linear Friction Model with Dilation Angle– shear on up inclined smooth plane

$$\tau = \sigma_n \tan (\varphi + i)$$





Mechanical and Hydraulic Properties

Bi-Linear Shear Strength Model

When the normal stress is increased above a critical value, shear stress can eventually be developed so high that it causes shear failure through the asperities. When such shearing through asperity occurs, the shear strength is somehow related to the shear strength of the materials of the asperities. Rock materials have higher cohesion and internal friction angle of generally around 30°.

Mechanical and Hydraulic Properties

Bi-Linear Shear Strength Model

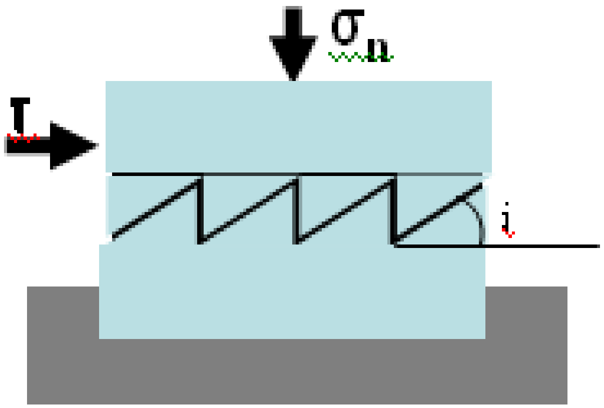
Therefore, shear strength for a rough fracture could exhibit two features, at low normal stress shearing by climbing the asperity angle, and at high stress shearing off the asperities. This leads to a bilinear shear strength model.

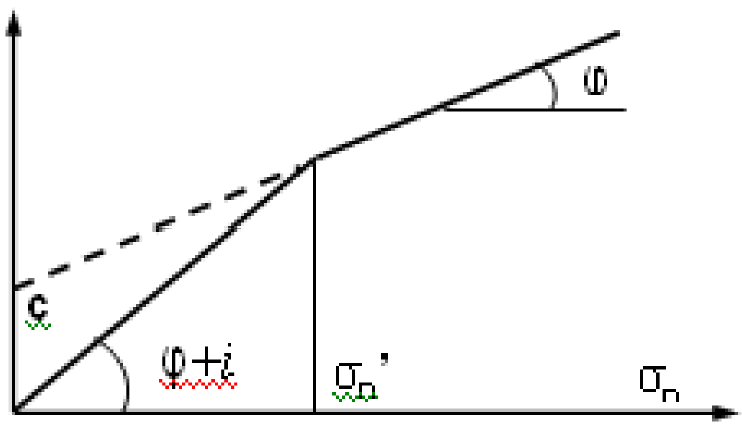
$$\tau = \sigma_n \tan(\varphi + i) \qquad \text{for } \sigma_n \leq \sigma_n'$$

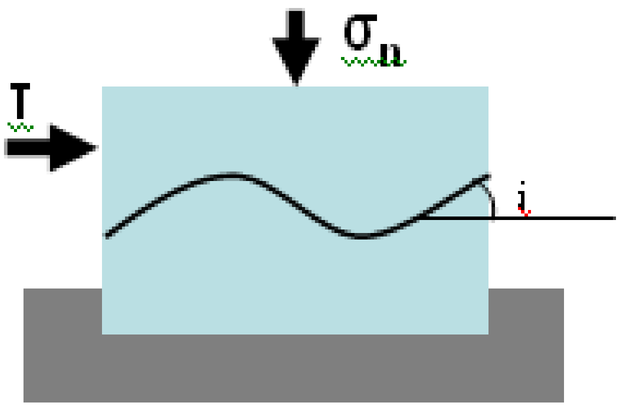
$$\tau = c + \sigma_n \tan\varphi \qquad \text{for } \sigma_n \geq \sigma_n'$$

σ_n' is the critical normal stress when shearing of asperity is assumed to start

Mechanical and Hydraulic Properties







Bi-Linear Shear Strength Model

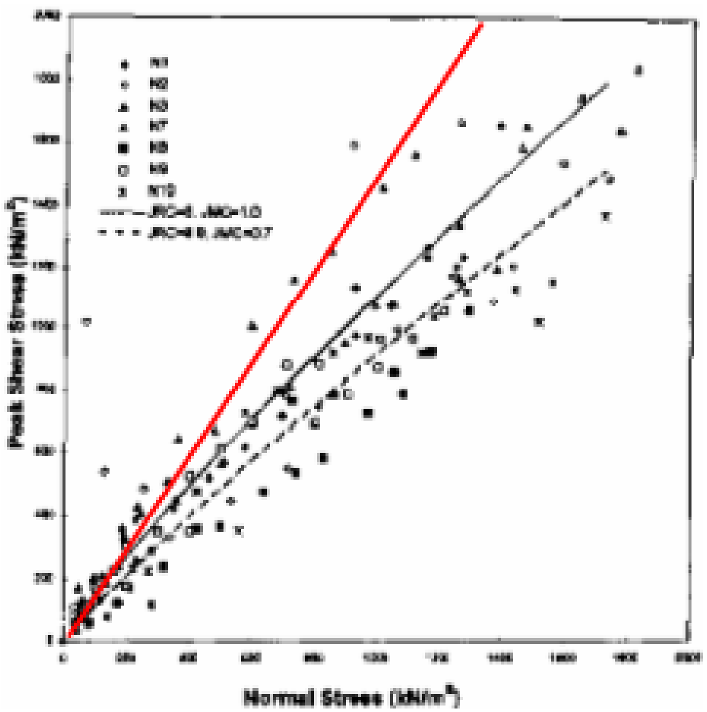
$\tau = \sigma_n \tan(\varphi + i)$ when $\sigma_n \leq \sigma_n'$

$\tau = c + \sigma_n \tan \varphi$ when $\sigma_n \geq \sigma_n'$

Mechanical and Hydraulic Properties

JRC-JCS Empirical Shear Strength Model

In reality, there is no clear boundary between shearing by climbing asperity and shearing off asperities. With increasing normal stress, asperity shearing off increases progressively. Therefore, the actual shear stress – normal stress relation is represented by a curve.



