# Analysis of Variance and Design of Experiments

## $2^n$ Factorial Experiments

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## Lecture 34

## Terminologies and Notations

Shalabh
Department of Mathematics and Statistics
Indian Institute of Technology Kanpur

Slides can be downloaded from http://home.iitk.ac.in/~shalab/sp

## Factorial experiments:

**Factorial experiments involve simultaneously more than one factor each at two or more levels.**

**Several factors affect simultaneously the characteristic under study in factorial experiments and the experimenter is interested in the main effects and the interaction effects among different factors.**

**First we consider an example to understand the utility of factorial experiments.**

# Factorial experiments:

**Example: Suppose the yield from different plots in an agricultural experiment depend upon**

**1. (i) variety of crop and (ii) type of fertilizer.**

**Both the factors are in the control of experimenter.**

**2. (iii) Soil fertility. This factor is not in the control of experimenter.**

# Factorial experiments:

**In order to compare the different crop varieties**

**- assign it to different plots keeping other factors like irrigation, fertilizer, etc. fixed and the same for all the plots.**

**- The conclusions for this will be valid only for the crops grown under similar conditions with respect to the factors like fertilizer, irrigation etc.**

# Factorial experiments:

**In order to compare different fertilizers (or different dosage of fertilizers)**

- **sow single crop on all the plots and vary the quantity of fertilizer from plot to plot.**
- **The conclusions will become invalid if different varieties of crop are sown.**
- **It is quite possible that one variety may respond differently than another to a particular type of fertilizer.**

# Factorial experiments:

**Through the factorial experiments, we can study the individual effect of each factor and interaction effect.**

**Now we consider a $2^2$ factorial experiment with an example and try to develop and understand the theory and notations through this example.**

# Factorial experiments:

**Factors**

**Levels**

**A general notation for representing the factors is to use capital letters, e.g., $A,B,C$ etc. and levels of a factor are represented in small letters.**

# Factorial experiments:

For example, if there are two levels of $A$, they are denoted as $a_0$ and $a_1$.

Similarly the two levels of $B$ are represented as $b_0$ and $b_1$.

Other alternative representation to indicate the two levels of $A$ is $0$ (for $a_0$) and 1 (for $a_1$).

The factors of $B$ are then 0 (for $b_0$) and 1 (for $b_1$).

# Factorial experiments:

**Treatment combinations are as follows:**

| Treatment combinations | | | | |
|---|---|---|---|---|
| | $a_0\ b_0$ | $a_0\ b_1$ | $a_1\ b_0$ | $a_1\ b_1$ |
| or | 0 0 | 0 1 | 1 0 | 1 1 |
| or | *I* | *b* | *a* | *ab* |

***I*** **denotes that both the treatments are at lower level –** $a_0\ b_0$ **or 0 0**

# Factorial experiments:

**(*ab*): Mean of all observations which receive the treatment combinations *ab*.**

**Note: An important point to remember is that the factorial experiments are conducted in a design of experiment. For example, the factorial experiment is conducted as an RBD.**

## Factorial experiments:

**For example, if there are**

- **two levels of $A$, they are denoted as $a_0$ and $a_1$.**
- **two levels of $B$ are represented as $b_0$ and $b_1$.**
- **two levels of $C$ are represented as $c_0$ and $c_1$.**

**Other alternative representation to indicate the two levels of $A$ is $0$ (for $a_0$) and 1 (for $a_1$).**

**The factors of $B$ are then 0 (for $b_0$) and 1 (for $b_1$).**

**The factors of $C$ are then 0 (for $c_0$) and 1 (for $c_1$).**

# Factorial experiments:

**Treatment combinations are as follows:**

| Treatment combinations | | | |
|---|---|---|---|
| $a_0\, b_0\, c_0$ | $a_0\, b_0\, c_1$ | $a_0\, b_1\, c_0$ | $a_0\, b_1\, c_1$ |
| $a_1\, b_0\, c_0$ | $a_1\, b_0\, c_1$ | $a_1\, b_1\, c_0$ | $a_1\, b_1\, c_1$ |

