Chapter 1
Introduction

Estimating the physical properties of water-bearing layers is an essential part of groundwater studies. A few physical properties and derived parameters of aquifers which appear in various equations are hydraulic conductivity, transmissivity, saturated thickness and storage coefficient. Transmissivity and storativity are of particular interest in studies of aquifers. Transmissivity '**T**' is the product of average hydraulic conductivity and saturated thickness of the aquifer, consequently, the transmissivity is the rate of flow under a hydraulic gradient equal to unity through a cross-section of unit width over the whole saturated thickness of water bearing layer. Storage coefficient or storativity '**S**' of an aquifer is the volume of water released from the storage per unit surface area of aquifer per unit decline in component of hydraulic head normal to the surface. It is a function of saturated thickness of aquifer.

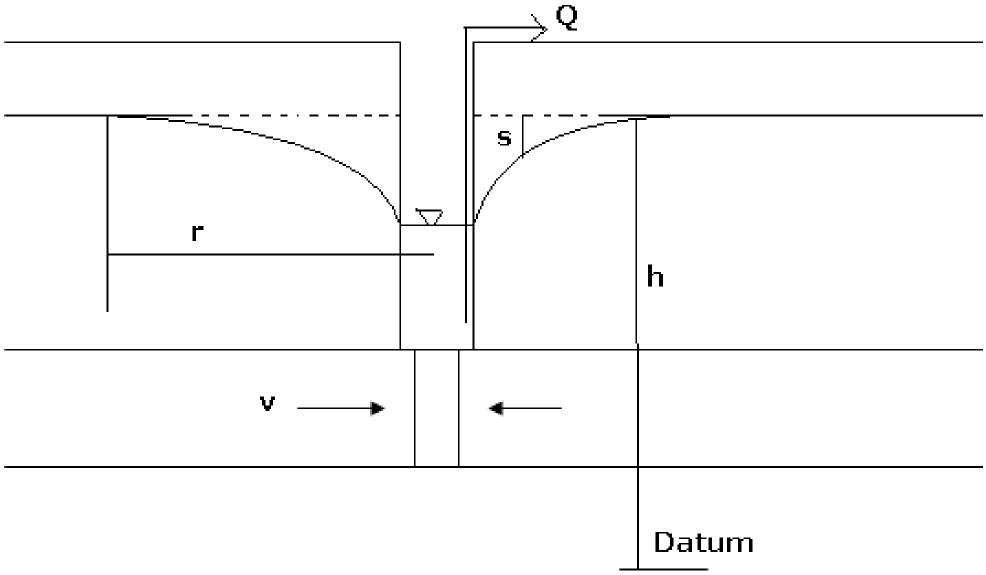

Figure 1.1: Unsteady flow to well in confined aquifer

One of the most effective ways of determining these properties is to conduct and analyze aquifer tests. Aquifer tests are used to determine field scale characteristics, particularly hydraulic transmissivity and storage coefficient. In literature aquifer tests based on the analysis of drawdown during pumping are commonly referred to as pumping tests. Owing to high costs of aquifer tests, it is often performed without piezometers, cutting costs and admitting certain, sometimes appreciable error. In order to distinguish from normal aquifer tests, such tests are called single-well tests. In these tests measurements are taken inside the pumped well. Well-flow equations are used in analysis of drawdown data generated. These equations are developed under a number of common assumptions and conditions like aquifer being homogenous and isotropic, infinite areal extent of the aquifer, complete penetration of aquifer by well, negligible storage in the well and constant rate of discharge accompanied by simultaneous decline in head . An analytical expression for drawdown in an observation well located at a distance of '**r**' from pumping well assuming an ideal confined aquifer was reported by Theis (1935) as:

$$s = \frac{Q}{4\pi T} W(u) \tag{1.1}$$

where 's' is the drawdown [L] from an initially horizontal piezometric surface; 'Q' is the constant pumping rate [L^3/T] and 'T' is the transmissivity [L^2/T] .The dimensionless parameter 'u' , which depends on time 't'[T], is defined as:

$$u_i = \frac{r^2 S}{4Tt_i} \tag{1.2}$$

where 'S' is the storage coefficient and 'W (u)' is the well function.

The well flow equation can be re-written in the following form:

$$s = \alpha . W(\frac{\beta}{t}) \tag{1.3}$$

$$\alpha = \frac{Q}{4\pi T} \quad \text{and} \quad \beta = \frac{r^2 S}{4T} \tag{1.4a}$$

$$T = \frac{Q}{4\pi\alpha} \quad \text{and} \quad S = \frac{\beta Q}{\alpha\pi r^2} \tag{1.4b}$$

$$W(u) = \int_u^\infty \frac{e^{-u}}{u} du \tag{1.5}$$

However for computational purposes in this work, the following approximations for W(u) as obtained by Allen(1954) and Hastings(1955) are used :

For u ≤ 1:

$$W(u) = -\ln(u) + a_o + a_1 u + a_2 u^2 + a_3 u^3 + a_4 u^4 + a_5 u^5 \tag{1.6a}$$

For u ≥ 1:

$$W(u) = \frac{1}{ue^u} \frac{b_0 + b_1 u + b_2 u^2 + b_3 u^3 + u^4}{c_0 + c_1 u + c_2 u^2 + c_3 u^3 + u^4} \tag{1.6b}$$

Where rounded off values of the constants are given by:

$a_0 = -0.57722$; $a_1 = 0.99999$; $a_2 = -0.24991$; $a_3 = 0.05519$;

$a_4 = -0.00976$; $a_5 = 0.00108$; $b_0 = 0.26777$; $b_1 = 8.63476$;

$b_2 = 18.05902$; $b_3 = 8.57333$; $c_0 = 3.95850$; $c_1 = 21.09965$;

$c_2 = 25.63296$; $c_3 = 9.57332$

Chapter 2

METHODS OF DETERMINING S AND T

Theis gives an analytical relationship to relate drawdown with the aquifer transmissivity (**T**) and storage coefficient (**S**). Since the well function is an integral function of S and T, it is not possible to determine explicitly the parameters S and T from a set of values of s and t at an observation well.