Mechanical and Hydraulic Properties

JRC-JCS Empirical Shear Strength Model

Based on extensive test results and noticing the progressive damage of asperities, Barton (1973) proposed that the peak shear strengths of joints could be represented by the empirical relation,

$$\tau = \sigma_n \tan [JRC \log_{10}(JCS/\sigma_n) + \varphi_r]$$

σ_n = effective normal stress, JRC = joint roughness coefficient, JCS = joint wall compressive strength, and φ_r = drained residual friction angle.

Mechanical and Hydraulic Properties

Comments on JRC-JCS Shear Strength Model

It has the basic form of friction law.

High JRC gives high dilation angle (i).

Higher JCS delays the reduction of i, since strong rock has less shearing off.

When σ_n is high approaching to JCS, the equation becomes the basic friction equation.

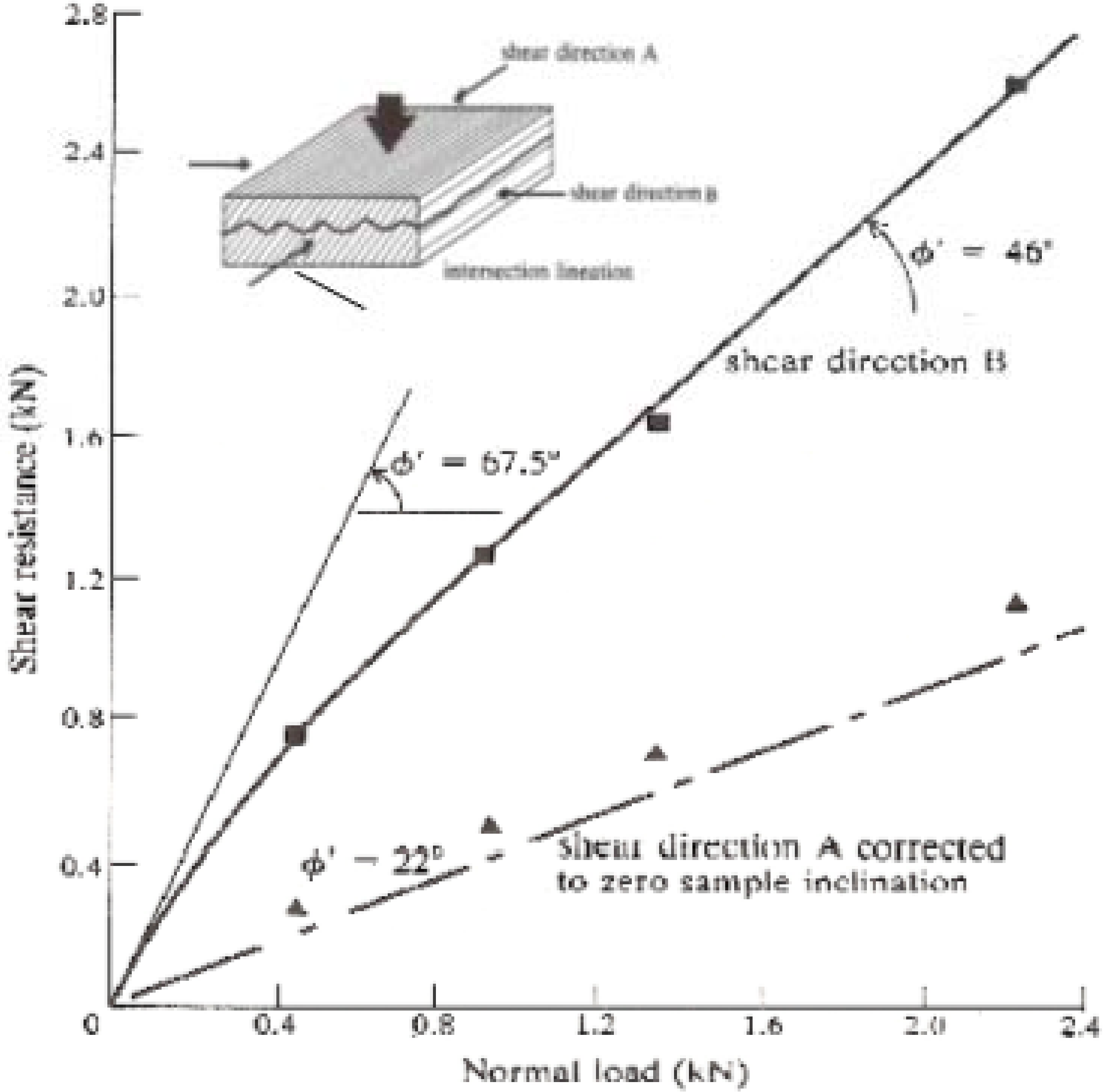
It is widely accepted and used in rock engineering.

