Assignment 2
MTH 314-Multivariate Analysis

All questions are in reference of T^2, so all terms have the usual meaning. The numerical-based questions have to be analyzed using any open-source software.

Q.1 Eight men received 2 mg/kg of a certain drug. The recorded changes in blood sugar and blood pressure (systolic and diastolic) are listed below.

Men No.	1	2	3	4	5	6	7	8
Blood sugar	30	90	-10	10	30	60	0	40
Blood Pressure (Systolic)	-8	7	-2	0	-2	0	-2	1
Blood Pressure (diastolic)	-1	6	4	2	5	3	4	2

Test $Ho: \mu = 0$ $vs.$ $H_1: \mu \neq 0$. Also construct 95% confidence interval for the case of systolic and diastolic blood pressure change only.

Q.2 Two measurements, stride and strain, were taken on eight dogs for two treatments which are given in the following table. The dogs were randomly assigned, four to each treatment. Treatment 1 was control and treatment 2 was a metal plate placed on the leg of dogs. Test $H_o: \mu_1 = \mu_2$ against $H_i: \mu_1 \neq \mu_2$. Also construct 95% confidence interval for $(\mu_1 - \mu_2)$. Assume equal covariance matrices of the populations.

	Contral				Plate			
dog	1	2	3	4	1	2	3	4
Stride	131.5	145	141	150	40.5	80	50	90
Strain	9	12	30	36	54	74.5	64.5	60.5

Q: 3 Consider the measurements in the following table on flea beetle Halticus for two species, Haltica oeracea and Haltica carduorum. These measurements are of the thorax length x_1 (in microns) and elytra length x_2 (.01 mm).

Haltica oleracea		Haltica Carduorum	
x_1	x_2	y_1	y_2
180	245	181	305
192	260	158	237
217	276	184	300
221	299	171	273
171	239	181	297
192	262	181	308
213	278	177	301
192	255	198	308
170	244	180	286
201	276	177	299
		176	317
		192	312
		164	265

Test $H_0: \mu_1 = \mu_2$ vs $H_1: \mu_1 \neq \mu_2$ assuming (i)equal and (ii) unequal covariance matrices of the populations. Does the conclusions in (i) and (ii) changes?

Q. 4 Tires were measured for their wear during the first 1000 revolutions, the second 1000 revolutions, and the third 1000 revolutions. Two types of filler in the tires were used, F_1 and F_2. The data are given in the following table. Is there a significant difference in the two fillers? If so, which periods differ? Assume equal covariance.

F_1 Period			F_2 Period		
1	2	3	1	2	3
194	192	141	239	127	90
208	188	165	189	105	85
233	217	171	224	123	79
241	222	201	243	123	110
265	252	207	243	117	100
269	283	191	226	125	75

Q.5 Glycine in the spinal cord of cates with local tetanus rigidity (see Sema and Kano, 1969): The left sides of the cats were considered a control, and the right sides have local tetanus rigidity. The amount of glycine present in the gray and white matter was recorded. The data are given in the following table. Is there a difference between control and tetanus for the two characteristics of gray and white matter? Assume equal covariance matrices for the papulations and analyze it using any open source software.

	Gray Matter		White Matter	
	Control (L)	Tetanus (R)	Control (L)	Tetanus (R)
1	5.7	4.6	3.0	3.6
2.	6.1	5.9	3.2	2.7
3.	5.6	5.3	2.7	2.9
4.	6.1	5.8	3.3	3.4
5.	6.7	6.6	2.9	3.8
6.	5.4	5.3	3.0	2.8
7.	5.9	5.5	3.3	3.1
8.	5.9	5.4	3.9	3.5
9.	5.7	5.2	2.8	3.6
10.	4.8	4.4	2.6	2.6
11.	5.8	5.2	2.7	3.2

Q. 6 Consider an i.i.d. sample of size 5 from a bivariate normal distribution

$$\underline{X} \sim N_2\left(\underline{\mu}, \begin{pmatrix} 3 & \rho \\ \rho & 1 \end{pmatrix}\right)$$

where ρ is a known parameter. Suppose $\underline{\bar{x}}' = (1, 0)$. For what value of ρ would $H_0 : \mu' = (0, 0)$ be rejected in favor of $H_1 : \mu' \neq (0, 0)$ (at 5% level of significance).

Q. 7 The following table gives the binding of Warfarin to albumin A/A and albumin A/Me variants.

A/A		A/Me	
x_1	x_2	x_1	x_2
2.04	98.7	1.69	98.3
1.70	98.2	1.00	97.8
1.72	98.5	1.16	97.5
1.01	98.5	1.18	97.7
1.21	98.2	0.92	96.7
1.22	98.1	1.06	96.7
1.69	97.2	1.24	96.3
1.40	96.7	1.05	96.9
1.88	97.2	0.68	97.6
1.92	97.6	1.69	97.4
2.07	97.8	1.68	97.7
1.41	97.9	1.38	96.8
1.42	97.2	0.87	96.9
		1.76	97.7
		1.68	96.2

(a) Perform a two-sample T^2 test on the above data assuming equal covariance. If there is a significant difference, for which characteristic does it occur ?