# Factorial experiments:

**Example:**

**Two factors: Irrigation (*I*) and Nitrogen (*N*).**

**Levels:**

**Irrigation has 2 levels: $I_0$ and $I_1$**

**Nitrogen has 3 levels: $N_0$, $N_1$ and $N_2$**

# Factorial experiments:

## We need two RBD

| | | |
|---|---|---|
| $N_0$ | $N_1$ | $N_2$ |
| $N_2$ | $N_0$ | $N_1$ |
| $N_1$ | $N_2$ | $N_0$ |

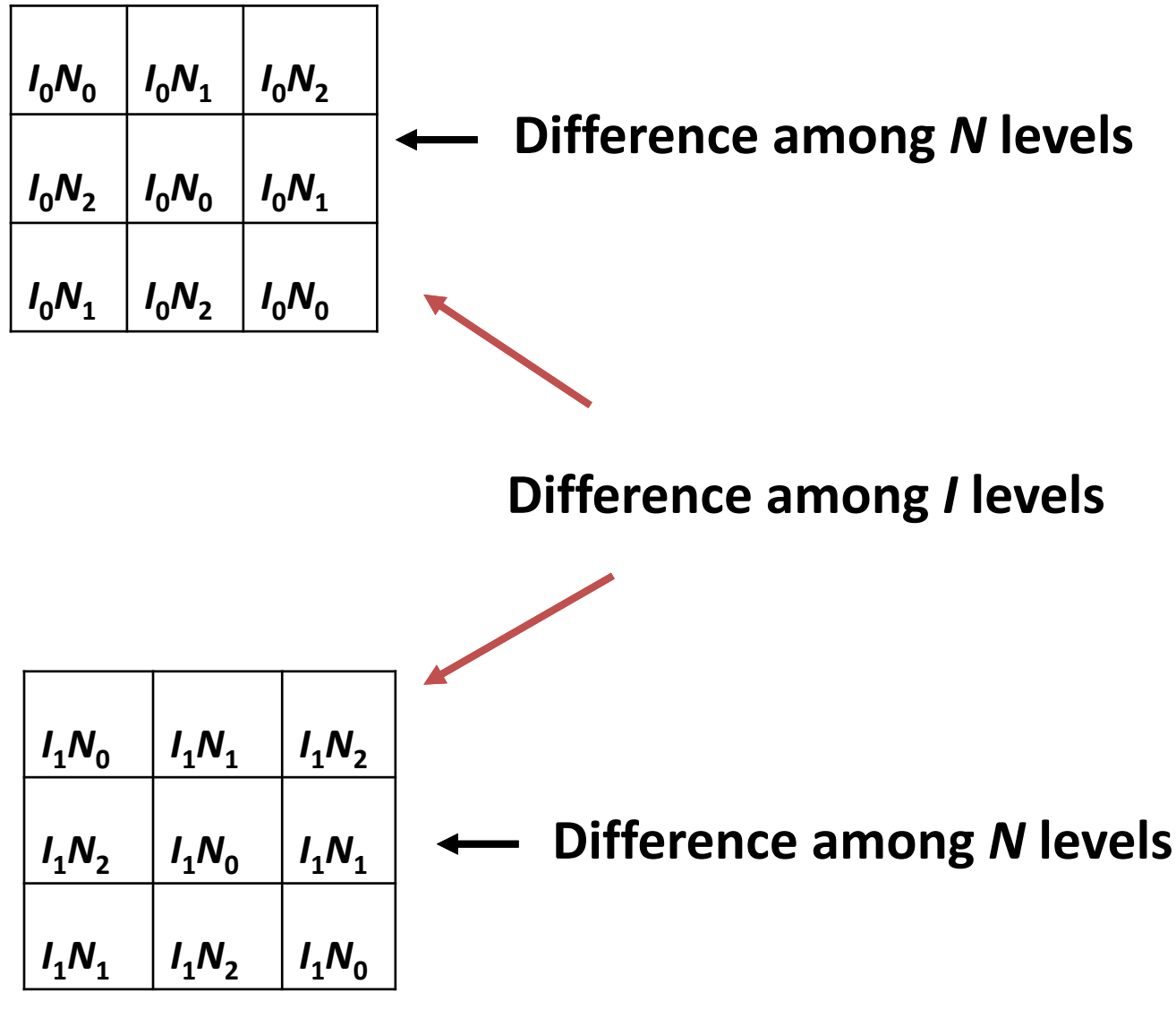
| | | |
|---|---|---|
| $N_0$ | $N_1$ | $N_2$ |
| $N_2$ | $N_0$ | $N_1$ |
| $N_1$ | $N_2$ | $N_0$ |