Mechanical and Hydraulic Properties

Factors affects Rock Joint Shear Strength

Direction

Joint surface profile is a 3D feature while shearing is a directional activity. Surface profile along a particular direction would be different along another direction and hence gives different shear strength.



Effect of shearing direction on shear strength of a joint in slate

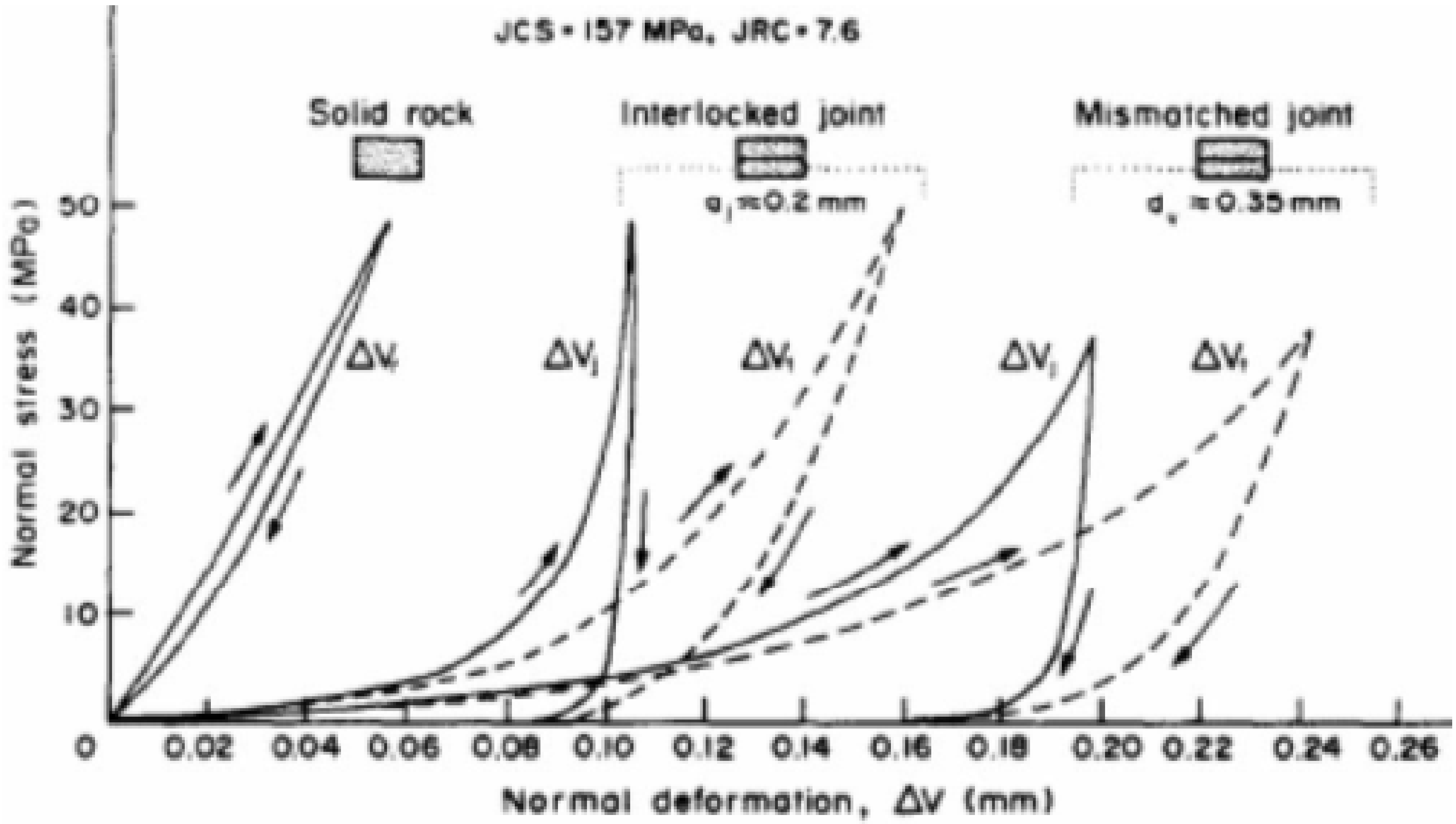
Mechanical and Hydraulic Properties

Factors affects Rock Joint Properties

