

Exploratory Statistical Data Analysis With R Software (ESDAR) Swayam Prabha

Lecture 6

Operations with Matrices in R

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Slides can be downloaded from
<http://home.iitk.ac.in/~shalab/sp>



Matrix

A matrix is a rectangular array with p rows and n columns.

An element in the i -th row and j -th column is denoted by X_{ij} (book version) or $X[i, j]$ ("program version"), $i = 1, 2, \dots, n, j = 1, 2, \dots, p$.

We consider only numerical matrices, whose elements are generally real numbers.

Matrix

In R, a 4×2 -matrix X can be created with a following command:

```
> x = matrix( nrow=4, ncol=2, data=c(11,12,13,  
14,15,16,17,18) )
```

```
> x
```

```
      [,1] [,2]  
[1,]  11  15  
[2,]  12  16  
[3,]  13  17  
[4,]  14  18
```

Matrix

We see:

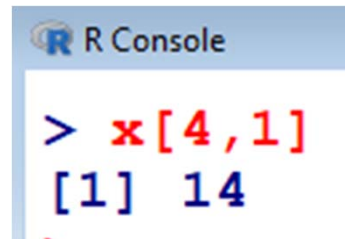
- The parameter `nrow` defines the row number of a matrix.
- The parameter `ncol` defines the column number of a matrix.
- The parameter `data` assigns specified values to the matrix elements.
- The values from the parameters are written column-wise in matrix.

Matrix

```
> x
      [,1] [,2]
[1,]  11  15
[2,]  12  16
[3,]  13  17
[4,]  14  18
```

One can access a single element of a matrix with `x[i,j]`:

```
> x[4,1]
[1] 14
```



Matrix

In case, the data has to be entered row wise, then a 4×2 -matrix X can be created with

```
> x = matrix( nrow=4, ncol=2,
data=c(11,12,13,14,15,16,17,18), byrow = TRUE)
> x
      [,1] [,2]
[1,]  11  12
[2,]  13  14
[3,]  15  16
[4,]  17  18
>
```

Matrix

```
R Console  
> x = matrix( nrow=4, ncol=2, data=c(11,12,13,14,15,16,17,18), byrow = TRUE)  
> x  
      [,1] [,2]  
[1,]  11  12  
[2,]  13  14  
[3,]  15  16  
[4,]  17  18  
>
```

Matrix

In case, the data has to be entered column wise, then a 4×2 -matrix

X can be created with

```
x = matrix( nrow=4, ncol=2,  
data=c(11,12,13,14,15,16,17,18),byrow = FALSE)  
> x
```

```
      [,1] [,2]  
[1,]  11  15  
[2,]  12  16  
[3,]  13  17  
[4,]  14  18
```


Matrix

```
R Console
> x = matrix( nrow=4, ncol=2, data=c(11,12,13,14,15,16,17,18),byrow = FALSE)
>
> x
      [,1] [,2]
[1,]  11  15
[2,]  12  16
[3,]  13  17
[4,]  14  18
>
```

Matrix

Transpose of a matrix X : X'

Consider the matrix

$$x = \begin{pmatrix} 11 & 15 \\ 12 & 16 \\ 13 & 17 \\ 14 & 18 \end{pmatrix}$$

```
> x = matrix( nrow=4, ncol=2, data=c(11,12,13,14,  
15,16,17,18), byrow = FALSE)
```

```
> x  
      [,1] [,2]  
[1,]  11  15  
[2,]  12  16  
[3,]  13  17  
[4,]  14  18
```

Matrix

Transpose of a matrix X : X'

```
> xt <- t(x)
```

```
> xt
```

```
      [,1] [,2] [,3] [,4]
[1,]  11  12  13  14
[2,]  15  16  17  18
```

R Console

```
> xt <- t(x)
```

```
> xt
```

```
      [,1] [,2] [,3] [,4]
[1,]  11  12  13  14
[2,]  15  16  17  18
>
```