(b) Because x_2 is actually a proportion, a variance stabilizing transform such as $y = arcsin(x_2/100)^{\frac{1}{2}}$ generally eliminates the dependence between the mean and the variance of x_2. Use a T^2 test based on x_1 and y to test for differences between the two albumin variants. What characteristics for difference are different, if any, for the two variants ?

(c) Conduct the analysis in (b) and (c), assuming unequal covariances and see if the conclusions change.

Q: 8 Let $x_1, ..., x_{19}$ be i.i.d. $N_3(\mu, \Sigma)$ The sample mean vector and the sample covariance matrix from the data is given by

$$\bar{x} = \begin{pmatrix} 194.5 \\ 136.9 \\ 185.9 \end{pmatrix} \quad S = \begin{pmatrix} 187.6 & 45.9 & 113.6 \\ & 69.2 & 15.3 \\ & & 239.9 \end{pmatrix}$$

(a) Test the hypothesis $H_0 : \mu_1 - 2\mu_2 + \mu_3 = 0$ and $\mu_1 - \mu_3 = 0$, against the alternative $H_1 : \mu_1 - 2\mu_2 + \mu_3 \neq 0$ and $\mu_1 - \mu_3 \neq 0$, at 5% level of significance.

(b) Obtain 95% simultaneous confidence intervals for $\mu_1 - 2\mu_2 + \mu_3$ and $\mu_1 - \mu_3$.

(c) Test $H_0 : \mu_1 = \mu_2 = \mu_3$ against the alternative hypothesis that at least one mean is different from others.

Q. 9 A treatment was given to six subjects and their response at times 0, 1, and 2 were recorded. The sample mean vector and sample covariance matrix are given by

$$\bar{x} = \begin{bmatrix} 5.50 \\ 10.67 \\ 19.17 \end{bmatrix}, S = \begin{bmatrix} 1.91.84.3 \\ 1.82.676.07 \\ 4.36.0717.37 \end{bmatrix}$$

respectively. Assume that the observation vectors are i.i.d. $N_3(\mu, \Sigma)$ where $\mu = (\mu_1, \mu_2, \mu_3)'$ are the population mean responses at times 0,1, and 2, respectively. We wish to know whether the treatment has been effective over time, namely, we wish to test the hypothesis

$$H: \begin{pmatrix} \mu_2 - \mu_1 \\ \mu_3 - \mu_2 \end{pmatrix} = 0$$

against the alternative that it is different from zero. With the above information, carry out a test and give a 95% confidence interval for $\mu_3 - \mu_2$.

Q. 10 The data in the following table represent eight cows' respiratory rates over 15 minutes after the administration of the Ketamin drug.

	Cow							
Time (minutes)	1	2	3	4	5	6	7	8
0	20	28	22	20	28	36	30	28
5	36	20	24	36	32	44	32	38
10	28	52	16	18	30	52	28	28
15	20	44	26	12	30	48	30	20

Assuming normality with mean vector $\mu = (\mu_1, \mu_2, \mu_3, \mu_4)$, test the hypothesis $H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4$ against the alternative hypothesis that at least one mean is different from others at a 5% level of significance.

Q. 11 Simulate a normal sample with $\mu = (1, 2)'$ & $\Sigma = \begin{pmatrix} 1 & 0.5 \\ 0.5 & 2 \end{pmatrix}$ and test $H_0 : 2\mu_1 - \mu_2 = 0.2$ first with Σ known and then with Σ unknown. Compare the results.

Q.12. Let $X \sim N_2(\underline{\mu}, \Sigma)$ where Σ is known to be $\Sigma = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$. We have an iid sample of size n=6 providing $\underline{x} = (1 \quad 0.5)$. Solve the following test problems:

$(a)\begin{matrix} H_0 : \underline{\mu} = (2, 2/3)' \\ H_1 : \underline{\mu} \neq (2, 2/3)' \end{matrix}$, $(b)\begin{matrix} H_0 : \mu_1 + \mu_2 = 7/2 \\ H_1 : \mu_1 + \mu_2 \neq 7/2 \end{matrix}$, $(c)\begin{matrix} H_0 : \mu_1 = \mu_2 = 1/2 \\ H_1 : \mu_1 = \mu_2 \neq 1/2 \end{matrix}$,

For each case, calculate the rejection region and comment on their size and location. If Σ is unknown and if $S = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$. Then solve (a)-(c) and compare the result.