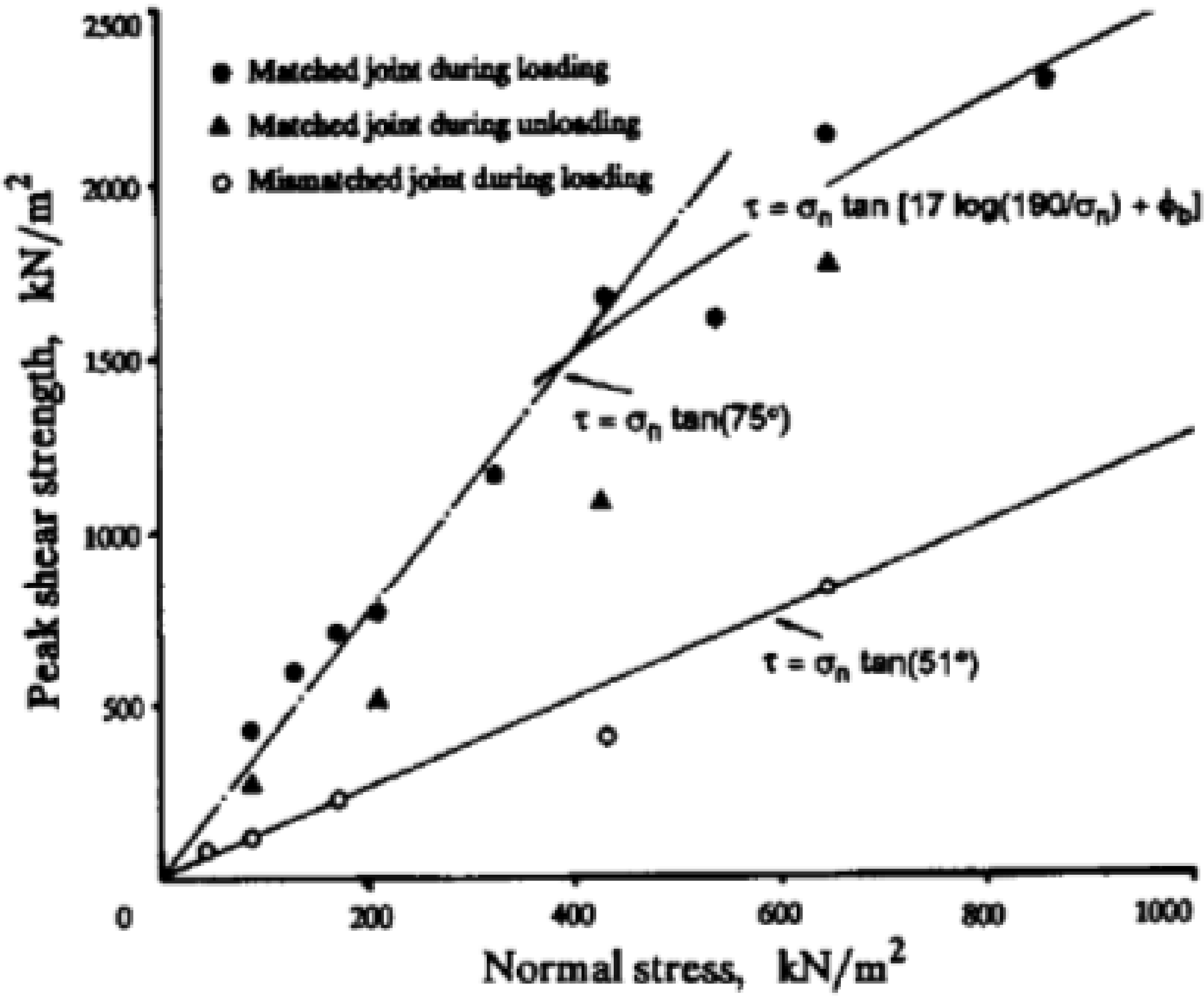
Matching and Mismatching

Natural joints suffer from weathering and alteration. It changes the degree of matching of joint surfaces. Mismatched joint generally have much lower shear strength than matched joints.

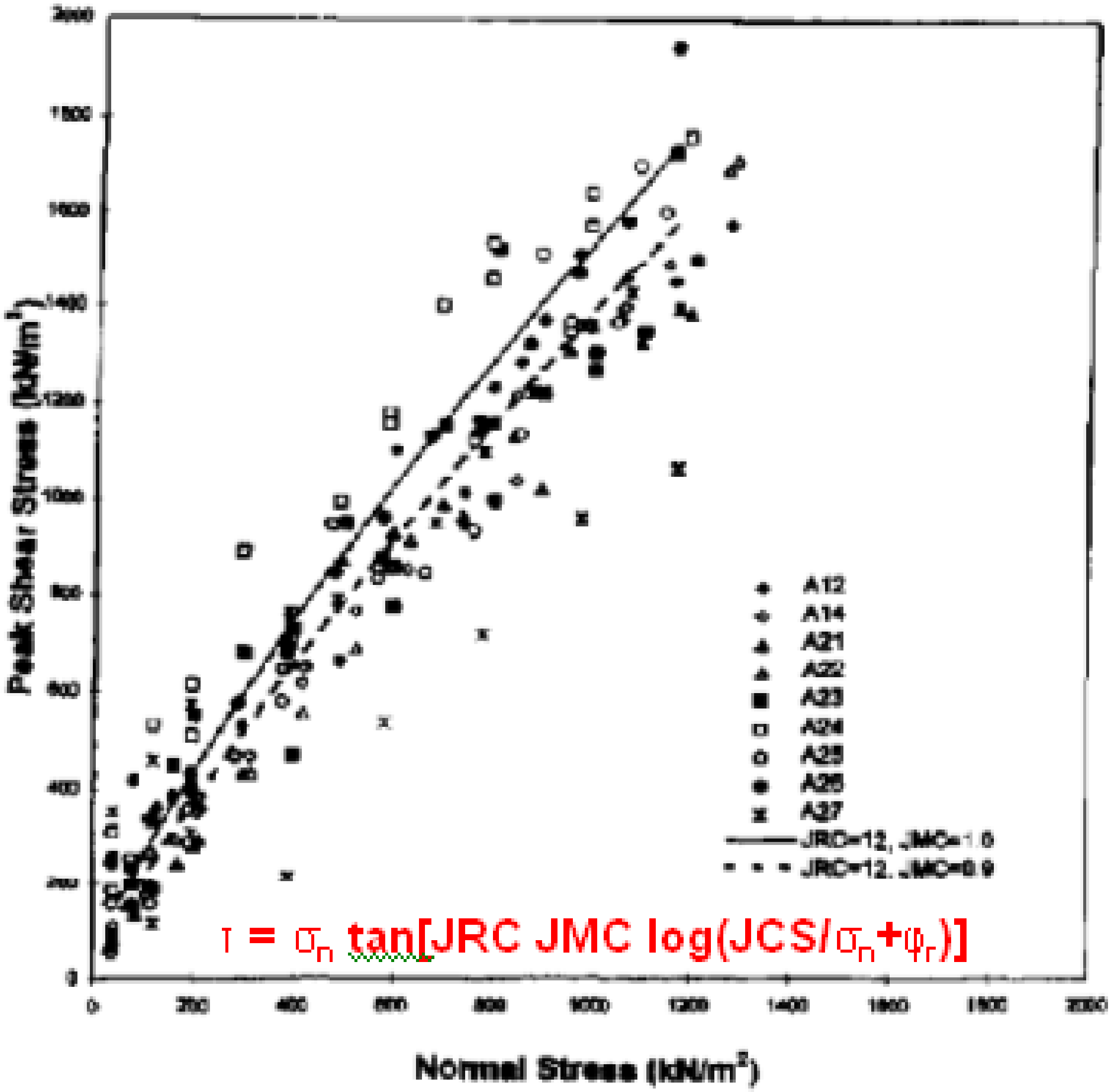
It is also evident from cyclic shear tests. Later cycle shear tests give lower strength than the first cycle – due to joint surface damage and reduced matching.