| | | |
|---|---|---|
| $I_0N_0$ | $I_0N_1$ | $I_0N_2$ |
| $I_0N_2$ | $I_0N_0$ | $I_0N_1$ |
| $I_0N_1$ | $I_0N_2$ | $I_0N_0$ |

| | | |
|---|---|---|
| $I_1N_0$ | $I_1N_1$ | $I_1N_2$ |
| $I_1N_2$ | $I_1N_0$ | $I_1N_1$ |
| $I_1N_1$ | $I_1N_2$ | $I_1N_0$ |

## Total number of plots required : 18

# Factorial experiments:



# Factorial experiments:

**Example:**

**Two factors: Irrigation (*I*) and Nitrogen (*N*).**

**Levels:**

**Irrigation has 2 levels: $I_0$ and $I_1$**

**Nitrogen has 2 levels: $N_0$ and $N_1$**

# Factorial experiments:

| | | Nitrogen | | | |
|---|---|---|---|---|---|
| | | $N_0$ $a_0$ | $N_1$ $a_1$ | Mean Effect | Main Effect |
| Irrigation | $I_0$ | $I_0N_0$ | $I_0N_1$ | $\frac{I_0N_0 + I_0N_1}{2}$ Average effect of *N* for irrigation level $I_0$ | Compare the group means (or totals) Main effect (Irrigation) = Average effect of *N* for irrigation level $I_0$ - Average effect of *N* for irrigation level $I_1$ $= \frac{(b) + (ab)}{2} - \frac{(1) + (a)}{2}$ = Main effect of *B* |
| | $b_0$ | $(a_0b_0) \equiv (1)$ | $(a_1b_0) \equiv (a)$ | $\frac{(1) + (a)}{2}$ | |
| | $I_1$ | $I_1N_0$ | $I_1N_1$ | $\frac{I_1N_0 + I_1N_1}{2}$ Average effect of *N* for irrigation level $I_1$ | |
| | $b_1$ | $(a_0b_1) \equiv (b)$ | $(a_1b_1) \equiv (ab)$ | $\frac{(b) + (ab)}{2}$ | |
| | | | | General Mean Effect $\frac{I_0N_0+I_0N_1+I_1N_0+I_1N_1}{4} = \frac{(1)+(a)+(b)+(ab)}{4}$ | |

