Synopsis

Technology development in aerospace engineering have created new avenues of research, particularly in the areas of health monitoring, vibration and shape control of flexible structures using the concept of smart or intelligent structures. Under external loading, conventional structures deform in a passive manner with no control over the configuration of the deformed state. Whereas in smart structures, the structural deformation is continuously monitored and controlled (using distributed sensors and actuators) to achieve the desired configuration of the deformed state of the structure. In principle, distributed sensing and actuation can be implemented by embedding piezoelectric material in the structure. A part from piezoelectric materials shape-Memory Alloys (SMA), electro-Rheological and Magneto-Rheological materials also have been used for smart structural applications. In this thesis, the focus is on electro-thermo-elastic formulation and analysis of smart structures having embedded piezoelectric materials.

Piezoelectricity represents the interaction between electrical and mechanicals characteristics of a material. Through piezoelectric material was discovered by curie brothers in 1880's, the potential for use in deformation control, health and usage monitoring of flexible structures is of recent origin. In piezoelectric materials an electrical voltage is developed when mechanical loading (pressure) is applied. This is known as direct piezoelectric effect. In the indirect or converse effect, piezoelectric materials undergo mechanical strain when a voltage is applied across them. The direct and converse effects from the fundamental basis for the use of piezoelectric materials, both as sensor and strain actuators. The electro-elastic characteristics of piezoelectric materials have been studied extensively by physicists and materials scientist. It is well established that the electrical properties of piezoelectric materials depend on temperature. In addition under cyclic variation of electrical field, material polarization exhibits a hysteresis effect leading to energy loss in the form of intense heating of the material. Essentially three types of physical behavior exist in a piezoelectric crystal. These are thermo-elasticity, piezoelectricity and pyroelectricity.

Mathematical formulation of electro-thermo-elasticity involves the basis understanding of electrodynamics, thermodynamics, heat transfer and continuum mechanics. The governing equations, constitutive equations and boundary conditions have to be obtained in a consistent manner to bring out the electro-thermo-elastic coupling effects while modeling the behaviour of the piezo continuum in a smart structure. The electro-magnetic phenomena associated with the motion of free charges. Hence, the electrical behaviour of the continuum is modeled in a quasi-static manner considering only the laws of electro-statics.

In the present formulation, the concepts of electrostatics of dielectrics are used to derive the expressions for the electrostatic forces and moments are incorporated in the thermo-elastic modeling of the continuum. The conversation laws of charges, mass, linear momentum, angular momentum and energy are applied to derive the governing equations. The relevant constitutive equations are obtained by applying the second law of thermodynamics (Clausius-Duhem inequality) incorporating a thermodynamic potential.

The complete electro-thermo-elastic problem involves thirty three unknowns and same number of equations. It is shown that the interaction of polarisation and electric field in the piezo material results in rendering the stress tensor non-symmetric. The antisymmetric part of the stress tensor is uniquely related to polarisation and electric field. It is brought out that the interaction of polarisation and electric field (a) results in non-liener distributed body force in equilibrium equations, and (b) introduce a non linear term in the constitutive relation between stress-strain-temprature and electric field. If the electric field inside the polarised medium is constant, the non-linear distributed body force becomes zero. If the polarisation and electric field vectors are parallel, then the distributed body couple vanishes, resulting in symmetric stress tensor. The governing equation are then reduced to five equations with independent unknowns namely, three displacements along the three Cartesian coordinate directions, one electric potential and one temperature variable.

The general electro-thermo-elastic formulation is applied to the problems of actuation, sensing and shape control of a smart beam with piezo-layers/piezo-patches. Reduced constitutive equation have been derived after incorporating the beam assumptions in the genral constitutive relations. Variational form of the problem has been abtained for several cases of study, namely, (i) linear electro-elastic analysis of surface mounted smart beams representing extension actuation scheme, (ii) linear electro-elastic analysis of sandwiched smart beam in shear actuation scheme, (iii) linear electro-thermo-elastic analysis of surface mounted smart beams, and (iv) non-linear electro-elastic analysis of surface mounted smart beam.

A layer-by-layer finite element modeling of the smart beams has been developed to properly account for the transfer of shear stress at the interface of material discontinuity across the thickness of the smart beams. Using layer-by-layer finite model, the linear and non-linear stiffness matrices for various problems of study have been developed.

For the purpose of validation of the formulation and solution technique, the results of the present study are first compared with those available in the literature for a few specialised cases of actuation and sensing of a smart beam. Subsequenty, several different problem of actuations of a smart beam having piezoelectric elements have been solved. The important observations of the study can be summarised as:

1. It is shown that layer-by-layer finite element modeling effective captures the continuity of shear stress across the interface between piezo-layers and the metallic material.
2. The present formulation and solution technique evaluates the induced potential in the piezo material due to both externally applied electric field and due to mechanical staining. With increase in strain in piozo material, the assumption of uniform electric field (E=voltage/thickness) is violated.
3. It is shown that interaction of polarisation and electric field results in an interesting step like transverse deformation in the case of a beam made of piozo material
4. The effects of various types of electrical configurations of the piexo layers (grounding, and equipotertial conditions) on the induced potential have been analysed. It is clearly brought out that the induced potential in the piezo material is a function of the strain at the given location.
5. A comparative study of various response quantities for the sensing and actuation problems of a given smart beam has brought out

the key difference in the induced shear stress between the piezo layer and the core material. For identical tip deflection, this difference in the nature of shear stress distribution is responsible for inducing a low potential in the piezo material in sensing mode in comparison to the actuation potential.

6. The current model can be effectively used to capture all the response quantities of a smart beam under sensing and actuation modes.

Using the electro-elastic and electro-elastic formulations, shape control of a smart cantilever beam acted upon by external menhanical and thermal loads has been performed. The results of the study indicate that the piezo patches can be effectively used to control the deformation of the structure. By suitable choice of piezo actuator length and thickness, the performance of the piezo actuators can be enhanced.

The effect of non-linearities due to the interaction of electric field and polarisation is analysed by solving an example problem of actuation of a smart beam. The results indicate that the non-linearities show a significant for high values of actuation voltage.

The thesis is organised in several chapters.

Chapter 1 presents the introduction, literature survey, motivation and objectives of the present study. In chapter 2, the governing equations, constitutive relations and boundary conditions for a genral electro-thermal-elastic continuum have been developed, starting from the basic principles. The non-linear effects due to the interaction of polarisation and electric field have been properly accounted. These equations have been specialised to solve problems related to electro-elastic and electro-thermal-elastic analysis of smart beams made of metallic host structure with embedded piezo patches. Chapter 3 deals with the description and formulation of layer-by-layer finite element analysis of smart beams. The results of the present study are presented in four chapters. The problems of actuation and sensing of single and multi-patched smart cantilever beams have been addressed in chapter 4 and 5 respectively. The distribution of induced potential in the piezo material due to externally applied potential and due to mechanical straining has been clearly brought out. Chapter 6 presents the studies on shape control of a smart beam under external mechanical and thermal loadings. In chapter 7, the influence of non-linearities due to polarisation and electric field interactions has been analysed. The example problem chosen for this study is the actuation problem of a smart beam with externally applied potential. The important conclusions of this study have been summarised in chapter 8.