Normal deformation of matched and mismatched joints

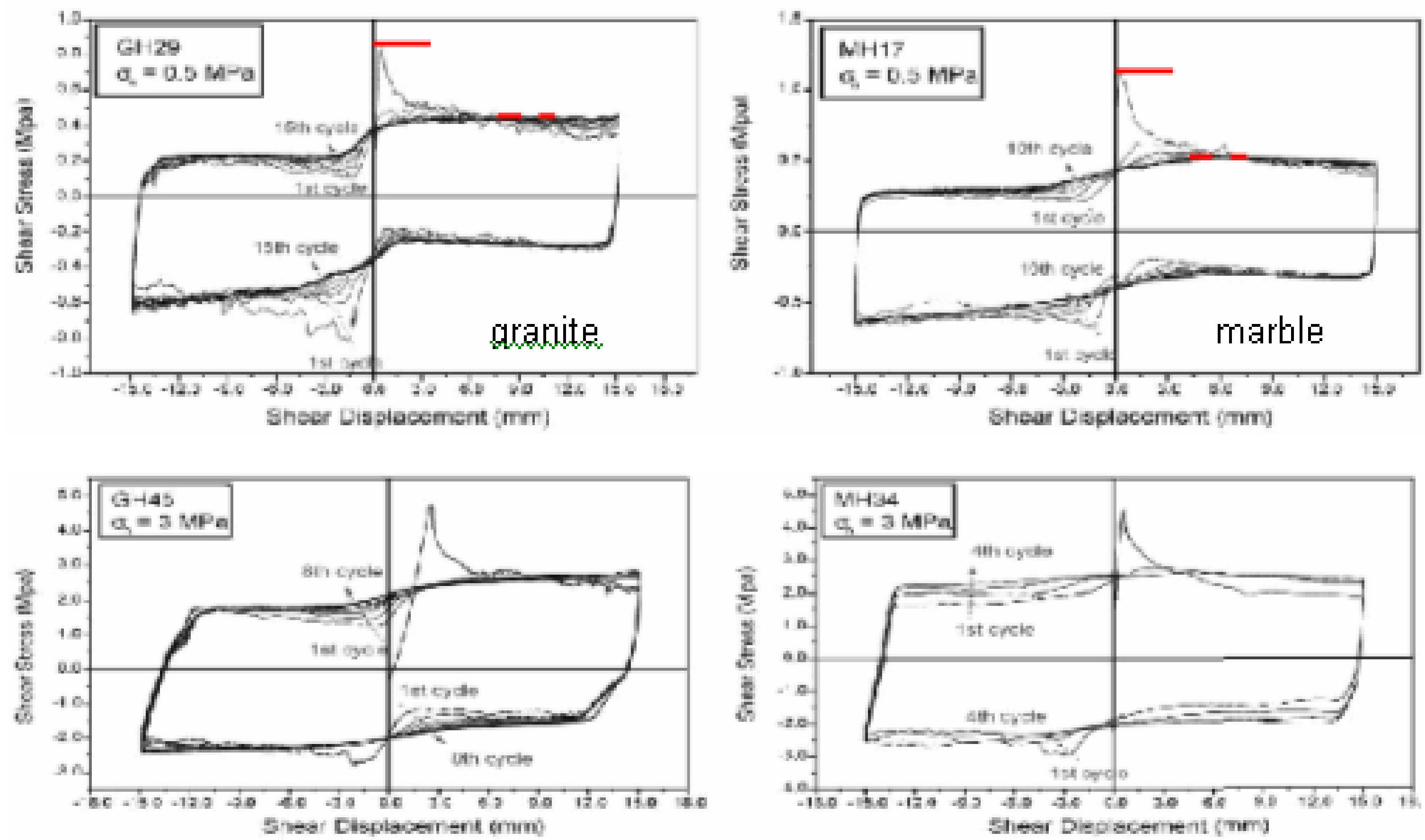


Shear strength of matched and mismatched joints in granite



$$\tau = \sigma_n \tan[JRC\ JMC \log(JCS/\sigma_n + \phi_r)]$$

Peak shear strength of joints in cyclic tests



Peak shear strength of joints in cyclic tests

Mechanical and Hydraulic Properties

Factors affecting Rock Joint Shear Strength

Water and Water Pressure

When a joint is wet, it has generally a lower friction angle than a dry joint. Shear strength of a wet joint is calculated use the wet (and drained) friction angle. If a joint is subjected to water pressure, normal stress in the shear strength equation is the effective normal stress, i.e., total stress – water pressure.