Several methods have been proposed to evaluate these parameters which include numerical approximations of well function, providing an initial estimate for iterative solution and slope matching techniques. However, methods involving initial guess of S and T may fail to converge if the initial guess is not sufficiently close to the actual values. Methods involving numerical approximations of well functions give different fit in different domains not necessarily covering the entire range of values. Slope matching technique involves two schemes. The slope of the drawdown is computed as a function of time, and a straight line is fitted through the resulting curve. The intercepts thus obtained give the time averaged values of transmissivity and storage coefficients. An alternative scheme is to use approximations for numerical differentiation and with the help of available discrete data, obtain the time varied transmissivity and storage coefficients.

2.1 Approximate Well Functions

Several high accuracy expressions for the well function have been developed. A criteria function f (e_i), measuring the error between the drawdown obtained using the approximated well function and the actual drawdown is minimized to estimate the aquifer parameters.

$$s_i = \frac{Q}{4\pi T} W(u_i) \tag{2.1}$$

$$E_i = s_i - \frac{Q}{4\pi T} W(u_i) \tag{2.2}$$

and

$$u_i = \frac{r^2 S}{4 T t_i} \tag{2.3}$$

S and T can be obtained by varying them so that the sum of square of errors for the entire data set is minimized. Considering the sum of square of errors overemphasizes large errors that are associated with the later part of the pump test. Thus, for the initial part of the pump test, data will not be properly used. Therefore proportionate error is used.

$$e_i = \frac{E_i}{s_i} \quad (2.4)$$

$$f(e_i) = (e_i^{-2} + e_c^{-2})^{-0.5} \quad (2.5)$$

where e_c is the proportionate cutoff error

Criterion function is chosen so that contribution of an erroneous observation is limited to e_c only.

2.2 Slope Matching Technique

In slope matching method, the Theis equation is rewritten as:

$$s = \alpha.W(\frac{\beta}{t}) \quad (2.6)$$

$$\alpha = \frac{Q}{4\pi T} \quad \text{and} \quad \beta = \frac{r^2 S}{4T} \quad (2.7)$$

$$T = \frac{Q}{4\pi\alpha} \quad \text{and} \quad S = \frac{\beta Q}{\alpha\pi r^2} \quad (2.8)$$

The slope matching methods are based on time derivative (slope) of the drawdown as obtained from Eq.(2.6)

$$\frac{\partial s}{\partial t} = \alpha \frac{\partial}{\partial t} W(\frac{\beta}{t}) = \alpha \frac{\partial W(\frac{\beta}{t})}{\partial(\frac{\beta}{t})} . \frac{\partial(\frac{\beta}{t})}{\partial t} \quad (2.9)$$

Using the relation

$$\frac{dW(u)}{du} = -\frac{e^{-u}}{u} \quad (2.10)$$

$$\frac{\partial s}{\partial t} = \frac{\alpha}{t} e^{\frac{-\beta}{t}} \quad (2.11)$$

or

$$t\frac{\partial s}{\partial t} = \alpha.e^{\frac{-\beta}{t}} \quad (2.12)$$

Using finite differences, the value of the left hand side can be obtained at any value of time and α and β are obtained using any two successive time values.

Different methods can be used to estimate S and T using differential equation Eq.(2.12). Straight line fit can be used to obtain an average value of the parameters. Central difference, finite difference involving logarithm of t, finite difference involving logarithm of t and s and many more can be used to estimate slope of drawdown curve.

Chapter 3

AQUIFER PARAMETER DETERMINATION: A NEW APPROACH

The methods that have been employed, more commonly, in determination of aquifer parameters are slope matching, use of numerical approximations to well function and iterative schemes involving initial guess of parameters S and T. All these methods arrive at the parameters with certain errors built in to the values on account of the method employed.

3.1 Problems with Earlier Methods

Slope matching methods require numerical differentiation of discrete data which may consist systematic and random errors associated with the experimental techniques. These random errors get magnified when derivatives are deployed to compute average of time varying parameters. Another problem related to differentiating discrete data is unequal spacing in the empirical data that would involve further complexities to computing derivates subject to the order of derivative technique being used. The slope matching formulations do not take into account and thus preclude finite difference approximation of parameters values, thereby predicting much larger values of S and T at large times. However this tradeoff is made in favor of the procedure becoming too involved. Other methods like non-linear least squares which require initial guesses of S and T may have convergence problems if these guess values are very different from the actual values. Methods which use approximations to the well function generally require tabulations of values of the well function for a wide range of values of non-dimensional ‘u’ making these computationally intensive. Also different approximations are valid only for a limited range of u and have different implications in terms of error.

Keeping in the backdrop, the problem of magnification of errors and stability encountered due to numerical differentiation of empirical data we propose a new scheme which employs integration of drawdown data in the process of determination of S and T. Integration of uncertain data is more forgiving on the errors, as summations tends to cancel out random positive and negative errors unlike differentiation which being subtractive tends to add negative and positive errors. Thus integration tends to smoothen out the perturbations in the field data. This is illustrated for a particular reported field data set taken from [Srivastava and Guzman-Guzman, 1994]. The original data set is slightly perturbed to demonstrate the effect of differentiation of propagation errors. Figure 3.1, 3.2 and 3.3 gives the pictorial illustration of the same.

