Exploratory Statistical Data Analysis With R Software (ESDAR)

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Lecture 26

Absolute Deviation in R and Measures Based on Squared Deviations

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Slides can be downloaded from

http://home.iitk.ac.in/~shalab/sp

Absolute Deviation

The absolute deviation of observations $x_1, x_2, \ldots, x_n$ around a value A

is defined as

- $$D(A) = \frac{1}{n}\sum_{i=1}^{n} |x_i - A|$$ **for discrete (ungrouped) data.**

- $$D(A) = \frac{1}{n}\sum_{i=1}^{K} f_i\,|x_i - A|$$ **for continuous (grouped) data.**

  where $n = \sum_{i=1}^{K} f_i$

Absolute Mean Deviation

The absolute deviation of observations $x_1, x_2, \ldots, x_n$ is minimum when measured around median, i.e., *A* is the median of data.

In this case, the absolute deviation is termed as absolute mean deviation and is defined as

- $D(\bar{x}_{med}) = \frac{1}{n}\sum_{i=1}^{n} \left| x_i - \bar{x}_{med} \right|$ **for discrete (ungrouped) data.**

- $D(\bar{x}_{med}) = \frac{1}{n}\sum_{i=1}^{K} f_i \left| x_i - \bar{x}_{med} \right|$ **for continuous (grouped) data.**

  where $n = \sum_{i=1}^{K} f_i$

Absolute Deviation and Absolute Mean Deviation

R command: **Ungrouped data**

Data vector: `x`

Absolute deviation for given *A*

```
mean(abs(x - A))
```

Absolute mean deviation

```
mean(abs(x - median(x)))
```

Absolute Deviation and Absolute Mean Deviation

Example: Ungrouped data

Following are the time taken (in seconds) by 20 participants in a race: 32, 35, 45, 83, 74, 55, 68, 38, 35, 55, 66, 65, 42, 68, 72, 84, 67, 36, 42, 58.

```
> time = c(32, 35, 45, 83, 74, 55, 68, 38, 35,
55, 66, 65, 42, 68, 72, 84, 67, 36, 42, 58)

> A = 10
> mean(abs((time - A)))  #Absolute deviation around A= 10
[1] 46

> median(time)
[1] 56.5

> mean(abs(time - median(time)))  # Absolute mean deviation around median
 [1] 14.5
```

Absolute Deviation and Absolute Mean Deviation

Example: Ungrouped data

R Console

```
> time
 [1] 32 35 45 83 74 55 68 38 35 55 66 65 42 68 72 84 67 36 42 58
> A=10
> A
[1] 10
> mean(abs(time - A))
[1] 46
>
> median(time)
[1] 56.5
> mean(abs(time - mean(time)))
[1] 14.5
> 
```

Absolute Deviation and Absolute Mean Deviation

R command: Ungrouped data and missing values

If data vector `x` has missing values as `NA`, say `xna`, then R command is

Absolute deviation for given *A*

```
mean(abs((xna - A)),na.rm=TRUE)
```

Absolute mean deviation

```
mean(abs((xna - median(xna, na.rm=TRUE))),
na.rm= TRUE)
```

Absolute Deviation and Absolute Mean Deviation

Example: **Handling missing values**

Suppose two data points are missing in the earlier example where the time taken (in seconds) by 20 participants in a race. They are recorded as NA

NA, NA, 45, 83, 74, 55, 68, 38, 35, 55, 66, 65, 42, 68, 72, 84, 67, 36, 42, 58.

```
> time.na = c(NA, NA, 45, 83, 74, 55, 68, 38,
35, 55, 66, 65, 42, 68, 72, 84, 67, 36, 42, 58)
```

Absolute Deviation and Absolute Mean Deviation

Example: Handling missing values

```
> time.na = c(NA, NA, 45, 83, 74, 55, 68, 38,
35, 55, 66, 65, 42, 68, 72, 84, 67, 36, 42, 58)

> A = 10

> mean(abs((time.na - A)), na.rm= TRUE)

[1] 48.5


> median(time.na, na.rm = TRUE)
[1] 61.5

> mean(abs((time.na - median(time.na, na.rm =
TRUE))), na.rm= TRUE)
[1] 13.38889
```

Absolute Deviation and Absolute Mean Deviation

Example: Handling missing values

```
> time.na
 [1] NA NA 45 83 74 55 68 38 35 55 66 65 42 68 72 84 67 36 42 58
> A =10
> A
[1] 10
> mean(abs((time.na - A)), na.rm= TRUE) #Absolute deviation around A= 10
[1] 48.5
>
> median(time.na, na.rm=TRUE)
[1] 61.5
> mean(abs((time.na - median(time.na, na.rm = TRUE))), na.rm= TRUE) # Absolute mean d$
[1] 13.38889
> |
```

Absolute Deviation and Absolute Mean Deviation

R command: Grouped data

Data vector: `x`

Frequency vector: `f`

Absolute deviation for given *A*

```
sum(f * abs(x - A))/sum(f)
```

Absolute mean deviation

```
sum(f * abs(x - xmedian))//sum(f)
```

Note: Median in this case is to be computed as `xmedian` using the median for grouped data separately.

