$2^3$ F.E $\rightarrow$ 3 factors: A, B, C.
- 8 treat combinations
A B C: to be confounded
Full factorial: $2^3 = 8$ plots per block
Confound A B C: size of block = 4 plots

\[ ABC = (a-1)(b-1)(c-1) \]
\[ = (a+b+c+abc) - ((1)+ab+bc+ac) \]

Defining contrast:

- Block 1
  - \[ a, b, c, abc \]
- Block 2
  - \[ (1) ab bc ac \]

Confounding arrangement
2³ Factorial Expt

ABC in confounded

\[ ABC = (a-1)(b-1)(c-1) \]

\[
= \left( a+b+c+abc \right) - \left( (1) + ab + bc + ac \right)
\]

Block 1

\[ a \ b \ c \ abc \]

Block 2

\[ (1) \ ab \ bc \ ac \]

Con founding arrangement

Arrangement of treatment combinations in different blocks, whereby some predetermined interactions as contrasts are confounded is called confounding arrangement.
Defining contrast:
The interactions which are confounded are called the defining contrasts of the confounding arrangement.

Requirements of conf. arrangement:

- Only predetermined interaction are confounded.
- Estimates of interactions which are not confounded are orthogonal whenever interactions are orthogonal.
- SS of ANOVA for unconfounded expt are still mutually orthogonal.
ABC and BCD: Defining Contrast

\[\begin{align*}
ABC \times BCD &= A B C \land c^2 D \\
&= A B \land 1 \land 1 \land D = AD \\
&= A B C \times B C D \times A B E \\
&= A^2 B^3 C^2 E \\
&= A \land B \land A \land C \land A B E \\
&= A^3 B^2 C E = A C E
\end{align*}\]
Generalized interaction:
Given any two interactions, the generalized interactions is obtained by multiplying the expressions in CAPITAL letters and ignoring all terms with an even exponent.

Eg. Generalized interaction of
- \(A\,B\,C\) & \(B\,C\,D\) is \(A\,B\,C\,D^2 = AD\)
- \(A\,B\), \(A\,C\), \(A\,B\,E\) is \(A\,B\,C\,E = ACE\)

Independent interaction:
A set of main effects and interaction contrasts is called independent if NO member of the set can be obtained as a generalized interaction of the other members of the set.

Eg. \(A\,B\), \(B\,C\), \(A\,D\) : independent set
\(A\,B\), \(B\,C\), \(C\,D\), \(A\,D\) : NOT independent set
because \((A\,B)(B\,C)(C\,D) = A\,B\,C\,D^2 = A\)
Orthogonality of treatment combination and Contrast

- The treatment combination $a^p b^q c^r \ldots$ is said to be orthogonal to the interaction $A^x B^y C^z \ldots$ if
  $$px + qy + rz + \ldots \text{ is divisible by } 2$$

Since $p, q, r, \ldots, x, y, z, \ldots$ are either 0 or 1, so check for even no. of letters in common

- Treat combination (1) is orthogonal to every interaction

- If $a^p b^q c^r \ldots$ and $a^{p'} b^{q'} c^{r'} \ldots$ are both orthogonal to $A^x B^y C^z \ldots$, then the product $a^{p+p'} b^{q+q'} c^{r+r'} \ldots$ is also orthogonal to $A^x B^y C^z \ldots$

- Similarly, if two interactions are orthogonal to a treatment combination their generalized interaction is orthogonal to it.
How to obtain confounding arrangement

- Suppose we wish to have a confounding arrangement in $2^b$ blocks of a $2^n$ factorial expmt.
- Block size = $2^{n-p}$ plots
- Total no. of interactions to be confounded = $2^b - 1$ (i.e. # of elements in defining contrasts = $2^b - 1$)

How?

If $p$ factors are to be confounded, then order interactions (main) effect with $p$ factor = $p_c$,

\[
\begin{array}{cccc}
 & \text{I} & \text{II} & \text{III} \\
\text{I} & u & u & u \\
\text{II} & u & u & u \\
\text{III} & u & u & u \\
\vdots & \ddots & \ddots & \ddots \\
\end{array}
\]

Total no. of factors to be confounded = $p_c_1 + p_c_2 + p_c_3 + \ldots + p_c_p = 2^b - 1$
- If any two interactions are confounded, their generalized interaction is also confounded.

- \( p \) out of \( (2^p - 1) \) defining contrasts are independent and rest are obtained as generalized interaction.

- No. of effects confounded automatically = \( 2^p - p - 1 \).