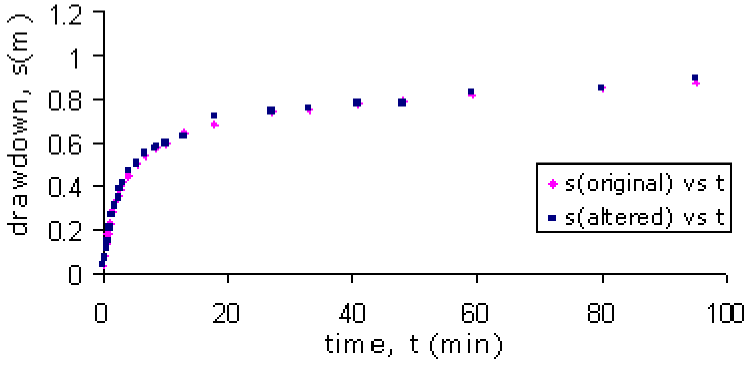

Figure 3.1 Drawdown curve

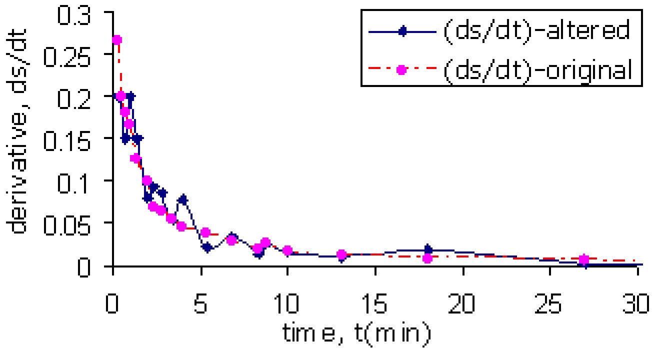

Figure 3.2 Plot of slope of drawdown curve using original drawdown data and the altered drawdown data

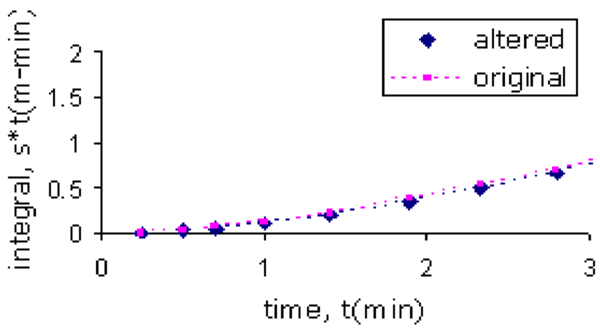

Figure 3.3 Plot of area under the drawdown curve using original drawdown data and the altered drawdown data

3.2 New Approach

A number of numerical schemes have been formulated to provide a method to calculate S and T. However efforts were focused on arriving at an analytical solution of the well function using analytical integration or analytical methods have not proved to be useful. Because of the imperfect nature of the integral and limited techniques of integration, no solution has yet been obtained. Our efforts directed towards the same also did not yield any solution. Different approximations of the well function have been proposed in the literature from time to time. Well functions with good degree of simplicity and scale covering entire range of time, t were adopted. Using these well functions, a time integration of a function F(s, t) was done which would give a simplified relation of α and β. This relationship together with s and t data gives equations from which we could calculate α and β. But the equations obtained using this method gave multiple roots of β for the same value of α which contradicts the assumption that both α and β are time independent and unique.

Lastly, a new approach involving use of numerical integration of drawdown data is proposed. In this approach using the Theis equation Eq.(1.3) and approximations for well function (1.7a and 1.7b) a relationship of the following form involving integrals is formulated:

$$\int G(s,t)dt = f(\alpha,\beta) * \int g(u)du \qquad (3.1)$$

where

$$u = \frac{\beta}{t}$$

This integration is done for the limits [(t, 2t), (2u, 4u)] and [(t, 4t), (u, 4u)] and the ratio of the two integrals is taken. This will eliminate f (α, β) and result into an equation of the form:

$$\frac{\int_{t}^{2t} G(s,t)dt}{\int_{t}^{4t} G(s,t)dt} = \frac{\int_{2u}^{4u} g(u)du}{\int_{u}^{4u} g(u)du} \qquad (3.2)$$

or

K(t) = F (u)

$$F(u) = \frac{\int_{2u}^{4u} g(u)du}{\int_{u}^{4u} g(u)du}$$

Using a particular value of t, the left hand side ratio of integrals, K (t) can be calculated using drawdown curve as shown in Figure 3.4 Since F(u) = K(t), this value is used to calculate the value of u using the curve between F(u) vs u, which is generated for a particular functional form of F(u) as shown in Figure 3.5. Once u is known for the given value of t, β can be calculated as β = ux4t.

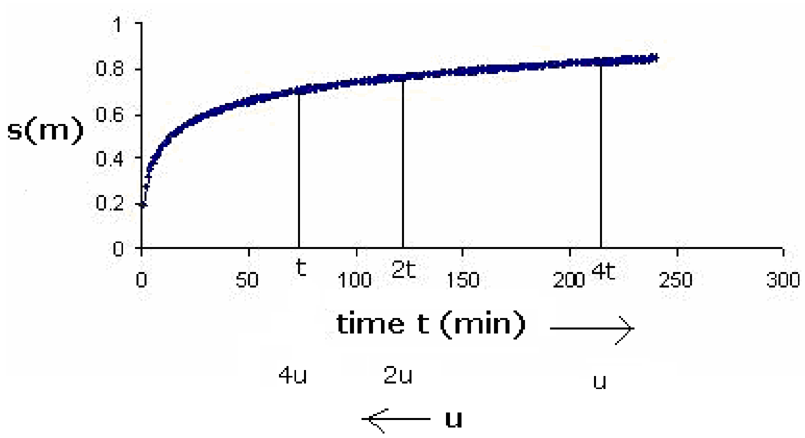

Figure 3.4 Drawdown Curve – plot of s (m) vs t (min)

