1.T Verify that \( y = x^2 \sin x \) and \( y = 0 \) both are solutions of the initial value problem
\[
x^2 y'' - 4xy' + (x^2 + 6)y = 0, \quad y(0) = y'(0) = 0.
\]
Does it contradict the uniqueness?

2.D Find general solution of the following differential equations given a known solution \( y_1 \):
\[
\text{(i) } x(1-x)y'' + 2(1-2x)y' - 2y = 0 \quad y_1 = 1/x
\]
\[
\text{(ii) } (1-x^2)y'' - 2xy' + 2y = 0 \quad y_1 = x
\]

3.D Verify that \( \sin x/\sqrt{x} \) is a solution of \( x^2 y'' + xy' + (x^2 - 1/4)y = 0 \) over any interval on the positive \( x \)-axis and hence find its general solution.

4.D Solve the following differential equations:
\[
\text{(i) } y'' - 4y' + 3y = 0 \quad \text{(ii) } y'' + 2y' + (\omega^2 + 1)y = 0, \quad \omega \text{ is real.}
\]

5.D Solve the following initial value problems:
\[
\text{(i) } y'' + 4y' + 4y = 0 \quad y(0) = 1, \quad y'(0) = -1
\]
\[
\text{(ii) } y'' - 2y' - 3y = 0 \quad y(0) = 1, \quad y'(0) = 3
\]

6.D The equation
\[
x^2 \frac{d^2 y}{dx^2} + ax \frac{dy}{dx} + by = 0,
\]
where \( a, b \) are constants, is called the Euler-Cauchy equation. Show that under the transformation \( x = e^t \) (when \( x > 0 \)) for the independent variable, the above reduces to
\[
\frac{d^2 y}{dt^2} + (a-1) \frac{dy}{dt} + by = 0,
\]
which is an equation with constant coefficients.
Hence solve: (i) \( x^2 y'' + 2xy' - 12y = 0 \) (ii) \( x^2 y'' + xy' + y = 0 \) (iii) \( x^2 y'' - xy' + y = 0 \)

7.T Find a particular solution of each of the following equations by the method of undetermined coefficients and hence find its general solution:
\[
\text{(i) } y'' + 4y = 2 \cos^2 x + 10e^x \quad \text{(ii) } y'' + y = \sin x + (1 + x^2)e^x
\]
\[
\text{(iii) } y'' - y = e^{-x}(\sin x + \cos x) \quad \text{(iv) } y'' - 3y'' - y' + 3y = x^2 e^x
\]

8.T By using the method of variation of parameters, find the general solution of:
\[
\text{(i) } y'' + 4y = 2 \cos^2 x + 10e^x \quad \text{(ii) } y'' + y = x \sin x
\]
\[
\text{(iii) } y'' + y = \cot^2 x \quad \text{(iv) } x^2 y'' - x(x+2)y' + (x+2)y = x^3, \quad x > 0.
\]
[Hint. \( y = x \) is a solution of the homogeneous part]

**Supplementary problems** from “Advanced Engg. Maths.” by E. Kreyszig (8th Edn.)

(a) Problem Set 2.1: Q. 6,7,10 \hspace{1cm} (b) Problem Set 2.2: Q.13,17,23,26,27,28
(c) Problem Set 2.3: Q.13,17,19,20 \hspace{1cm} (d) Problem Set 2.6: Q.2,6,14
(e) Problem Set 2.7: Q.12,16,18 \hspace{1cm} (f) Problem Set 2.8: Q.16
(g) Problem Set 2.9: Q.9,13,21 \hspace{1cm} (h) Problem Set 2.10: Q.5,10,15
(i) Problem Set 2.13: Q.4,11,18,20 (a.1,a.3,b.) \hspace{1cm} (j) Problem Set 2.14: Q.14,16,20