Analysis of Variance

LECTURE - 1

FACTORIAL EXPERIMENTS

Dr. Shalabh
Department of Mathematics and Statistics
Indian Institute of Technology Kanpur

Factorial experiments involve simultaneously more than one factor each at two or more levels.

Several factors affect simultaneously the characteristic under study in factorial experiments and the experimenter is interested in the main effects and the interaction effects among different factors.

First we consider an example to understand the utility of factorial experiments.

Example: Suppose the yield from different plots in an agricultural experiment depend upon

1. (i) variety of crop and (ii) type of fertilizer.

Both the factors are in the control of experimenter.

2. (iii) Soil fertility. This factor is not in the control of experimenter.

In order to compare the different crop varieties

- assign it to different plots keeping other factors like irrigation, fertilizer, etc. fixed and the same for all the plots.
- The conclusions for this will be valid only for the crops grown under similar conditions with respect to the factors like fertilizer, irrigation etc.

In order to compare different fertilizers (or different dosage of fertilizers)

- sow single crop on all the plots and vary the quantity of fertilizer from plot to plot.
- The conclusions will become invalid if different varieties of crop are sown.
- It is quite possible that one variety may respond differently than another to a particular type of fertilizer.

Through the factorial experiments, we can study the individual effect of each factor and interaction effect.

Now we consider a 2^2 factorial experiment with an example and try to develop and understand the theory and notations through this example.

Factors

Levels

A general notation for representing the factors is to use capital letters, e.g., A,B,C etc. and levels of a factor are represented in small letters.

For example, if there are two levels of A, they are denoted as a_0 and a_1.

Similarly the two levels of B are represented as b_0 and b_1.

Other alternative representation to indicate the two levels of A is 0 (for a_0) and 1 (for a_1).

The factors of B are then 0 (for b_0) and 1 (for b_1).

(*ab*): Mean of all observations which receive the treatment combinations *ab*.

Note: An important point to remember is that the factorial experiments are conducted in a design of experiment. For example, the factorial experiment is conducted as an RBD.

Example:

Two factors: Irrigation (I) and Nitrogen (N).

Levels:

Irrigation has 2 levels: I_0 and I_1

Nitrogen has 3 levels: N_0, N_1 and N_2

We need two RBD

N_0	N_1	N_2
N_2	N_0	N_1
N_1	N_2	N_0

N_0	N_1	N_2
N_2	N_0	N_1
N_1	N_2	N_0

I_0N_0	I_0N_1	I_0N_2
I_0N_2	I_0N_0	I_0N_1
I_0N_1	I_0N_2	I_0N_0

I_1N_0	I_1N_1	I_1N_2
I_1N_2	I_1N_0	I_1N_1
I_1N_1	I_1N_2	I_1N_0

Total number of plots required : 18

I_0N_0	I_0N_1	I_0N_2
I_0N_2	I_0N_0	I_0N_1
I_0N_1	I_0N_2	I_0N_0

Difference among N levels

Difference among I levels

I_1N_0	I_1N_1	I_1N_2
I_1N_2	I_1N_0	I_1N_1
I_1N_1	I_1N_2	I_1N_0

Difference among N levels