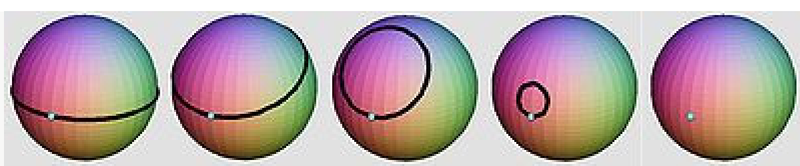
(From Wikipedia)

In topology, a topological space is called **simply connected** (or **1-connected**) if it is path-connected and every path between two points can be continuously transformed, staying within the space, into every other while preserving the two endpoints in question (see below for an informal discussion).

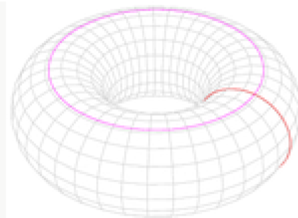
Informal discussion

Informally, a thick object in our space is simply connected if it consists of one piece and does not have any "holes" that pass all the way through it. For example, neither a doughnut nor a coffee cup (with handle) is simply connected, but a hollow rubber ball *is* simply connected. In two dimensions, a circle is not simply connected, but a disk and a line are. Spaces that are connected but not simply connected are called **non–simply connected** or, in a somewhat old-fashioned term, **multiply connected**.



A sphere is simply connected because every loop can be contracted (on the surface) to a point.

Notice that the definition only rules out "handle-shaped" holes. A sphere (or, equivalently, a rubber ball with a hollow center) *is* simply connected, because any loop on the surface of a sphere can contract to a point, even though it has a "hole" in the hollow center. The stronger condition, that the object have no holes of *any* dimension, is called contractibility.



A torus is not simply connected. Neither of the colored loops can be contracted to a point without leaving the surface.

- The Euclidean plane $\mathbf{R}^2$ is simply connected, but $\mathbf{R}^2$ minus the origin (0,0) is not. If $n > 2$, then both $\mathbf{R}^n$ and $\mathbf{R}^n$ minus the origin are simply connected.
- Analogously: the n-dimensional sphere S^n is simply connected if and only if $n \geq 2$.
- Every convex subset of $\mathbf{R}^n$ is simply connected.
- A torus, the (elliptic) cylinder, the Möbius strip and the Klein bottle are not simply connected.
- Every topological vector space is simply connected; this includes Banach spaces and Hilbert spaces.
- The special orthogonal group SO(n,**R**) is not simply connected for $n \geq 2$; the special unitary group SU(n) is simply connected.

Formally, a homogony between two continuous functions f and g from a topological space X to a topological space Y is defined to be a continuous function $H : X \times [0,1] \to Y$ from the product of the space X with the unit interval [0,1] to Y such that, if $x \in X$ then $H(x,0) = f(x)$ and $H(x,1) = g(x)$.

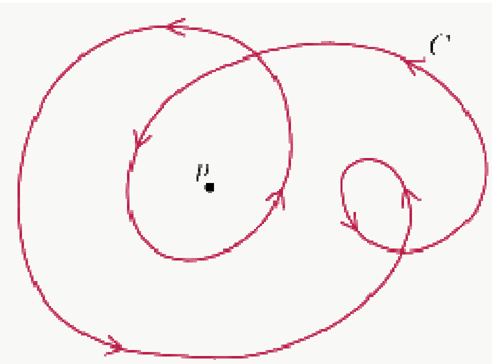
If we think of the second parameter of H as time then H describes a *continuous deformation* of f into g: at time 0 we have the function f and at time 1 we have the function g.

An alternative notation is to say that a homotopy between two continuous functions $f, g : X \to Y$ is a family of continuous functions $h_t : X \to Y$ for $t \in [0,1]$ such that $h_0 = f$ and $h_1 = g$, and the map $t \mapsto h_t$ is continuous from [0,1] to the space of all continuous functions $X \to Y$. The two versions coincide by setting $h_t(x) = H(x,t)$.

