Exploratory Statistical Data Analysis With R Software (ESDAR)

Swayam Prabha

Lecture 37

Association of Discrete Variables

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Slides can be downloaded from

http://home.iitk.ac.in/~shalab/sp

Association between Two Discrete Variables

Example: Suppose we want to know if boys and girls have any inclination to choose between mathematics and biology.

If there is no discrimination, we expect that the total number of boys and girls opting for mathematics and biology should be nearly the same.

Data on such issues are obtained as frequency.

A measure based on frequency data or summarized frequency data is needed to study the association between two such variables.

Association between Two Discrete Variables

Suppose the data is obtained as follows:

Student number	1	2	3	4	5	6	7	8	9	10
Gender M: male F: female	M	F	M	M	F	F	F	M	M	F
Subject Math: Mth Biology: Bio	Bio	Bio	Mth	Mth	Mth	Bio	Bio	Mth	Mth	Mth

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Data can be summarized as follows

	Male Students	Female Students	Total (Rows)
Math	$n_{11} = 4$	$n_{12} = 2$	$n_{1+} = 6$
Biology	$n_{21} = 1$	$n_{22} = 3$	$n_{2+} = 4$
Total (Columns)	$n_{+1} = 5$	$n_{+2} = 5$	$n = 10$

Students preferring maths

Students preferring biology

Male Students preferring maths and biology

Female Students preferring maths and biology

This is a 2 x 2 contingency table.

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n_{ij} : Frequency in $(i, j)^{th}$ cell

$n_{1+} = n_{11} + n_{12}$: Row total (1^{st} row of data)

$n_{2+} = n_{21} + n_{22}$: Row total (2^{nd} row of data)

$n_{+1} = n_{11} + n_{21}$: Column total (1^{st} column of data)

$n_{+2} = n_{12} + n_{22}$: Column total (2^{nd} column of data)

$n = n_{11} + n_{12} + n_{21} + n_{22} = n_{1+} + n_{2+} = n_{+1} + n_{+2}$ = Total frequency

Association between Two Discrete Variables

In general, let *X* and *Y* be two discrete variables

$x_1, x_2, \ldots, x_k$: *k* classes of *X*

$y_1, y_2, \ldots, y_l$: *l* classes of *Y*

n_{ij} : Frequency of $(i, j)^{th}$ cell corresponding to (x_i, y_j)

$$i = 1,2,\ldots,k; \quad j = 1,2,\ldots,l;$$

This frequencies can be presented in the following *k* x *l* contingency table.

Association between Two Discrete Variables

k x *l* Contingency Table

		Y					Total (Rows)
		y_1	⋯	y_j	⋯	y_l	
X	x_1	n_{11}	⋯	n_{1j}	⋯	n_{1l}	n_{1+}
	⋮	⋮	⋱	⋮	⋱	⋮	⋮
	x_i	n_{i1}	⋯	n_{ij}	⋯	n_{il}	n_{i+}
	⋮	⋮	⋱	⋮	⋱	⋮	⋮
	x_k	n_{k1}	⋯	n_{kj}	⋯	n_{kl}	n_{k+}
Total (Columns)		n_{+1}	⋯	n_{+j}	⋯	n_{+l}	n

Marginal frequency

$$n_{i+} = \sum_{j=1}^{l} n_{ij}$$

Marginal frequency

$$n_{+j} = \sum_{i=1}^{k} n_{ij}$$

$$n = \sum_{i=1}^{k} n_{i+} = \sum_{j=1}^{l} n_{+j} = \sum_{i=1}^{k}\sum_{j=1}^{l} n_{ij}$$

Total frequency

Association between Two Discrete Variables

When the data on two variables are summarized in a contingency table, there are several characteristics of the data can be studied.

$$n_{i+} = \sum_{j=1}^{l} n_{ij}, \quad n_{+j} = \sum_{i=1}^{k} n_{ij}, \quad n = \sum_{i=1}^{k} n_{i+} = \sum_{j=1}^{l} n_{+j} = \sum_{i=1}^{k}\sum_{j=1}^{l} n_{ij}$$

n_{ij} : Absolute frequencies

: Represents joint frequency distribution of *X* and *Y*

Joint frequency distribution tells how the values of both the variables behave jointly.

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n_{i+} : Represents marginal frequency distribution of *X*

n_{+j} : Represents marginal frequency distribution of *Y*

Marginal frequency distribution tells how the values of one variable behave in the joint distribution.

If relative frequency is used instead of absolute frequency, then the similar information is provided by the

- **joint relative frequency distribution,**
- **marginal relative frequency distribution, and**
- **conditional relative frequency distribution.**

Association between Two Discrete Variables

$f_{ij} = \frac{n_{ij}}{n}$: Relative frequency

: Represents joint relative frequency distribution of *X* and *Y.*

$f_{i|j}(X|\ Y = y_j) = \frac{n_{ij}}{n_{+j}}$: Conditional <u>frequency distribution</u> of *X* given $Y = y_j$

$f_{j|i}(Y|\ X = x_i) = \frac{n_{ij}}{n_{i+}}$: Conditional <u>frequency distribution</u> of *Y* given $X = x_i$

<u>Conditional frequency distribution</u> tells how the values of one variable behave when another variable is kept fixed.

Association between Two Discrete Variables

$f_{i+} = \sum_{i=1}^{l} f_{ij}$: **Marginal relative frequency distribution of *X***

$f_{+j} = \sum_{i=1}^{k} f_{ij}$: **Marginal relative frequency distribution of *Y***

$f_{i|j}(X \mid Y)$: **Conditional relative frequency distribution of *X* given $Y = y_j$**

$f_{j|i}(Y \mid X)$: **Conditional relative frequency distribution of *Y* given $X = x_i$**