MTH203: Assignment-5

1.T Verify that $y = x^2 \sin x$ and $y = 0$ both are solutions of the initial value problem

$$x^2y'' - 4xy' + (x^2+6)y = 0, \quad y(0) = y'(0) = 0.$$

Does it contradict the uniqueness?

2.D Find general solution of the following differential equations given a known solution y_1:

(i) $x(1-x)y'' + 2(1-2x)y' - 2y = 0$ $\qquad y_1 = 1/x$

(ii) $(1-x^2)y'' - 2xy' + 2y = 0$ $\qquad y_1 = x$

3.D Verify that $\sin x/\sqrt{x}$ is a solution of $x^2y'' + xy' + (x^2 - 1/4)y = 0$ over any interval on the positive x-axis and hence find its general solution.

4.D Solve the following differential equations:

(i) $y'' - 4y' + 3y = 0$ $\qquad$ (ii) $y'' + 2y' + (\omega^2+1)y = 0, \quad \omega$ is real.

5.D Solve the following initial value problems:

(i) $y'' + 4y' + 4y = 0$ $\qquad y(0) = 1,\ y'(0) = -1$

(ii) $y'' - 2y' - 3y = 0$ $\qquad y(0) = 1,\ y'(0) = 3$

6.D The equation

$$x^2\frac{d^2y}{dx^2} + ax\frac{dy}{dx} + by = 0,$$

where a, b are constants, is called the Euler-Cauchy equation. Show that under the transformation $x = e^t$ (when $x > 0$) for the independent variable, the above reduces to

$$\frac{d^2y}{dt^2} + (a-1)\frac{dy}{dt} + by = 0,$$

which is an equation with constant coefficients.

Hence solve: (i) $x^2y'' + 2xy' - 12y = 0$ (ii) $x^2y'' + xy' + y = 0$ (iii) $x^2y'' - xy' + y = 0$

7.T Find a particular solution of each of the following equations by the method of undetermined coefficients and hence find its general solution:

(i) $y'' + 4y = 2\cos^2 x + 10e^x$ $\qquad$ (ii) $y'' + y = \sin x + (1+x^2)e^x$

(iii) $y'' - y = e^{-x}(\sin x + \cos x)$ $\qquad$ (iv) $y''' - 3y'' - y' + 3y = x^2e^x$

8.T By using the method of variation of parameters, find the general solution of:

(i) $y'' + 4y = 2\cos^2 x + 10e^x$ $\qquad$ (ii) $y'' + y = x \sin x$

(iii) $y'' + y = \cot^2 x$ $\qquad$ (iv) $x^2y'' - x(x+2)y' + (x+2)y = x^3, \quad x > 0.$

[Hint. $y = x$ is a solution of the homogeneous part]

Supplementary problems from "Advanced Engg. Maths." by E. Kreyszig (8^{th} Edn.)

(a) Problem Set 2.1: Q. 6,7,10 $\qquad$ (b) Problem Set 2.2: Q.13,17,23,26,27,28

(c) Problem Set 2.3: Q.13,17,19,20 $\qquad$ (d) Problem Set 2.6: Q.2,6,14

(e) Problem Set 2.7: Q.12,16,18 $\qquad$ (f) Problem Set 2.8: Q.16

(g) Problem Set 2.9: Q.9,13,21 $\qquad$ (h) Problem Set 2.10: Q.5,10,15

(i) Problem Set 2.13: Q.4,11,18,20 (a.1,a.3,b.) $\qquad$ (j) Problem Set 2.14: Q.14,16,20