[edit]Properties

Continuous functions *f* and *g* are said to be homotopic if and only if there is a homotopy *H* taking *f* to *g* as described above. Being homotopic is an equivalence relation on the set of all continuous functions from *X* to *Y*. This homotopy relation is compatible with function composition in the following sense: if $f_1, g_1 : X \to Y$ are homotopic, and $f_2, g_2 : Y \to Z$ are homotopic, then their compositions $f_2 \circ f_1$ and $g_2 \circ g_1 : X \to Z$ are also homotopic.

The term winding number *may also refer to the rotation number of an iterated map.*



This curve has winding number two around the point *p*.

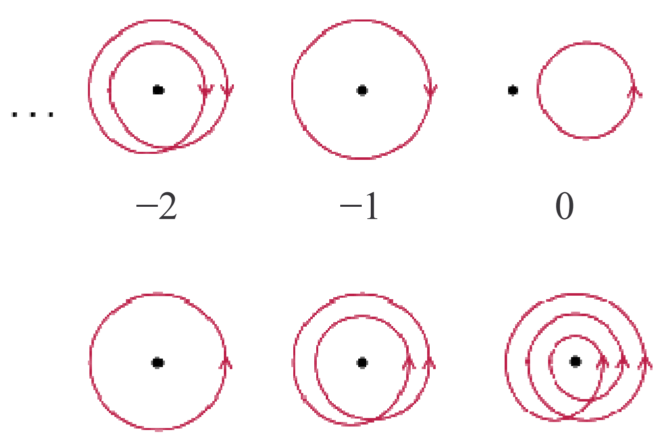
In mathematics, the **winding number** of a closed curve in the plane around a given point is an integer representing the total number of times that curve travels counterclockwise around the point. The winding number depends on the orientation of the curve, and is negative if the curve travels around the point clockwise.

Winding numbers are fundamental objects of study in algebraic topology, and they play an important role in vector calculus, complex analysis, geometric topology, differential geometry, and physics.

Suppose we are given a closed, oriented curve in the *xy* plane. We can imagine the curve as the path of motion of some object, with the orientation indicating the direction in which the object moves. Then the **winding number** of the curve is equal to the total number of counterclockwise rotations that the object makes around the origin.

When counting the total number of rotations, counterclockwise motion counts as positive, while clockwise motion counts as negative. For example, if the object first circles the origin four times counterclockwise, and then circles the origin once clockwise, then the total winding number of the curve is three.

Using this scheme, a curve that does not travel around the origin at all has winding number zero, while a curve that travels clockwise around the origin has negative winding number. Therefore, the winding number of a curve may be anyinteger. The following pictures show curves with winding numbers between −2 and 3:



Formal definition

A curve in the *xy* plane can be defined by parametric equations:

$$x = x(t) \quad \text{and} \quad y = y(t) \qquad \text{for } 0 \leq t \leq 1.$$

If we think of the parameter *t* as time, then these equations specify the motion of an object in the plane between *t* = 0 and *t* = 1. The path of this motion is a curve as long as the functions *x*(*t*) and *y*(*t*) are continuous. This curve is closed as long as the position of the object is the same at *t* = 0 and *t* = 1.

We can define the **winding number** of such a curve using the polar coordinate system. Assuming the curve does not pass through the origin, we can rewrite the parametric equations in polar form:

$$r = r(t) \quad \text{and} \quad \theta = \theta(t) \qquad \text{for } 0 \leq t \leq 1.$$

The functions *r*(*t*) and *θ*(*t*) are required to be continuous, with *r* > 0. Because the initial and final positions are the same, *θ*(0) and *θ*(1) must differ by an integer multiple of 2*π*. This integer is the winding number:

$$\text{winding number} = \frac{\theta(1) - \theta(0)}{2\pi}.$$

This defines the winding number of a curve around the origin in the *xy* plane. By translating the coordinate system, we can extend this definition to include winding numbers around any point *p*.

To define the *n*-th homotopy group, the base point preserving maps from an *n*-dimensional sphere (with base point) into a given space (with base point) are collected into equivalence classes, called **homotopy classes.** Two mappings are **homotopic** if one can be continuously deformed into the other. These homotopy classes form a group.