MTH203: Assignment-3

1.D Solve the following linear first order differential equations using the method of variation of parameters:

(i) $xy' - 2y = x^4$ (ii) $y' + (\cos x)y = \sin x \cos x$

[Method: To solve $y' + P(x)y = R(x)$, first find the general solution $y_h(x) = cy_1(x)$ of the homogeneous equation $y' + P(x)y = 0$. Assume that $y_p(x) = u_1(x)y_1(x)$ is a (particular) solution of the original equation. Find u_1 and the general solution of the original equation is given by $y(x) = y_h(x) + y_p(x)$]

2.D Solve $y' + y = x - 1$, $y' + y = \cos 2x$. Hence solve $y' + y = cos^2 x - x/2$

3.T Show that the set of solutions of the homogeneous linear equation, $y' + P(x)y = 0$ on an interval $I = [a, b]$ form a vector subspace W of the real vector space of continuous functions on I. What is the dimension of W?

4.T Let $f(x, y)$ be continuous on the closed rectangle $R : |x - x_0| \le a,\ |y - y_0| \le b$.

(i) Show that y is a solution of the initial value problem $y' = f(x, y),\ y(x_0) = y_0$ iff

$$y(x) = y_0 + \int_{x_0}^{x} f[t, y(t)]\, dt.$$

(ii) Let $|f(x, y)| \le M$ and $y_n(x) = y_0 + \int_{x_0}^{x} f[t, y_{n-1}(t)]\, dt$, with $y_0(x) = y_0$. Show by the method of induction that $|y_n(x) - y_0| \le b$ for $|x - x_0| \le h$, where $h = \min\{a, b/M\}$.

5.D Use Picard's method of successive approximation to solve the following initial value problems and compare these results with the exact solutions:

(i) $y' = 2\sqrt{x},\ y(0) = 1$ (ii) $y' + xy = x,\ y(0) = 0$ (iii) $y' = 2\sqrt{y}/3,\ y(0) = 0$

6.D Apply (i) Euler method and (ii) improved Euler method to compute $y(x)$ at $x = 0.2, 0.4, 0.6, 0.8, 1.0$ for the initial value problem: $y' = xy + xy^2,\ y(0) = 1$. Compare the errors in each point with the exact solution.

7.T Solve $y' = (y - x)^{2/3} + 1$. Show that $y = x$ is also a solution. What can be said about the uniqueness of the initial value problem consisting of the above equation with $y(x_0) = y_0$, where (x_0, y_0) lies on the line $y = x$.

8.T Discuss the existence and uniqueness of the solution of the inital value problem

$$(x^2 - 2x)y' = 2(x - 1)y, \qquad y(x_0) = y_0.$$

Supplementary problems from "Advanced Engg. Maths." by E. Kreyszig (8^{th} Edn.)

(i) Page 58–59, Q.1,2,5,14,18,19

(ii) Page 951, Q.1,2,6,7