# Factorial experiments:

| | | Nitrogen | | | |
|---|---|---|---|---|---|
| | | $N_0$ $a_0$ | $N_1$ $a_1$ | $N_1 - N_0$ | Average Interaction Effect |
| Irrigation | $I_0$ | $I_0N_0$ | $I_0N_1$ | $I_0N_1 - I_0N_0$ Effect of *N* for irrigation level $I_0$ | Compare the effect of *N* at different levels of *I*. $\frac{\left[\begin{array}{l}(\text{Effect of } N \text{ for irrigation level } I_1 \\ - \text{ Effect of } N \text{ for irrigation level } I_0)\end{array}\right]}{2}$ = Effect of *N* for different levels of *I* $= \frac{(ab)-(b)}{2} - \frac{(a)-(1)}{2}$ Interaction effect of *A* and *B* |
| | $b_0$ | $(a_0b_0) \equiv (1)$ | $(a_1b_0) \equiv (a)$ | $(a) - (1)$ | |
| | $I_1$ | $I_1N_0$ | $I_1N_1$ | $I_1N_1 - I_1N_0$ Effect of *N* for irrigation level $I_1$ | |
| | $b_1$ | $(a_0b_1) \equiv (b)$ | $(a_1b_1) \equiv (ab)$ | $(ab) - (b)$ | |
| | | | | General Mean Effect $\frac{I_0N_0+I_0N_1+I_1N_0+I_1N_1}{4} = \frac{(1)+(a)+(b)+(ab)}{4}$ | |

# Factorial experiments:

| | | Nitrogen | | | |
|---|---|---|---|---|---|
| | | $N_0$ | $a_0$ | $N_1$ | $a_1$ |
| Irrigation | $I_0$ $b_0$ | $I_0N_0$ | $(a_0b_0) \equiv (1)$ | $I_0N_1$ | $(a_1b_0) \equiv (a)$ |
| | $I_1$ $b_1$ | $I_1N_0$ | $(a_0b_1) \equiv (b)$ | $I_1N_1$ | $(a_1b_1) \equiv (ab)$ |
| **Main effect** | | $\frac{I_0N_0 + I_1N_0}{2}$ Average effect of *I* for nitrogen level $N_0$ $\frac{(1)+(b)}{2}$ | | $\frac{I_0N_1 + I_1N_1}{2}$ Average effect of *I* for nitrogen level $N_1$ $\frac{(a)+(ab)}{2}$ | |

**Main effect**

Compare the group means (or totals)

Main effect (Nitrogen) = Main effect of *A* =

Average effect of irrigation at $N_1$ level

- Average effect of irrigation at $N_0$ level $= \frac{(a)+(ab)}{2} - \frac{(1)+(b)}{2}$

**General Mean Effect**

$\frac{I_0N_0+I_0N_1+I_1N_0+I_1N_1}{4} = \frac{(1)+(a)+(b)+(ab)}{4}$

# Factorial experiments:

| | | Nitrogen | |
|---|---|---|---|
| | | $N_0$ $a_0$ | $N_1$ $a_1$ |
| Irrigation | $I_0$ $b_0$ | $I_0N_0$ $(a_0b_0) \equiv (1)$ | $I_0N_1$ $(a_1b_0) \equiv (a)$ |
| | $I_1$ $b_1$ | $I_1N_0$ $(a_0b_1) \equiv (b)$ | $I_1N_1$ $(a_1b_1) \equiv (ab)$ |
| Main effect | | $\frac{I_0N_0 + I_1N_0}{2}$ Average effect of *I* for nitrogen level $N_0$ $\frac{(1) + (b)}{2}$ | $\frac{I_0N_1 + I_1N_1}{2}$ Average effect of *I* for nitrogen level $N_1$ $\frac{(a) + (ab)}{2}$ |
| $I_1$ - $I_0$ | | $I_1N_0 - I_0N_0$ : Effect of *I* for nitrogen at $N_0$ level $(b) - (1)$ | $I_1N_1 - I_0N_1$ : Effect of *I* for nitrogen at $N_1$ level $(ab) - (a)$ |
| Average Interaction Effect | | Compare the effect of *I* at different levels of *N*. $\frac{[\text{Effect of } I \text{ for irrigation level } N_1 - \text{Effect of } I \text{ for irrigation level } N_0)]}{2}$ $= \frac{(ab) - (a)}{2} - \frac{(b) - (1)}{2}$ Interaction effect of *A* and *B* | |