Assignment -1

MTH 314-Multivariate Analysis 2024

1. Draw random samples of sizes 5, 50, 200, 500, and 1000 from $N_3(\underline{\mu}, \Sigma)$

where $\underline{\mu} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$, $\Sigma = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 3 & 2 \\ 1 & 2 & 2 \end{pmatrix}$

and (i) establish $E(\bar{\underline{X}}) = \underline{\mu}$ and $E(S) = \Sigma$. Use any s/w language to do this.

2. Partition the random vector $\underline{X} = (X_1, X_2, X_3, X_4, X_5)'$ of order 5×1 into two components $\underline{X} = (X_1, X_2)'$ such that $\underline{X}_1 = (X_1, X_4)'$ and $\underline{x}_2 = (X_2, X_3, X_5)'$. The random vector $\underline{X}$ is having a multivariate normal distribution $N_5(\underline{\mu}, \Sigma)$ with mean vector and covariance matrix as follows:

$$\underline{\mu} = \begin{pmatrix} 2 \\ 15 \\ 6 \\ 20 \\ -10 \end{pmatrix}, \quad \Sigma = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 10 & 0 & 0 \\ 0 & 0 & 0 & 20 & 0 \\ 0 & 0 & 0 & 0 & 30 \end{pmatrix}$$

(i) Write the suitable partitioned vectors μ_1 , μ_2 and matrices $\Sigma_{11}, \Sigma_{12}, \Sigma_{12}$ and Σ_{21}

(ii) Compute $E(\underline{X}_1 | \underline{X}_2 = \underline{x}_2)$, $E(\underline{X}_2 | \underline{X}_1 = \underline{x}_1)$, $V(\underline{X}_1 | \underline{X}_2 = \underline{x}_2)$, and $V(X_2 | \underline{X}_2 = \underline{x}_2)$

(iii) Simulate $\underline{X}_1 | \underline{X}_2 = \underline{x}_2$, and $\underline{X}_2 | \underline{X}_1 = \underline{x}_2$ by draw random samples of sizes 5, 50, 200, 500, and 1000 from $N(\underline{\mu}, \Sigma)$ and establish the unbiased properties of the estimators of $E(\underline{X}_1 | \underline{X}_2 = \underline{x}_2)$, $E(\underline{X}_2 | \underline{X}_1 = \underline{x}_1)$, $V(\underline{X}_1 | \underline{X}_2 = \underline{x}_2)$, and $V(X_2 | \underline{X}_2 = \underline{x}_2)$

3. Draw the contour diagram of $\underline{X} = (X_1, X_2)'$ from $N_2(\underline{\mu}, \Sigma)$ by generating the random numbers where $\underline{\mu} = \begin{pmatrix} 2 \\ 10 \end{pmatrix}$, $\Sigma = \begin{pmatrix} 2 & 3 \\ 3 & 15 \end{pmatrix}$.

4. Let the random vector $\underline{X} = (X_1, X_2, X_3)'$ is having a multivariate normal distribution $N_3(\underline{\mu}, \Sigma)$. Find the distribution of $\begin{pmatrix} X_1 - X_2 \\ X_2 - X_3 \end{pmatrix}$.

5. If the random vector $\underline{X}$ is having a multivariate normal distribution $N_p(\underline{\mu}, \Sigma)$ with $|\Sigma| \neq 0$, Show that the joint density can be written as the product of marginal densities of $\underline{X}_1$ and $\underline{X}_2$ if $\Sigma_{12} = 0$ where $\underline{X}_1$ and $\underline{X}_2$ are of orders $q \times 1$ and $((p - q) \times 1)$ respectively, and Σ_{12} is of order $((p - q) \times q)$.

6. Suppose $X^{(1)}$ and $X^{(2)}$ of q and (p - q) components, respectively have the density

$$\frac{1A1^{1/2}}{(2\pi)^{\frac{p}{2}}} exp\left(\frac{Q}{2}\right), \quad \text{where}$$

$$Q = (x^{(1)} - \mu^{(1)})'A_{11}(x^{(1)} - \mu^{(1)}) + (x^{(1)} - \mu^{(1)})'A_{12}(x^{(2)} - \mu^{(2)})$$
$$+(x^{(2)} - \mu^{(2)})'A_{21}(x^{(1)} - \mu^{(1)})+(x^{(2)} - \mu^{(2)})'A_{22}(x^{(2)} - \mu^{(2)}).$$

Show that Q can be written as $Q_1 + Q_2$, where

$$Q_1 = \left[(x^{(1)} - \mu^{(1)}) + A_{11}{}^{-1}A_{12}(x^{(2)} - \mu^{(2)})\right]' A_{11} \left[(x^{(1)} - \mu^{(1)}) + A_{11}{}^{-1}A_{12}(x^{(2)} - \mu^{(2)})\right],$$

$$Q_2 = (x^{(2)} - \mu^{(2)})'(A_{22} - A_{21}A_{11}^{-1}A_{12})(x^{(2)} - \mu^{(2)}).$$

(i) Show that the marginal density of $X^{(2)}$ is $\frac{|A_{22}-A_{22}A_{11}^{-1}A_{12}|^{1/2}}{(2\pi)^{\frac{p-q}{2}}} exp\left[-\frac{Q_2}{2}\right]$.

(ii) Show that the conditional density of $X^{(1)}$ given $x^{(2)}$ is $\frac{|A_{11}|^{1/2}}{(2\pi)^{\frac{q}{2}}} exp\left[-\frac{Q_1}{2}\right]$.

7. Describe the set up of finding the marginal distribution of a submatrices of $A \sim W(\Sigma, n)$ when the partitioning of Z_α is partitioned in two subvectors with q and (p- q) elements such that the partitions are not independent. Find the marginal density of $A_{11}$and A_{22} in this case where $A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}$.

8. Given $A \sim W(\Sigma, n)$, find the density of inverted Wishart Distribution.