2^3 F.E → 3 factors : A, B, C.

→ 8 treat combinations

ABC : to be confounded

Full factorial : $2^3 = 8$ plots per block

Confound ABC : size of block = 4 plots

$$ABC = (a-1)(b-1)(c-1)$$

$$= \boxed{(a + b + c + abc)} - \boxed{((1) + ab + bc + ac)}$$

Block 1 | Block 2

$\boxed{a, b, c, abc}$ | $\boxed{(1)\ ab\ bc\ ac}$

Defining contrast

confounding arrangement

2^3 Factorial Expt

ABC is confounded : Defining contrast

$$ABC = (a-1)(b-1)(c-1)$$

$$= \underbrace{(a+b+c+abc)}_{\text{Bl. 1}} - \underbrace{((1)+ab+bc+ac)}_{\text{Bl. 2}}$$

a	b	c	abc

Block 1

(1)	ab	bc	ac

Block 2

Confounding arrangement

Arrangement of treatment combinations in diff. blocks, whereby some predetermined interactions ~~an~~ contrasts are confounded is called confounding arrangement

Defining contrast :

The interactions which are confounded are called the defining contrasts of the confounding arrangement

Requirements of conf. arrangement

- Only predetermined interaction are confounded
- Estimates of interactions which are not confounded are orthogonal whenever interactions are orthogonal
- SS of ANOVA for unconfounded exp^nt are still mutually orthogonal

ABC and BCD : Defining contrast

$ABC \times BCD$

$= AB^2C^2D \qquad B^2 \equiv 1$

$= A.1.1.D = \underline{\underline{AD}}$

$ABC \times BCD \times ABE$

$= A^2B^3C^2E$

$= 1.B^2.B.1.E = \underline{\underline{B.E}}$

$AB \times AC \times ABE$

$= A^3B^2CE = \underline{\underline{ACE}}$

Generalized interaction :

Given any two interactions, the generalized interactions is obtained by multiplying the expressions in CAPITAL letters and ignoring all terms with an even exponent.

Eg. Generalized interaction of

- ABC & BCD is $AB^2C^2D = AD$
- AB, AC, ABE is $A^3B^2CE = ACE$

Independent interaction :

A set of main effects and interaction contrasts is called independent if NO member of the set can be obtained as a generalized interaction of the other members of the set.

Eg. AB, BC, AD : independent set

AB, BC, CD, AD : NOT independent set

because $(AB)(BC)(CD) = AB^2C^2D = A$

Orthogonality of treatment combinations and Contrast

- The treat. combination $a^p b^q c^r \ldots$ is said to be orthogonal to the interaction $A^x B^y C^z \ldots$ if $px + qy + rz + \ldots$ is divisible by 2

[Since $p, q, r, \ldots, x, y, z, \ldots$ are either 0 or 1 so check for even no. of letters in common

- Treat combination (1) is orthogonal to every interaction
- If $a^p b^q c^r \ldots$ and $a^{p'} b^{q'} c^{r'} \ldots$ are both orthogonal to $A^x B^y C^z \ldots$, then the product $a^{p+p'} b^{q+q'} c^{r+r'} \ldots$ is also orthogonal to $A^x B^y C^z \ldots$
- Similarly, if two interactions are orthogonal to a treat. combination their generalized interaction is orthogonal to it.

How to obtain confounding arrangement

- Suppose we wish to have a conf. arrangement in 2^p blocks of a 2^n factorial expt.
- Block size $= 2^{n-p}$ plots
- Total no. of interactions to be confounded $= 2^p - 1$
  (i.e. # of elements in defining contrasts $= 2^p - 1$)

How ?

If p factors are to be confounded, then

I order interactions (main) effect with p factor $= {}^pC_1$

II " " " " " $= {}^pC_2$

III " " " " " $= {}^pC_3$

⋮

p " " " " " $= {}^pC_p$

Total no. of factors to be confounded

$$= {}^pC_1 + {}^pC_2 + {}^pC_3 + \cdots + {}^pC_p = 2^p - 1$$

- If any two interactions are confounded, their generalized interaction is also confounded
- p out of $(2^p - 1)$ defining contrasts are independent and rest are obtained as generalized interactio
- No. of effects confounded automatically $= 2^p - p - 1$

So confound only $(p-1)$ effects

Example: Consider

2^5 factorial $\boxed{(n = 5)} \rightarrow$ factors A, B, C, D,

Confounded in $2^3 = 8$ blocks $\boxed{(p = 3)}$

of size $2^{5-3} = 4$ each $\boxed{(2^{n-p})}$.