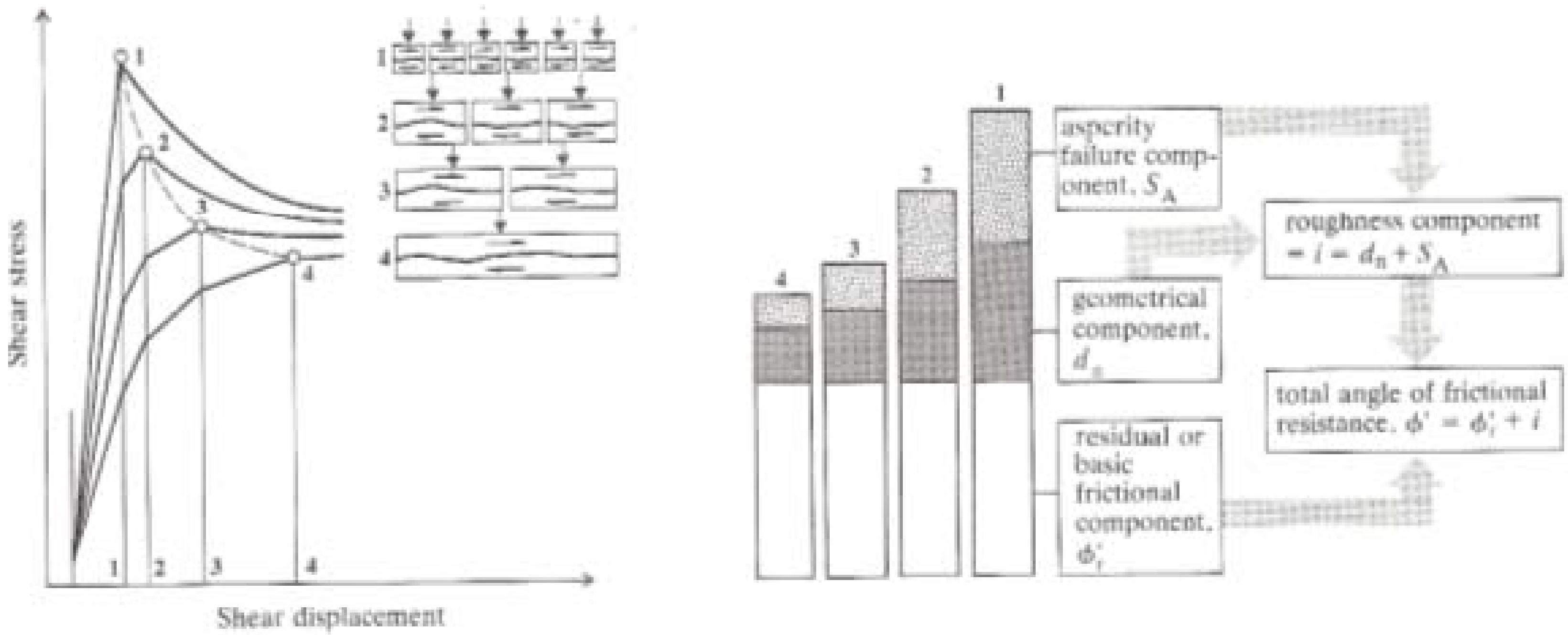
Mechanical and Hydraulic Properties

Factors affecting Rock Joint Shear Strength

Scale Effects

Shear strength of rough joint is scale dependent. As the scale increases, the steeper asperities shear off and the inclination of the controlling roughness decreases. Similarly, the asperity failure component of roughness decreases with increasing scale because the material compressive strength, JCS, decreases with increasing size

Mechanical and Hydraulic Properties



Influence of scale on the three components of discontinuity shear strength

Mechanical and Hydraulic Properties

Flow between Parallel Plates

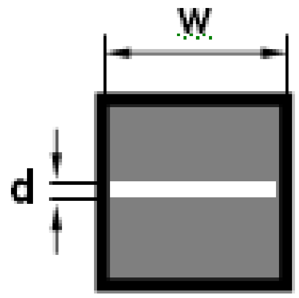
For flow of a viscous fluid through a narrow interspace between two closely spaced parallel plates (e.g., rock joints), Darcy's flow law is applicable when the flow is laminar.

Mechanical and Hydraulic Properties

Flow between Parallel Plates

Intrinsic permeability (K) and coefficient of permeability (k) for laminar flow between parallel plates (aperture d):

$K = d^2 / 12$, $k = g\, d^2 / 12\, \nu$



This is often called the "parallel plate theory" in the flow mechanics of rock joints.