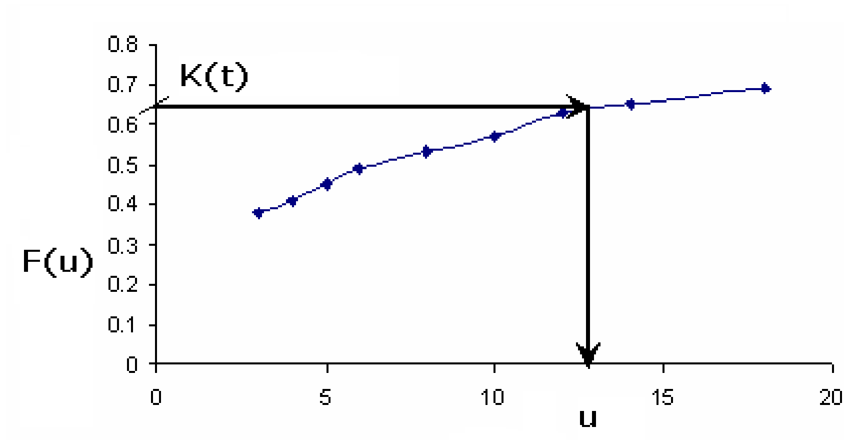

Figure 3.5 F(u) vs u where u = β/t

Using the pumping data used earlier, it is demonstrated that integration technique does not lead to magnification of error in comparison to the derivates used in slope matching equations. The functional form of K(t) is calculated by integrating the pumping data from the interval t_1 to $2t_1$ and t_1 to $4t_1$ and taking the ratio of the two integrals. This is done both on the original pumping data and the altered data to show the deviation of the results obtained. Similarly % deviation is observed for the slope of the drawdown of the altered from the original data. Figure 3.6 shows the variation of K(t) and Figure 3.7 and Figure 3.8 shows the % deviations.

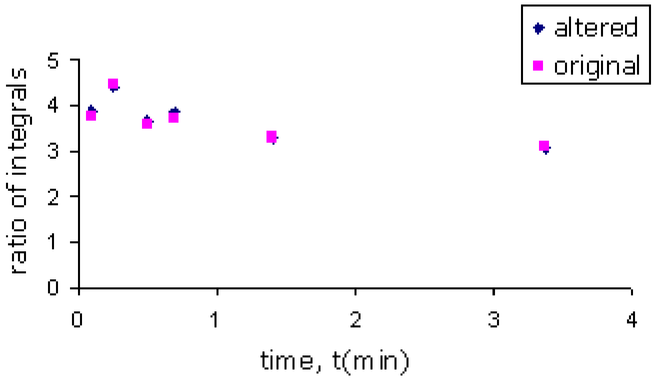

Figure 3.6 Graphical representation of the functional form K(t) using the original pumping test data and the altered data.

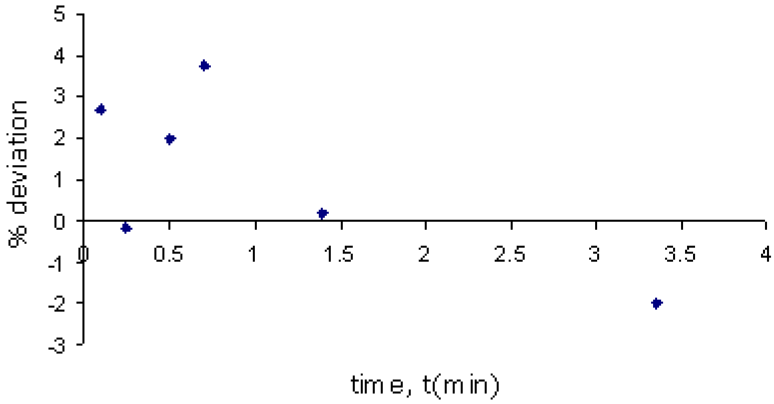

Figure 3.7 Plot of % deviation of F(u) for the altered data from the F(u) of the original data

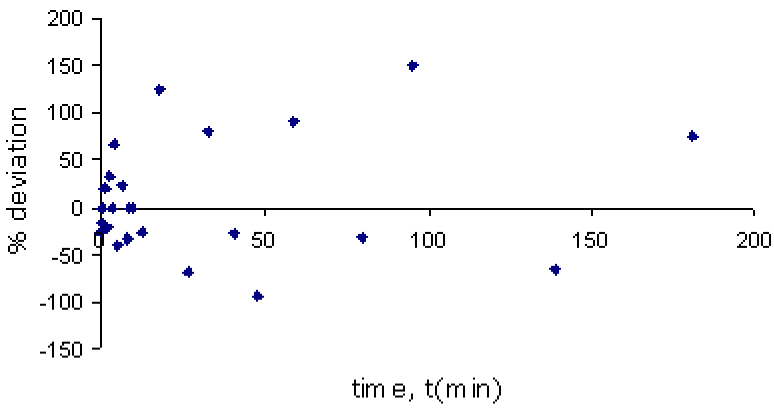

Figure 3.8 Plot of % deviation of slope of drawdown for the altered data from the original

Calculations and Results

Initially the analytical methods were used to calculate the integral and therefore varied forms of G(s, t) were deployed to ease the calculations. But due to the inclusion of e^u or log (u) in combination with polynomial functions of u, the inefficacy of analytical methods forced us to switch over to numerical methods. For calculations using the proposed method, G(s, t) has been taken as simply s and thereby g (u) as W $(u)/u^2$. W (u) has been computed using Eq.(1.6a) and Eq.(1.6b).