Matrix

Multiplication of a matrix with a constant

```
> x = matrix(nrow=4, ncol=2,  
data=c(11,12,13,14,15,16,17,18), byrow=T )
```

```
> x  
      [,1] [,2]  
[1,]  11  12  
[2,]  13  14  
[3,]  15  16  
[4,]  17  18
```

```
R Console  
> x = matrix(nrow=4, ncol=2, data=c(11,12,13,14,15,16,17,18), byrow=T )  
> x  
      [,1] [,2]  
[1,]  11  12  
[2,]  13  14  
[3,]  15  16  
[4,]  17  18  
>
```

Matrix

Multiplication of a matrix with a constant

```
> x = matrix(nrow=4, ncol=2,  
data=c(11,12,13,14,15,16,17,18), byrow=T )
```

```
> x
```

```
      [,1] [,2]  
[1,]  11  12  
[2,]  13  14  
[3,]  15  16  
[4,]  17  18
```

```
> 4*x
```

```
      [,1] [,2]  
[1,]  44  48  
[2,]  52  56  
[3,]  60  64  
[4,]  68  72
```

Matrix

Multiplication of a matrix with a constant

```
R Console
> x = matrix(nrow=4, ncol=2, data=c(11,12,13,14,15,16,17,18), byrow=T )
> x
      [,1] [,2]
[1,]   11   12
[2,]   13   14
[3,]   15   16
[4,]   17   18
>
> 4*x
      [,1] [,2]
[1,]   44   48
[2,]   52   56
[3,]   60   64
[4,]   68   72
>
```

Matrix

Matrix multiplication: operator %*%

Consider the multiplication of X' with X

```
> xtx = t(x) %*% x
```

```
> xtx
```

```
      [,1] [,2]  
[1,]  804  860  
[2,]  860  920
```

```
R Console  
> xtx = t(x) %*% x  
> xtx  
      [,1] [,2]  
[1,]  804  860  
[2,]  860  920  
>
```

Matrix

Matrix multiplication: operator %*%

```
> y = matrix(nrow=2, ncol=2,  
data=c(11,12,13,14), byrow=T )
```

```
> z = matrix(nrow=2, ncol=2, data=c(21,22,23,  
24), byrow=T )
```

<pre>> y</pre>	<pre>> z</pre>
<pre> [,1] [,2]</pre>	<pre> [,1] [,2]</pre>
<pre>[1,] 11 12</pre>	<pre>[1,] 21 22</pre>
<pre>[2,] 13 14</pre>	<pre>[2,] 23 24</pre>

Matrix

Matrix multiplication: operator %*%

```
> y%*%z
      [,1] [,2]
[1,]  507  530
[2,]  595  622
```

```
R Console
> y%*%z
      [,1] [,2]
[1,]  507  530
[2,]  595  622
>
```

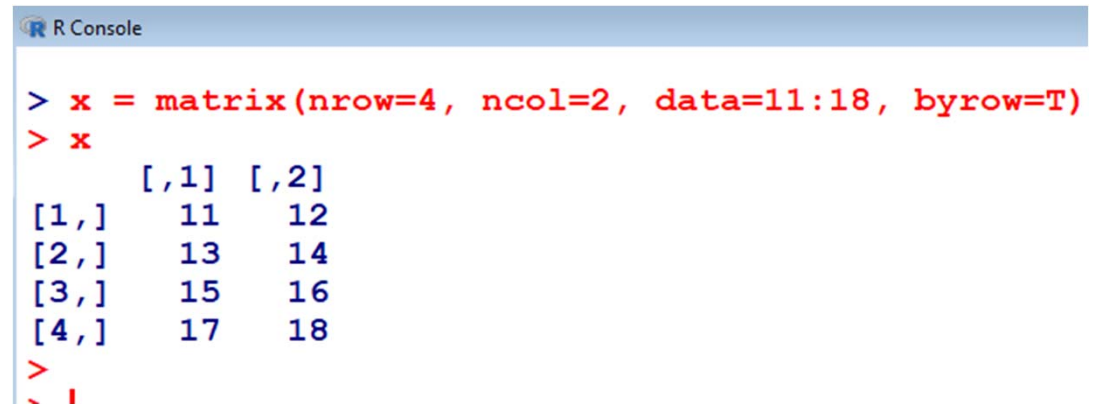
Matrix

Addition and subtraction of matrices (of same dimensions) can be executed with the usual operators + and -

```
> x = matrix(nrow=4, ncol=2, data=11:18,  
             byrow=T)
```

```
> x
```