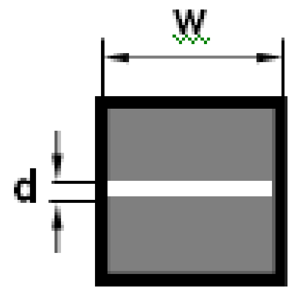
Mechanical and Hydraulic Properties

Flow between Parallel Plates

Flow $Q = A\, i\, g\, d^2 / 12\, \nu$

A is flow area, and $A = w\, d$, it gives

$Q = w\, i\, g\, d^3 / 12\, \nu$



The equation is identified as the "cubic flow law" for flow of fluid through parallel plates (and rock joints).

Mechanical and Hydraulic Properties

Flow in Rock Joints

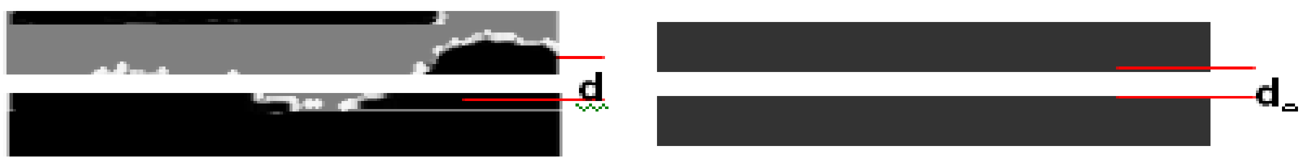
The parallel plates theory is for ideal smooth parallel plates and laminar flow. Rock joints have rough surfaces, are not smooth. It is found the equation is possible to apply to rough joints with modifications, to account for the deviations from the ideal conditions, e.g., the effects of joint roughness and flow path.

Mechanical and Hydraulic Properties

Flow in Rock Joints

For rock joints, equivalent hydraulic aperture (d_e) is used, instead of the aperture of smooth plates.

$d_e = f\ d$



d is the real (average mechanical) aperture of rock joint, and f is a factor that accounts for deviations from the ideal conditions assumed in the parallel smooth plate, and $f \leq 1$.

Mechanical and Hydraulic Properties

Flow in Rock Joints

For a given joint, f is a constant at different apertures, if joint surface profile remains the same.

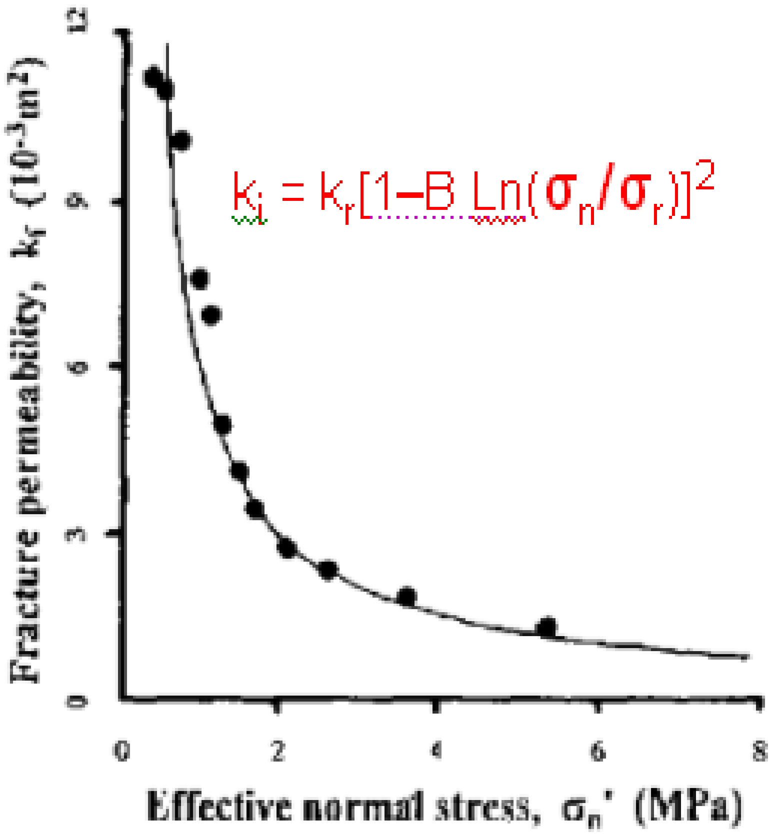
Joints with higher roughness have low f, i.e., rougher joints deviate more from smooth parallel plates. Hydraulic aperture (d_e), real aperture (d) and joint roughness (JRC) can be related as:

$$d_e = JRC^{2.5} / (d/d_e)^2$$

Mechanical and Hydraulic Properties

Rock joint permeability and hydraulic aperture changes with effective normal stress.

Joint permeability reduces asymptotically and approaches to zero with increasing effective normal stress.