For the purposes of computations, a synthetic s vs t curve was generated using MATLAB. Values of α and β for this computations are taken as 0.1212 and 0.1248 respectively. This s vs t curve was integrated within the time limits t to 2t and t to 4t and their ratio, K (t) was computed and plotted against t. Similarly F(u) was computed as ratio of integral of $W(u)/u^2$ in the limits (2u,4u) to (u,4u). Plots obtained are shown in Figure 4.1 and Figure 4.2.

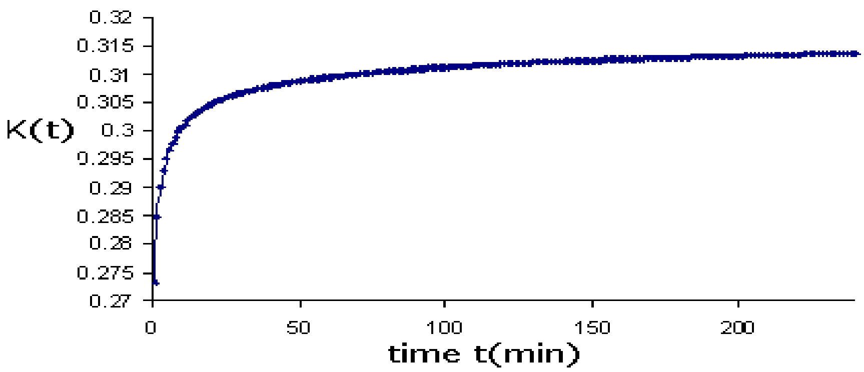

Figure 4.1: Curve between the ratio of integral of s on t for the range (t, 2t) to (t, 4t) vs t

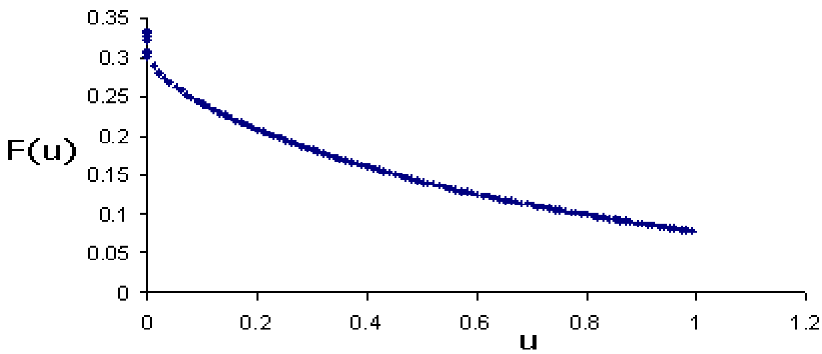

Figure 4.2: F (u) vs u generated from Well function approximation

The limiting values of F (u) are calculated for u approaching 0 and infinity and they come out to be 0.3333 and 0 near 0 and infinity respectively. These limiting values match with the values obtained from numerical integration using trapezoidal rule with well function approximations.

Values of α and β are calculated for different times ranging from 1 to 20 minutes. These values together with values of s, t, F(u),W(u) and u are tabulated in Table 4.1.

Table 4.1: Values of α and β for t = 1 to 20 mins.

t(min)	s(m) at t	F(u)	u	W(u)	4t	s at 4t	β	α
1	0.19693	0.2732	0.03231	2.8872091	4	0.35403	0.12924	0.122620144
1.5	0.24128	0.2805	0.02206	3.258708	6	0.40193	0.13236	0.123340292
2	0.27372	0.2848	0.01698	3.5154071	8	0.43618	0.13584	0.124076667
2.5	0.29929	0.2877	0.015	3.6374289	10	0.46285	0.15	0.127246474
3	0.32041	0.2899	0.01233	3.8307919	12	0.48469	0.14796	0.126524752
3.5	0.33838	0.2916	0.01099	3.9445093	14	0.5032	0.15386	0.127569734
4	0.35403	0.2929	0.01	4.0379252	16	0.51925	0.16	0.12859327
5	0.38033	0.295	0.008992	4.1431717	20	0.5461	0.17984	0.131807233
6	0.40193	2.97E-01	0.00746	4.3284259	24	0.56807	0.17904	0.131241706
8	0.43618	2.99E-01	0.00608	4.5316013	32	0.60278	0.19456	0.133016998
10	0.46285	0.3005	0.00582	4.5750465	40	0.62973	0.2328	0.137644502
12	0.48469	0.3017	0.0056	4.6133608	48	0.65177	0.2688	0.141278783
14	0.5032	0.3026	0.00499	4.7280831	56	0.67041	0.27944	0.141793193
16	0.51925	0.30342	0.004275	4.8820216	64	0.68656	0.2736	0.140630268
20	0.5461	0.30464	0.004076	4.929491	80	0.71355	0.32608	0.144751254

These values show increasing trends in β as t increases. As per the assumptions of this method α and β are constants. But this variation of β can be explained by the fact that F(u) becomes almost constant for these values of t and hence u derived from F(u) vs u curve remains almost constant, thereby β increases with increasing time. However, for times t ≤ 1, values of β obtained by this method are close to the assumed value with a maximum deviation of 4.89%. Values of α obtained are also close with a maximum deviation of 1.2% from the assumed value.

