Analysis of Variance and Design of Experiments

Experimental Designs and Their Analysis

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Lecture 17

Basics of Design of Experiments

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Design of Experiment:

Design of experiment means how to design an experiment in the sense that how the observations or measurements should be obtained to answer a query in a valid, efficient and economical way.

The designing of the experiment and the analysis of obtained data are inseparable.

Design of Experiment:

If the experiment is designed properly keeping in mind the question, then the data generated is valid and proper analysis of data provides the valid statistical inferences.

If the experiment is not well designed, the validity of the statistical inferences is questionable and may be invalid.

It is important to understand first the basic terminologies used in the experimental design.

Experimental unit:

For conducting an experiment, the experimental material is divided into smaller parts and each part is referred to as an experimental unit.

The treatment is randomly assigned to the experimental units.

The phrase “randomly assigned” is very important in this definition.

Experiment:

A way of getting an answer to a question which the experimenter wants to know.

Treatment:

Different objects or procedures which are to be compared in an experiment are called treatments.

Sampling unit:

The object that is measured in an experiment is called the sampling unit. This may be different from the experimental unit.

Factor:

A factor is a variable defining a categorization.

A factor can be fixed or random in nature.

Replication:

It is the repetition of the experimental situation by replicating the experimental unit.

Experimental error:

The unexplained random part of the variation in any experiment is termed as experimental error.

An estimate of experimental error can be obtained by replication.

Treatment design:

A treatment design is the manner in which the levels of treatments are arranged in an experiment.

Treatment design: Example

- **Suppose some varieties of fish food is to be investigated on some species of fishes.**
- **The food is placed in the water tanks containing the fishes.**
- **The response is the increase in the weight of fish.**
- **The experimental unit is the tank, as the treatment is applied to the tank, not to the fish.**
- **Note that if the experimenter had taken the fish in hand and placed the food in the mouth of fish, then the fish would have been the experimental unit as long as each of the fish got an independent scoop of food.**

Design of experiment:

One of the main objectives of designing an experiment is how to verify the hypothesis in an efficient and economical way.

In the contest of the null hypothesis of equality of several means of normal populations having the same variances, the analysis of variance technique can be used.

Note that such techniques are based on certain statistical assumptions. If these assumptions are violated, the outcome of the test of a hypothesis then may also be faulty and the analysis of data may be meaningless.

Design of experiment:

So the main question is how to obtain the data such that the assumptions are met and the data is readily available for the application of tools like analysis of variance.

The designing of such a mechanism to obtain such data is achieved by the design of the experiment.

After obtaining the sufficient experimental unit, the treatments are allocated to the experimental units in a random fashion.

Design of experiment:

Design of experiment provides a method by which the treatments are placed at random on the experimental units in such a way that the responses are estimated with the utmost precision possible.

Principles of experimental design:

There are three basic principles of design which were developed by Sir Ronald A. Fisher.

(i) Randomization

(ii) Replication

(iii) Local control

Principles of experimental design: Randomization

The principle of randomization involves the allocation of treatment to experimental units at random to avoid any bias in the experiment resulting from the influence of some extraneous unknown factor that may affect the experiment.

In the development of analysis of variance, we assume that the errors are random and independent.

In turn, the observations also become random. The principle of randomization ensures this.

Principles of experimental design: Randomization

The random assignment of experimental units to treatments results in the following outcomes.

a) It eliminates systematic bias.

b) It is needed to obtain a representative sample from the population.

c) It helps in distributing the unknown variation due to confounded variables throughout the experiment and breaks the confounding influence.

Principles of experimental design: Randomization

Randomization forms a basis of a valid experiment but replication is also needed for the validity of the experiment.

If the randomization process is such that every experimental unit has an equal chance of receiving each treatment, it is called complete randomization.

Principles of experimental design: Replication

In the replication principle, any treatment is repeated a number of times to obtain a valid and more reliable estimate than which is possible with one observation only.

Replication provides an efficient way of increasing the precision of an experiment. The precision increases with the increase in the number of observations.

Replication provides more observations when the same treatment is used, so it increases precision.

Principles of experimental design: Replication

For example, if the variance of *x* is σ^2 than variance of the sample mean based on *n* observation is $\frac{\sigma^2}{n}$. So as *n* increases, $Var(\bar{x})$ decreases.

Principles of experimental design: Local Control (Error control)

The replication is used with local control to reduce the experimental error.

For example, if the experimental units are divided into different groups such that they are homogeneous within the blocks, then the variation among the blocks is eliminated and ideally, the error component will contain the variation due to the treatments only. This will, in turn, increase the efficiency.

Complete and incomplete block designs:

In most of the experiments, the available experimental units are grouped into blocks having more or less identical characteristics to remove the blocking effect from the experimental error. Such design is termed as block designs.

The number of experimental units in a block is called the block size.

If size of block = number of treatments

and each treatment in each block is randomly allocated,

then it is a full replication and the design is called a complete block design.

Complete and incomplete block designs:

In case, the number of treatments is so large that a full replication in each block makes it too heterogeneous with respect to the characteristic under study, then smaller but homogeneous blocks can be used.

In such a case, the blocks do not contain a full replicate of the treatments.

Experimental designs with blocks containing an incomplete replication of the treatments are called incomplete block designs.