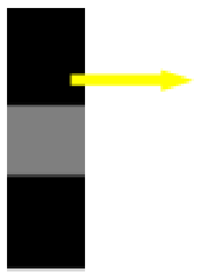


Mechanical and Hydraulic Properties

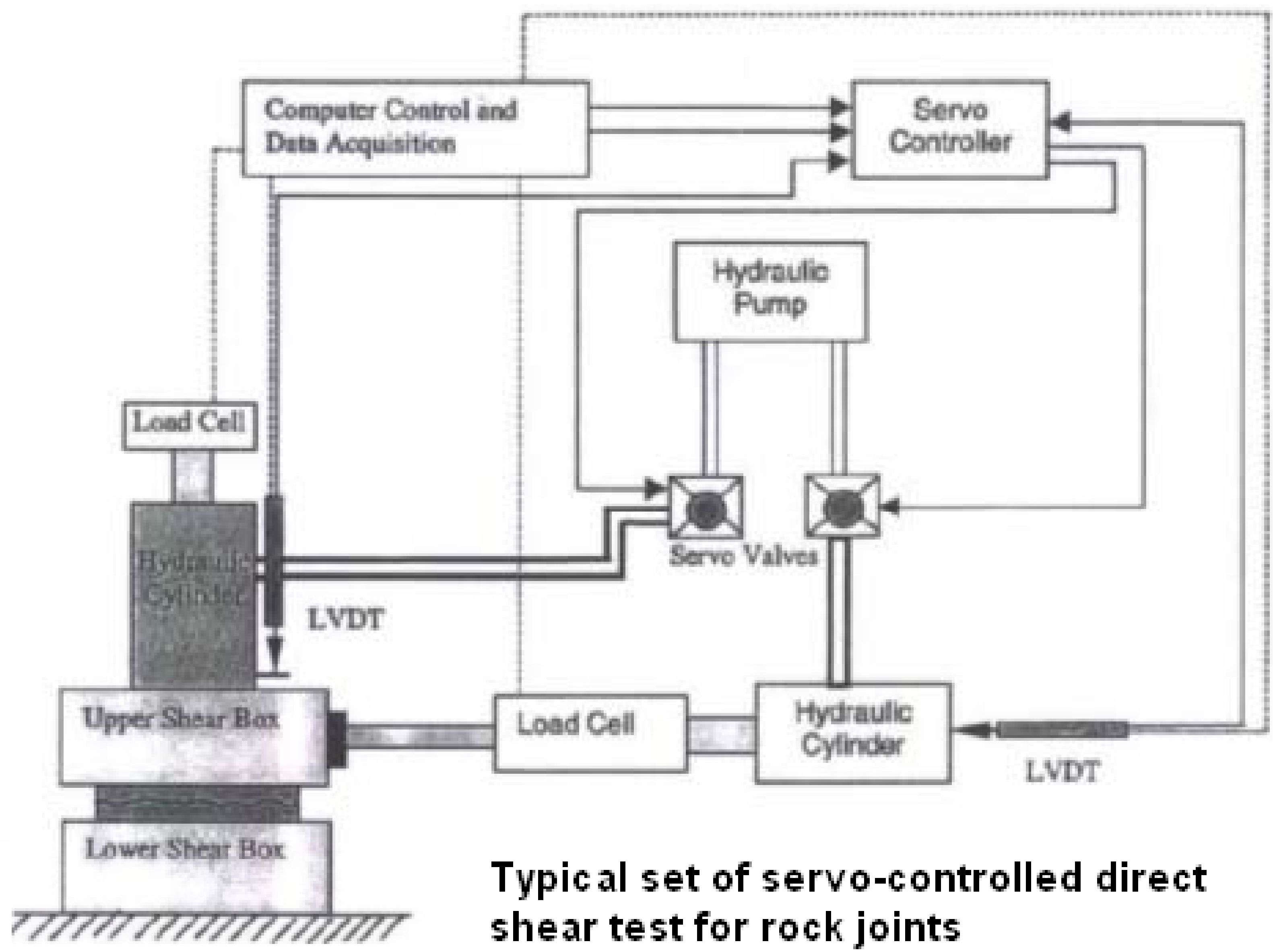
Shear, Aperture and Permeability

For an originally matched and closed joint, shear will start to general separation of the joint surface and creating larger aperture and high permeability. When shear starts, dilation occurs due to climbing effects. The climbing effects may be less obvious if the joint is under high normal stress. In this case, asperities would be crashed and crashed particles may be filled in the joint. This may still result in increasing of permeability but not as significant as in the case of climbing.

Mechanical and Hydraulic Properties



Change of aperture with shearing, due to asperity climbing. Change of aperture will lead to significant change of permeability.

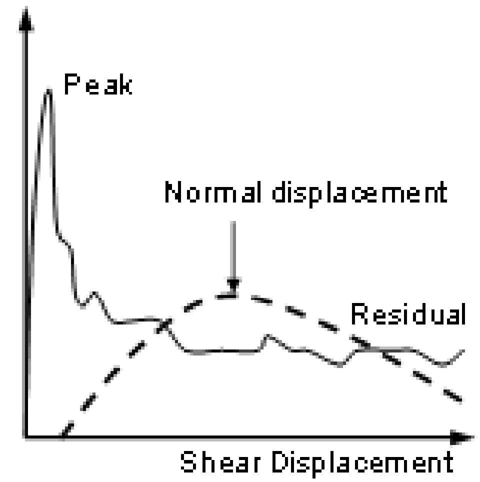


Typical set of servo-controlled direct shear test for rock joints

Mechanical and Hydraulic Properties

Typical fracture shear test results

Normal stress	Peak shear strength	Residual shear strength	Displacements at peak shear strength	
σ_n (MPa)	τ_p (MPa)	τ_r (MPa)	Normal v (mm)	Shear u (mm)
0.25	0.25	0.15	0.54	2.00
0.50	0.50	0.30	0.67	2.50
1.00	1.00	0.60	0.65	3.20
2.00	1.55	1.15	0.45	3.60
3.00	2.15	1.70	0.30	4.00
4.00	2.60	--	0.15	4.20



Typical shear test result at a constant normal stress

Mechanical and Hydraulic Properties

Typical layout of field shear tests in an adit.