	[,1]	[,2]
[1,]	11	12
[2,]	13	14
[3,]	15	16
[4,]	17	18



```
R Console  
> x = matrix(nrow=4, ncol=2, data=11:18, byrow=T)  
> x  
      [,1] [,2]  
[1,]  11  12  
[2,]  13  14  
[3,]  15  16  
[4,]  17  18  
>  
>
```

Matrix

Addition and subtraction of matrices (of same dimensions!) can be executed with the usual operators + and -

Create another matrix.

```
> 5*x
      [,1] [,2]
[1,]  55  60
[2,]  65  70
[3,]  75  80
[4,]  85  90
```

```
R Console
> 5*x
      [,1] [,2]
[1,]  55  60
[2,]  65  70
[3,]  75  80
[4,]  85  90
>
```

Matrix

Addition and subtraction of matrices (of same dimensions!) can be executed with the usual operators + and -

```
> x + 5*x
      [,1] [,2]
[1,]   66  72
[2,]   78  84
[3,]   90  96
[4,]  102 108
```

```
> x - 5*x
      [,1] [,2]
[1,]  -44 -48
[2,]  -52 -56
[3,]  -60 -64
[4,]  -68 -72
```

```
R Console
> x + 5*x
      [,1] [,2]
[1,]   66  72
[2,]   78  84
[3,]   90  96
[4,]  102 108
>
```

```
R Console
> x - 5*x
      [,1] [,2]
[1,]  -44 -48
[2,]  -52 -56
[3,]  -60 -64
[4,]  -68 -72
>
```

Matrix

Matrix Addition:

```
> y = matrix(nrow=2, ncol=2,  
data=c(11,12,13,14), byrow=T )
```

```
> z = matrix(nrow=2, ncol=2, data=c(21,22,23,  
24), byrow=T )
```

<pre>> y</pre>	<pre>> z</pre>
<pre> [,1] [,2]</pre>	<pre> [,1] [,2]</pre>
<pre>[1,] 11 12</pre>	<pre>[1,] 21 22</pre>
<pre>[2,] 13 14</pre>	<pre>[2,] 23 24</pre>

Matrix

Matrix Addition and Subtraction:

```
> y+z
      [,1] [,2]
[1,]   32  34
[2,]   36  38
```

```
> y-z
      [,1] [,2]
[1,]  -10  -10
[2,]  -10  -10
```

```
R Console
> y+z
      [,1] [,2]
[1,]   32  34
[2,]   36  38
>
> y-z
      [,1] [,2]
[1,]  -10  -10
[2,]  -10  -10
>
```

Matrix

Access to rows, columns or submatrices:

```
R Console
> x = matrix(nrow=4, ncol=2, data=11:18, byrow=T)
> x
      [,1] [,2]
[1,]   11   12
[2,]   13   14
[3,]   15   16
[4,]   17   18
```

```
> x[4,]
[1] 17 18
```

```
R Console
> x[4,]
[1] 17 18
```

```
> x[,2]
[1] 12 14 16 18
```

```
R Console
> x[,2]
[1] 12 14 16 18
```

Matrix

Access to rows, columns or submatrices:

```
R Console  
  
> x = matrix(nrow=4, ncol=2, data=11:18, byrow=T)  
> x  
      [,1] [,2]  
[1,]   11  12  
[2,]   13  14  
[3,]   15  16  
[4,]   17  18  
>
```

```
> x[1:3, 1:2]  
      [,1] [,2]  
[1,]   11  12  
[2,]   13  14  
[3,]   15  16
```

```
R Console  
  
> x[1:3, 1:2]  
      [,1] [,2]  
[1,]   11  12  
[2,]   13  14  
[3,]   15  16  
>
```