So confound only \( (p-1) \) effects.

**Example:** Consider

2^5 factorial \((n = 5)\) → factors A, B, C, D, confounded in \( 2^3 = 8 \) blocks \((p = 3)\) of size \( 2^{5-3} = 4 \) each \((2^{n-p})\).

Then

# of defining contrasts = \( 2^3 - 1 = 7 \) (\( 2^p - 1 \)).

# of independent contrasts out of 7 defining contrasts = 3 (\( p \)).
Choose any $p = 3$ independent contrasts, say

(i) $ACE$  
(ii) $ABDE$  
(iii) $CDE$

Then 7 defining contrasts are

(iv) $ACE \times ABDE = A^2BCDE^2 = BCD$
(v) $ACE \times CDE = AC^2DE^2 = AD$
(vi) $ABDE \times CDE = ABCD^2E^2 = ABC$
(vii) $ACE \times ABDE \times CDE = A^2BC^2D^2E^3 = BE$

If we choose another set of independent contrasts as

(i) $ABCD$  
(ii) $ACDE$  
(iii) $ABCDE$

Then defining contrasts are

(iv) $ABCD \times ACDE = A^2BC^2D^2E = BE$
(v) $ABCD \times ABCDE = A^2B^2C^2D^2E = E$
(vi) $ACDE \times ABCDE = A^2BC^2D^2E^2 = B$
(vii) $ABCD \times ACDE \times ABCDE = A^3B^2C^3D^3E$

Note: Main effects are confounded with $ACD$. 
As a rule
→ try to confound, as far as possible higher order interactions only because they are difficult to interpret
→ choose independent contrasts to be confounded carefully
→ it is possible that in trying to confound only higher order interactions, some main effects may get confounded as well.

Now divide \(2^n\) treats combi. into \(2^k\) groups of \(2^{n-k}\) combinations each, each group going into one block.

Group containing the combination (1) is called the principal block or key block.
How to obtain principal block:

It contains all treat combs, which are orthogonal to \( p \) independent defining contrasts

- Write treat combs in standard order
- Check each one of them for orthogonality
- If two treat combs belong to principal block, their product also belongs to principal block.

When few combs of principal block have been determined, many of others can be obtained by multiplication rule.
Example: Consider $2^5$ factorial in $2^3$ groups $(b = 3)$.

→ Main effects: A, B, C, D, E

→ Suppose we want to confound 3 effects:
  1. $AD$
  2. $BE$
  3. $ABC$

→ Other generalized interactions are automatically confounded:
  4. $AD \times BE = ADBE$
  5. $AD \times ABC = BCD$
  6. $BE \times ABC = ACE$
  7. $AD \times BE \times ABC = CDE$

→ Write treatment combs in standard order:

1. a  b  ab  c  ac  bc  abc
d  ad  bd  abd  cd  acd  bcd  abcd
e  ae  be  abe  ce  ace  bce  abce
d e  ade  bde  abde  cde  acde  bcde  abcde
Place a treat combo in the principal block if it has an even no. of letters in common with the confounded effects (AD, BE and ABC)

Principal block has

(i) , acd , bce , abde = (acd) x (bce)

Note that in

AD, BE, ABC

& (i), acd, bce, abde

Even no. of alphabets are common between them

If independent defining contrasts are ACD, ABCD, ABCDE, then principal block has

(i) , ac , ad , cd = (ac) x (cd)
→ obtain other blocks of configuration arrangement from principal block by multiplying the combinations of principal block by a combination not occurring in it

OR SIMPLY
choose treat combs not occurring in it and multiply with them in the principal block. Choose only DISTINCT blocks
obtain other blocks by multiplying by a, b, ac, c, ac, bc, abc

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>ab</th>
<th>c</th>
<th>ac</th>
<th>bc</th>
<th>abc</th>
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<td></td>
<td>cde</td>
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</table>

Choose any other combination – some block will be repeated e.g. multiply by ae, gives ae, cde, bcd, abc, bd – last block
ANOVA

<table>
<thead>
<tr>
<th>Effects</th>
<th>S.S.</th>
<th>D.F.</th>
<th>F</th>
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<tr>
<td>Unconfounded effects</td>
<td>SS</td>
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<td>F = \frac{SS}{SSE} \sim F(1, df_e)</td>
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<tr>
<td>Confounded effects</td>
<td>SS</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Residuals</td>
<td>SSE</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

Total

\[ TSS \]

\[ df \]

Same