Table 4.2: Values of α and β for t = 0.04 to 1

t(min)	s(m) at t	F(u)	u	W(u)	4t	s(m) at 4t	β	α
0.04	0.001358	0.10121	0.7802	0.3219605	0.16	0.039037	0.124832	0.1212478
0.08	0.011093	0.16223	0.391	0.7176981	0.32	0.087196	0.12512	0.12149398
0.12	0.024875	0.1917	0.2633	1.004175	0.48	0.12288	0.126384	0.12236911
0.16	0.039037	0.20928	0.1968	1.2358726	0.64	0.1507	0.125952	0.12193814
0.2	0.052493	0.22112	0.1568	1.4264252	0.8	0.17341	0.12544	0.12156964
0.24	0.06499	0.22974	0.1329	1.5695495	0.96	0.19257	0.127584	0.12269125
0.28	0.076531	0.23635	0.1102	1.7354748	1.12	0.20914	0.123424	0.12050881
0.32	0.087196	0.24161	0.0989	1.8329331	1.28	0.22372	0.126592	0.12205574
0.36	0.097078	0.24593	0.08691	1.9507193	1.44	0.23674	0.1251504	0.12136036
0.4	0.10627	0.24954	0.07822	2.0477261	1.6	0.2485	0.125152	0.12135412
0.44	0.11485	0.25262	0.07093	2.1385332	1.76	0.25922	0.1248368	0.12121392
0.48	0.12288	0.25529	0.06553	2.2124988	1.92	0.26908	0.1258176	0.12161815
0.52	0.13044	0.25763	0.06054	2.2868665	2.08	0.27819	0.1259232	0.1216468
0.56	0.13757	0.2597	0.05614	2.3580482	2.24	0.28667	0.1257536	0.12157088
0.6	0.14431	0.26156	0.05267	2.4184734	2.4	0.29459	0.126408	0.12180824
0.64	0.1507	0.26322	0.04905	2.4861498	2.56	0.30203	0.125568	0.12148504
0.68	0.15678	0.26474	0.04695	2.5278564	2.72	0.30904	0.127704	0.12225378
0.72	0.16258	0.26612	0.04314	2.6087635	2.88	0.31566	0.1242432	0.12099985
0.76	0.16812	0.26738	0.04096	2.6584834	3.04	0.32195	0.1245184	0.12110288
0.8	0.17341	0.26855	0.03979	2.686317	3.2	0.32792	0.127328	0.12207048
0.84	0.17849	0.26962	0.03896	2.7065833	3.36	0.33361	0.1309056	0.12325872
0.88	0.18337	0.27063	0.03629	2.7749562	3.52	0.33905	0.1277408	0.12218211
0.92	0.18806	0.27156	0.03497	2.8107111	3.68	0.34425	0.1286896	0.1224779
0.96	0.19257	0.27243	0.03349	2.8524998	3.84	0.34924	0.1286016	0.12243296
1	0.19693	0.27325	0.0323	2.8875088	4	0.35403	0.1292	0.12260742

4.1 Analytical Solution for F (u)

In the above formulation the values of K (t) and F (u) are both generated by numerically integration. K (t) can be calculated only numerically by integrating drawdown data. However F(u) can be calculated analytically. But for this there is a need to adopt different form of G(s, t) and g(u) in Eq. (3.2) that would make the functional form g(u) integrable. This is done by using G(s, t) as s/t^2 and g(u) as simply W(u). The analytical solution given in this section will be valid only for this specific functional form of G(s, t) corresponding to which g(u) has become well function itself. The derivation of this solution is as follows.

$$\frac{\int_t^{2t} \frac{s}{t^2} dt}{\int_t^{4t} \frac{s}{t^2} dt} = \frac{\int_{2u}^{4u} W(u) du}{\int_u^{4u} W(u) du} \quad (4.1)$$

where

$$\int W(u) du = [uW(u) - e^{-u}] \quad (4.2)$$

$$F(u) = \frac{4uW(4u) - 2uW(2u) - e^{-4u} + e^{-2u}}{4uW(4u) - uW(u) - e^{-4u} + e^{-u}} \quad (4.3)$$

For u < 0.01 this analytical function can be rewritten with the help of approximation of well function from Eq. (1.6a) as:

$$F(u) = \frac{-2\gamma - 4u \ln 4u + 2u \ln 2u - e^{-4u} + e^{-2u}}{-3\gamma - 4u \ln 4u + u \ln u - e^{-4u} + e^{-u}} \quad (4.4)$$

where

$$W(u) = -0.5771 - \ln u \quad (4.5)$$

The plot of F(u) vs u is shown in Figure 4.3. The limiting values of F(u) at u approaching to zero and infinity are calculated from the analytical solution and these come out to be 0.6667 near 0 and 0 near infinity. These values are in agreement with the plot of F(u) vs u.

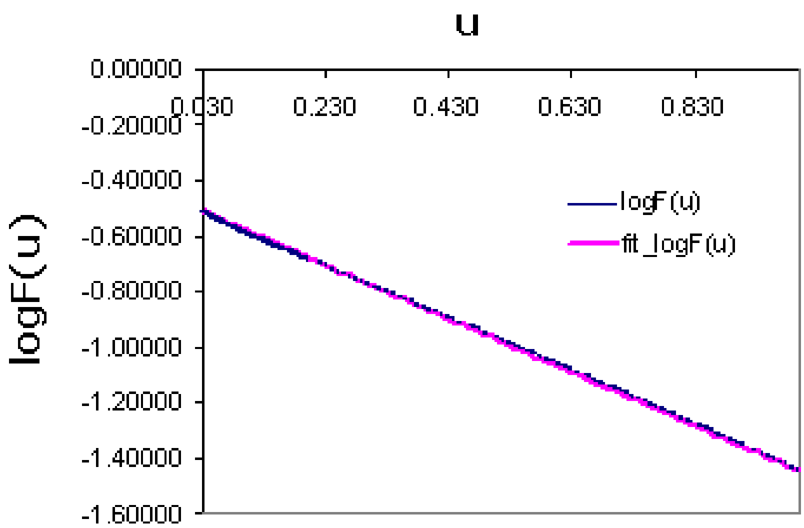

Figure 4.3: Plot of logF(u) and linear fit vs u

This methods aims at obtaining values of α and β, for this calculation, value of K(t) can be easily obtained for a given t, but F(u) is an implicit function of u and it is difficult to obtain value of u analytically from this functional form. Therefore a fit of F(u) vs u is obtained that can give an explicit functional dependence of F(u) on u, and using which u can be back calculated for a particular F(u)=K(t). For this curve, exponential and rational functions appear to be good fits. Different functional forms of rational and exponential or exponential and rational combined are tried. On plotting lnF(u) vs u, the curve obtained is linear for the range u > 0.03 and shows some curvature for u < 0.03. Since the method works well for low values of t (i.e. higher values of u), therefore we concentrate only on u > 0.03. The best fit obtained for u > 0.03 is:

$$F(u) = \exp(-0.485493 - 0.96u) \tag{4.6}$$

Therefore

$$u = \frac{-\ln F(u) - 0.485493}{0.96} \tag{4.7}$$

The RMSE and maximum percentage deviation for the lnF(u) fit are 0.006822 and 1.32888 respectively. Error calculations in u obtained will be as follows:

$$\frac{\Delta u}{u} = \frac{-\Delta \ln F(u)}{0.96u} \tag{4.8}$$

Therefore the percentage deviation for u can be calculated from this equation. The maximum percentage deviation for u comes out to be equal to 9.85068. Even a small percentage deviation in F(u) leads to larger percentage deviation in u primarily because in this region the curve is almost flat. Better estimates of u are obtained for u > 0.15.

Using the synthetic data set of s vs t, values of K(t) are obtained. Using these values of K(t) as F(u) in the analytical solution u is calculated which gives us the estimates of α and β. Percentage deviation of α and β are calculated and the maximum % deviation are 4.0096 and 8.76673 respectively. The average values of α and β obtained are 0.12298 and 0.12874 respectively. These results are summarised in Table 4.3.

Table 4.3: Values of α and β obtained using the fitting curve for u

t	K(t)	u_calc	β	α	%dev in α	%dev in β
0.04000	0.29379	0.77021	0.12323	0.11905	-1.77565	-1.25574
0.08000	0.42483	0.38601	0.12352	0.12004	-0.95966	-1.02212
0.12000	0.47813	0.26290	0.12619	0.12222	0.84159	1.11363
0.16000	0.50725	0.20131	0.12884	0.12380	2.14404	3.23627
0.20000	0.52584	0.16382	0.13105	0.12484	3.00019	5.01155
0.24000	0.53886	0.13834	0.13281	0.12549	3.54261	6.41545
0.28000	0.54857	0.11974	0.13411	0.12587	3.85661	7.45616
0.32000	0.55614	0.10546	0.13499	0.12605	3.99879	8.16476
0.36000	0.56223	0.09412	0.13553	0.12606	4.00960	8.59526
0.40000	0.56726	0.08484	0.13574	0.12596	3.92468	8.76673
0.44000	0.57150	0.07708	0.13566	0.12576	3.75874	8.70403
0.48000	0.57514	0.07047	0.13530	0.12547	3.52196	8.41152
0.52000	0.57829	0.06478	0.13474	0.12512	3.23830	7.96320
0.56000	0.58107	0.05978	0.13391	0.12471	2.89572	7.30164
0.60000	0.58353	0.05538	0.13292	0.12425	2.51848	6.50323
0.64000	0.58573	0.05146	0.13174	0.12376	2.11009	5.56271
0.68000	0.58771	0.04795	0.13041	0.12323	1.67617	4.49880
0.72000	0.58951	0.04476	0.12891	0.12266	1.20285	3.29470
0.76000	0.59116	0.04185	0.12722	0.12205	0.69954	1.94122
0.80000	0.59266	0.03921	0.12547	0.12143	0.19296	0.53793
0.84000	0.59405	0.03677	0.12355	0.12077	-0.35269	-1.00500
0.88000	0.59534	0.03451	0.12148	0.12008	-0.92289	-2.66408
0.92000	0.59653	0.03243	0.11934	0.11938	-1.49998	-4.37324
0.96000	0.59764	0.03049	0.11709	0.11866	-2.09907	-6.17400
1.00000	0.59868	0.02868	0.11473	0.11790	-2.71957	-8.06942

4.2 Error Propagation of α and β

This section focuses on how α and β vary with different u and how can one derive a relationship of percentage deviation in α and β with u.

Since

$$\beta = 4ut \tag{4.9}$$

$$\frac{\Delta\beta}{\beta} = \frac{\Delta u}{u} \tag{4.10}$$

Therefore error made in calculation of F(u) from the fit results in error in estimation of u and whatever error is made in this u is directly transferred to the estimation of β.

From Eq.(1.3)

$$s = \alpha W(u)$$

$$\frac{\Delta\alpha}{\alpha} = -\frac{\Delta W(u)}{W(u)} \tag{4.11}$$

Using

$$\Delta W(u) = \frac{\partial W}{\partial u}\Delta u$$

$$\frac{\Delta\alpha}{\alpha} = \frac{e^{-u}}{W(u)}\frac{\Delta u}{u} \tag{4.12}$$

Table 4.4 Error propagation in α and β.