Then # of defining contrasts $= 2^3 - 1 = 7$ $\boxed{(2^p - 1)}$

\# of independent contrasts out of 7 defining contrasts $= 3$ $\boxed{(p)}$

Choose any $p = 3$ independent contrasts, say

(i) ACE (ii) $ABDE$ (iii) CDE

then 7 defining contrasts are

(iv) $ACE \times ABDE = A^2BCDE^2 = BCD$

(v) $ACE \times CDE = AC^2DE^2 = AD$

(vi) $ABDE \times CDE = ABCD^2E^2 = ABC$

(vii) $ACE \times ABDE \times CDE = A^2BC^2D^2E^3$

$= BE$

If we choose another set of indep. contrasts as

(i) $ABCD$ (ii) $ACDE$ (iii) $ABCDE$

then defining contrasts are

(iv) $ABCD \times ACDE = A^2BC^2D^2E = BE$

(v) $ABCD \times ABCDE = A^2B^2C^2D^2E = E$

(vi) $ACDE \times ABCDE = A^2BC^2D^2E^2 = B$

(vii) $ABCD \times ACDE \times ABCDE = A^3B^2C^3D^3E$

Note: Main effects are confounded here $= ACD$

As a rule

→ Try to confound, as far as possible higher order interactions only because they are difficult to interpret

→ Choose independent contrasts to be confounded carefully

→ It is possible that in trying to confound only higher order interactions, some main effects may get confounded as well

Now divide 2^n treats combi. into 2^p groups of 2^{n-p} combinations each, each group going into one block.

Group containing the combination (1) is called the principal block or key block

How to obtain principal block:

It contains all treat combis. which are orthogonal to p independent defining contrasts

- write treat combs. in standard order
- Check each one of them for orthogonality
- If two treat combs. belong to principal block, their product also belongs to principal block.

[When few combs. of principal block have been determined, many of others can be obtained by multiplication rule]

Example: Consider 2^5 factorial (n = 5) in 2^3 groups ($p = 3$)

→ Main effects : A, B, C, D, E

→ Suppose want to confound 3 effects
(i) AD (ii) BE (iii) ABC

→ other generalized interactions are automatically confounded

(iv) AD × BE = ADBE

(v) AD × ABC = BCD

(vi) BE × ABC = ACE

(vii) AD × BE × ABC = CDE

→ write treat combs in standard order

(1)	a	b	ab	c	ac	bc	abc
d	ad	bd	abd	cd	acd	bcd	abcd
e	ae	be	abe	ce	ace	bce	abce
de	ade	bde	abde	cde	acde	bcde	abcde

→ Place a treat combo in the principal block if it has an even no. of letters in common with the confounded effects (AD, BE and ABC)

→ Principal block has

(1), acd, bce, abde = (acd) x (bce)

Note that in
AD, BE, ABC
& (1), acd, bce, abde
Even no. of alphabets are common between them

If independent defining contrasts are ACD, ABCD, ABCDE, then principal block has

(1), ac, ad, cd = (ac) x (cd)

→ obtain other blocks of conf. arrangement from principal block by multiplying the combinations of principal block by a combination not occurring in it

OR SIMPLY

choose treat combs not occurring in it and multiply with them in the principal block. Choose only DISTINCT blocks

obtain other blocks by multiplying by a, b, ac, c, ac, bc, abc

(1)	a	b	ab	c	ac	bc	abc
acd	cd	abcd	bcd	ad	d	abd	bd
bce	abce	ce	ace	be	abe	e	ae
abde	bde	ade	de	abcde	bcde	acde	cde
1	2	3	4	5	6	7	8

Blocks

Choose any other combination → some block will be repeated

Eg multiply by ae, gives ae, cde, abc, bd → last block

ANOVA

Effects.	S.S.	D.F.	F
Unconfounded effects	SS	1	$F = \frac{SS}{SSE^*}$
	SS	1	$\sim F(1, df_e^*)$
	⋮		
	SS	1	
Confounded effects	SS	1	
	SS	1	
	⋮		
SSE^*	SS	1	df_e^*
Residuals	SSE	df_e	
Total	TSS	df	

same