

MTH203N: Assignment-I

1.D Classify each of the following differential equations as linear, nonlinear and specify the order

$$(i) \quad y'' + y \sin x = 0 \quad (ii) \quad y'' + x \sin y = 0 \quad (iii) \quad y' = \sqrt{1+y}$$
$$(iv) \quad y'' + (y')^2 + y = x \quad (v) \quad y'' + xy' = \cos y' \quad (vi) \quad (xy')' = xy$$

2.T Find the differential equation of each of the following one-parameter families of plane curves:

$$(i) \quad xy^2 - 1 = cy \quad (ii) \quad cy = c^2x + 5 \quad (iii) \quad y = ax^2 + be^{2x}$$
$$(iv) \quad \text{Circles of unit radius with centres on } y\text{-axis} \quad (v) \quad y = a \sin x + b \cos x + b,$$

where a, b and c are constants.

3.D Verify that the given function in the left is the general solution to the corresponding differential equation in the right.

$$(i) \quad x^3 + y^3 = 3cxy \quad x(2y^3 - x^3)y' = y(y^3 - 2x^3)$$
$$(ii) \quad y = ce^{-x} + x^2 - 2x + 4 \quad y' + y = x^2 + 2$$
$$(iii) \quad y = cx - c^2 \quad y'^2 - xy' + y = 0$$

4.T Verify that $y = -1/(x + c)$ is general solution of $y' = y^2$. Find particular solutions such that (i) $y(0) = 1$, and (ii) $y(0) = -1$. In both the cases, find the largest interval I on which y is defined.

5.D Consider the differential equations $y' = \alpha y$, $x > 0$, where α is a constant. Show that

- (i) if $\phi(x)$ is any solution and $\psi(x) = \phi(x)e^{-\alpha x}$, then $\psi(x)$ is a constant;
- (ii) if $\alpha < 0$, then every solution tends to zero as $x \rightarrow \infty$.

6.D Reduce the differential equation $y' = f\left(\frac{ax + by + m}{cx + dy + n}\right)$, $ad - bc \neq 0$ to a separable form. Also discuss the case of $ad = bc$.

7.T For each of the following differential equations, draw several isoclines with appropriate lineal elements and hence sketch some solution curves

$$(i) \quad y' = x \quad (ii) \quad y' = -x/y$$

Supplementary problems from “Advanced Engg. Maths.” by E. Kreyszig (8th Edn.)

- (i) Page 8–9, Q.9,12,17
- (ii) Page 13, Q.9,18
- (iii) Page 18, Q.7,10,14,20,22
- (iv) Page 23–24, Q.1,2,4,11,13