t	W(u)	%dev in u	dev_cal α	%dev in α	%dev in β
0.04000	0.32209	-1.25574	-1.78719	-1.77565	-1.25574
0.08000	0.71944	-1.02212	-0.9619	-0.95966	-1.02212
0.12000	1.0139	1.113629	0.846893	0.841594	1.113629
0.16000	1.2434	3.236272	2.141641	2.144035	3.236272
0.20000	1.4308	5.011548	2.996699	3.000186	5.011548
0.24000	1.5889	6.41545	3.545458	3.542612	6.41545
0.28000	1.7256	7.456157	3.86529	3.856609	7.456157
0.32000	1.8459	8.164763	4.012284	3.998794	8.164763
0.36000	1.9533	8.595265	4.035073	4.009602	8.595265
0.40000	2.0503	8.766729	3.954988	3.924681	8.766729
0.44000	2.1388	8.704027	3.791007	3.758739	8.704027
0.48000	2.2201	8.411522	3.550364	3.521963	8.411522
0.52000	2.2953	7.963205	3.267313	3.238296	7.963205
0.56000	2.3653	7.301638	2.919696	2.895725	7.301638
0.60000	2.4306	6.503235	2.539994	2.518477	6.503235
0.64000	2.492	5.562706	2.126014	2.110088	5.562706
0.68000	2.5498	4.498805	1.685251	1.67617	4.498805
0.72000	2.6045	3.294701	1.211357	1.202854	3.294701
0.76000	2.6563	1.941218	0.701404	0.699539	1.941218
0.80000	2.7056	0.537928	0.191215	0.192955	0.537928
0.84000	2.7526	-1.005	-0.3518	-0.35269	-1.005
0.88000	2.7974	-2.66408	-0.91917	-0.92289	-2.66408
0.92000	2.8404	-4.37324	-1.48832	-1.49998	-4.37324
0.96000	2.8815	-6.174	-2.07412	-2.09907	-6.174
1.00000	2.9211	-8.06942	-2.6776	-2.71957	-8.06942

Eq.(4.12) is obtained with an assumption that there is no error in the drawdown data. This assumption is good for only demonstrative purpose because drawdown data is synthetically generated. However if the field data is used then there will be an additional error term including s also. The results obtained from the synthetic data are in accordance with the error estimation done. This is shown in Table 4.4.

True value of u is calculated from Eq.(4.9) using the value of β equal to 0.1248. If one wants to calculate the values of α and β with high accuracy, then the values of u should be read from the plot of F(u) vs u, so that the error in estimation of u from analytical fitting could be ruled out. The calculations of α and β using F(u) vs u plot are shown in Table 4.5.

Table 4.5 Calculation of α and β using F(u) vs u curve and synthetic drawdown data.

t	K(t)	u	s at 4t	W(u)	β	α
0.04000	0.29379	0.7743	0.039037	0.32546	0.123888	0.119944
0.08000	0.42483	0.3805	0.087196	0.73621	0.12176	0.118439
0.12000	0.47813	0.2592	0.12288	1.0163	0.124416	0.120909
0.16000	0.50725	0.1969	0.1507	1.2355	0.126016	0.121975
0.20000	0.52584	0.157	0.17341	1.4253	0.1256	0.121666
0.24000	0.53886	0.1295	0.19257	1.5923	0.12432	0.120938
0.28000	0.54857	0.1125	0.20914	1.717	0.126	0.121805
0.32000	0.55614	0.09812	0.22372	1.8401	0.125594	0.12158
0.36000	0.56223	0.0861	0.23674	1.9593	0.123984	0.120829
0.40000	0.56726	0.0782	0.2485	2.048	0.12512	0.121338
0.44000	0.57150	0.0706	0.25922	2.1429	0.124256	0.120967
0.48000	0.57514	0.0649	0.26908	2.2216	0.124608	0.12112
0.52000	0.57829	0.06016	0.27819	2.2928	0.125133	0.121332
0.56000	0.58107	0.05615	0.28667	2.3579	0.125776	0.121579
0.60000	0.58353	0.052	0.29459	2.4306	0.1248	0.121201
0.64000	0.58573	0.04877	0.30203	2.4916	0.124851	0.121219
0.68000	0.58771	0.04655	0.30904	2.536	0.126616	0.121861
0.72000	0.58951	0.04337	0.31566	2.6037	0.124906	0.121235
0.76000	0.59116	0.04045	0.32195	2.6705	0.122968	0.120558
0.80000	0.59266	0.03923	0.32792	2.6999	0.125536	0.121456
0.84000	0.59405	0.03703	0.33361	2.7555	0.124421	0.121071
0.88000	0.59534	0.03564	0.33905	2.7924	0.125453	0.121419
0.92000	0.59653	0.03442	0.34425	2.826	0.126666	0.121815
0.96000	0.59764	0.03309	0.34924	2.8641	0.127066	0.121937
1.00000	0.59868	0.03213	0.35403	2.8926	0.12852	0.122392

The values of α and β obtained using F(u) vs u curve are much closer to the actual values than the values obtained using linear fit. The average values of α and β are 0.121223 and 0.125131 respectively. Maximum percentage deviations are 2.278 and 2.98 respectively.

Chapter 5
Conclusions

A new approach for determination of confined aquifer parameters has been discussed. The method requires that field conditions satisfy Theis' assumptions. In the foregoing discussion it has been shown that:

- Using this method β and α can be calculated quite accurately for $0.01<u<1$
- Applicability of this method in terms of time range t is contingent on the value of β, larger the value of beta larger is time range for which it is applicable.
- Using tabulated values of well function and analytical expression developed the values of β and α can be estimated quickly and has onsite applicability.
- For the range $u>0.03$, a fit has been generated for u and F(u) values from which u can be calculated using F(u)

Early drawdown data i.e. for which $u>0.01$, has often been considered unimportant in determination of aquifer parameters. In many cases of pump tests, substantial data have $u > 0.01$, especially when a pump test is for short duration and the observation well is at a large distance from the pumping well. Another important point is that early drawdown data are not corrupted by the effect of hydrological boundaries, also because of certain factors such as pump failure and time and resource constraints, long duration pump tests sometimes are not feasible, in all such cases this method can be effective in determination of aquifer parameters from early drawdown data.

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