Absolute Deviation and Absolute Mean Deviation

Example: Grouped data

Following are the time taken (in seconds) by 20 participants in a race: 32, 35, 45, 83, 74, 55, 68, 38, 35, 55, 66, 65, 42, 68, 72, 84, 67, 36, 42, 58.

```
> time = c(32, 35, 45, 83, 74, 55, 68, 38, 35,
55, 66, 65, 42, 68, 72, 84, 67, 36, 42, 58)
```

Absolute Deviation and Absolute Mean Deviation

Example: Grouped data

Considering the data as grouped data, we can present the data as

Class intervals	Mid point	Absolute frequency (or frequency)
31 – 40	35.5	5
41 – 50	45.5	3
51 – 60	55.5	3
61 – 70	65.5	5
71 – 80	75.5	2
81 - 90	85.5	2
	Total	20

We need to find the frequency vector and median.

Absolute Deviation and Absolute Mean Deviation

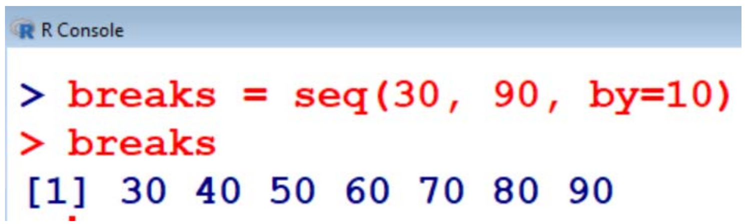
Example: Grouped data - Obtaining frequencies:

Create a sequence starting from 30 to 90 at an interval of 10 integers denoting the width.

```
breaks = seq(30, 90, by=10)
```

 # Sequence of 10 integers at interval of 10

```
> breaks = seq(30, 90, by=10)
> breaks
[1] 30 40 50 60 70 80 90
```

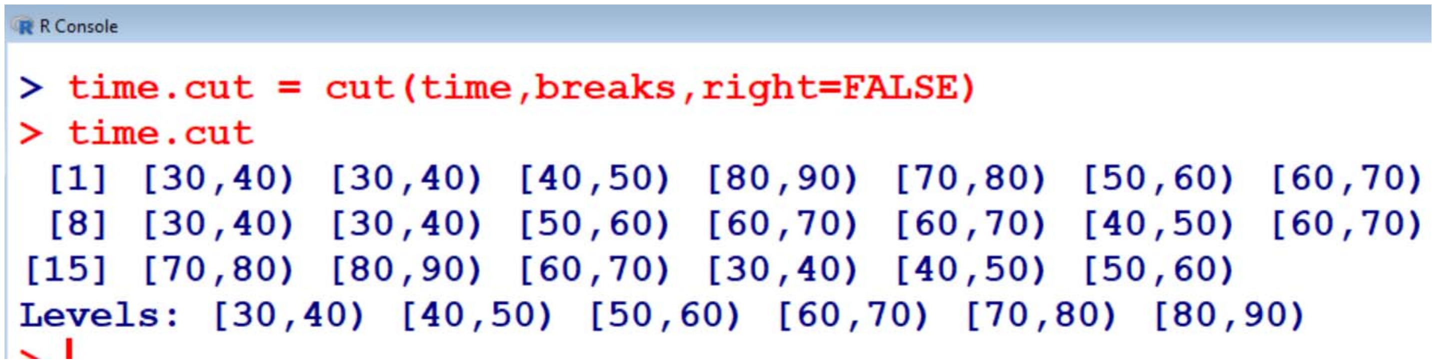


Absolute Deviation and Absolute Mean Deviation

Example: Grouped data - Obtaining frequencies:

Now we classify the time data according to the width intervals with `cut`.

```
> time.cut = cut(time,breaks,right=FALSE)
> time.cut
 [1] [30,40) [30,40) [40,50) [80,90) [70,80) [50,60) [60,70)
 [8] [30,40) [30,40) [50,60) [60,70) [60,70) [40,50) [60,70)
[15] [70,80) [80,90) [60,70) [30,40) [40,50) [50,60)
Levels: [30,40) [40,50) [50,60) [60,70) [70,80) [80,90)
```



Absolute Deviation and Absolute Mean Deviation

Example: Grouped data - Obtaining frequencies:

Frequency distribution

```
> table(time.cut)
time.cut
[30,40) [40,50) [50,60) [60,70) [70,80) [80,90)
      5       3       3       5       2       2
```

Extract frequencies from frequency table using command

```
> f = as.numeric(table(time.cut))
> f
[1] 5 3 3 5 2 2
```

Absolute Deviation and Absolute Mean Deviation

Example: Grouped data - Obtaining mid points:

Mid points, as obtained from the frequency table, are

```
> x = c(35,45,55,65,75,85)

> x
[1] 35 45 55 65 75 85
```

Note that the mid points are obtained from the frequency table obtained from the R software

```
[30,40) [40,50) [50,60) [60,70) [70,80) [80,90)
```

Absolute Deviation and Absolute Mean Deviation

Example: Grouped data – Obtaining median

Obtain median from the frequency table using

Median class (m = 3) : 50 – 60

Lower limit of class (e_m) = e_3 = 50

Frequency of median class (f_m) = f_3 = 3/20

Width of median class (d_m) = d_3 = 50 – 60 = 10

$$\bar{x}_{med} = e_m + \frac{0.5 - \sum_{i=1}^{m-1} f_i}{f_m} d_m$$

$$= 50 + \frac{0.5 - \left(\frac{5}{20} + \frac{3}{20} \right)}{3/20} \times 10$$

$$\approx 56.66$$

Absolute Deviation and Absolute Mean Deviation

Example: Grouped data

```
> f = c(5,3,3,5,2,2)

> x = c(35,45,55,65,75,85)

> xmedian = 56.66

> A = 10

> sum(f * abs(x - A))/sum(f)  #Absolute deviation
[1] 46                              around A= 10

> sum(f * abs(x - xmedian))/sum(f)  # Absolute
[1] 14.166                  mean deviation around median
```

Absolute Deviation and Absolute Mean Deviation

Comparison of results:

Ungrouped data

```
> mean(abs((time - A)))
[1] 46
```
\# Absolute deviation around *A*= 10

```
> mean(abs(time - median(time)))
 [1] 14.5
```
\# Absolute mean

Grouped data

```
> sum(f * abs(x - A))/sum(f)
[1] 46
```
\# Absolute deviation around *A*= 10

```
> sum(f * abs(x - xmedian))/sum(f)
[1] 14.166
```
\# Absolute mean deviation around median

Measure of Variation Based on Ungrouped (Discrete) Data

Observations on a variable X are obtained as $x_1, x_2, \ldots, x_n$.

Mean Squared Error

We considered the absolute deviation values $|x_i - A|$ in absolute deviation. Instead of this, consider squared values of deviations $(x_i - A)^2$ around any point A.

Then the mean squared error (MSE) with respect to A is defined as

- $s^2(A) = \frac{1}{n}\sum_{i=1}^{n}\left(x_i - A\right)^2$ **for discrete (ungrouped) data.**

Measure of Variation Based on Grouped (Continuous) data

Observations on a variable X are obtained and tabulated in K class intervals in a frequency table as follows. The mid points of the intervals are denoted by $x_1, x_2, \ldots, x_k$ which occur with frequencies $f_1, f_2, \ldots, f_K$ respectively and $n = f_1 + f_2 + \ldots + f_K$.

Class intervals	Mid point (x_i)	Absolute frequency (f_i)
$e_1 - e_2$	$x_1 = (e_1 + e_2)/2$	f_1
$e_2 - e_3$	$x_2 = (e_2 + e_3)/2$	f_2
…	…	…
$e_{K-1} - e_K$	$x_K = (e_{K-1} + e_K)/2$	f_K

Mean Squared Error

We considered the absolute deviation of values $|x_i - A|$ around the mid values of class intervals x_i. Instead of this, consider squared values of deviations $(x_i - A)^2$ around any point A.

Then the mean squared error (MSE) with respect to A is defined as

- $s^2(A) = \frac{1}{n}\sum_{i=1}^{K} f_i \left(x_i - A\right)^2$ **for continuous (grouped) data.**

  where $n = \sum_{i=1}^{K} f_i$