Muon Tomography for Detection of SNMs: A Reconstruction Algorithm using MATLAB

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Abstract- In this paper, a reconstruction algorithm for muon tomography has been formulated and studied. Muon is a subatomic particle that can penetrate 15 m of water, 1.3 m of lead and tens of meters of rocks or other materials before getting absorbed or deflected. This makes them highly suitable for employing them in the detection of deep hidden nuclear materials. In muon tomography, the muon's trajectory is measured before and after it penetrates the object. The angle of deflection and the point of deflection of muons are found and these data are integrated from various trajectories. We propose an algorithm that finds the angle and the point using an iterative method. The iteration is performed to find new points and redraw a muon's entry and exit trajectory till the two meet or shortest distance between the two is less than the dimension of a voxel. Then the scattering can be assigned to the voxel in that region. The angle of deflection is dependent on atomic number of the material. Consequently, a three dimensional image of the object or container under investigation is formed, based on the density and the atomic number of the material.

Introduction

U-235 and Pu-239 are designated as Special Nuclear Materials by NRC (Nuclear Regulatory Commission of US) in view of their strategic significance. Highly enriched uranium contains more than 20% U-235, in nuclear weapons its enrichment is greater than 90%. There have been several reports of international smuggling of HEU or WGPu, posing serious security threat. The problem is aggravated by the difficulty with the detection of SNMs especially HEU since its radioactivity is very low. It is marked by relatively low radiation energy signatures (gamma rays) that can be easily attenuated (typically the gamma rays emitted from HEU are attenuated around ten times by a material of thickness 0.3 cm). This makes HEU difficult to detect. The established methods employing x-rays or gamma rays are capable but not as efficient for detecting properly hidden or shielded materials. This is where muons can be used to make a significant contribution to SNM detection.

Muon tomography (MT) is a method which uses cosmic ray muons to generate three dimensional (3-D) images of the investigated volume. The basis of the reconstruction is the Coulomb scattering of muons with the material inside the volume being investigated [1]. Muon is an elementary particle carrying a unit negative charge with a mass of 105.7 MeV/c^2 (around 200 times the mass of electron) [9]. The upper atmosphere is continuously bombarded by cosmic ray protons, this impact produces pions [9]. The pions decay quickly into muons and keep coming towards the original trajectory of protons. Muons travel at nearly 95% of speed of light [9]. Due to their large mass and velocity they are highly penetrating (even greater than X-rays). And this makes muons suitable for tomographic application, especially for detecting special nuclear materials (SNMs) like HEU and WGPu. Due to their low radiation energy, SNMs like HEU are very easy to shield. Properly shielded HEU in small amount would possibly go undetected by X-ray scanners. On the other hand, if muons are employed to detect them, the detection possibility increases manifold. But muon tomography is not as established as x-ray or gamma ray tomography

The present detection technique uses several planar arrays of detectors (GEM or drift tubes) placed above and below the investigated volume [1-4]. The muon flux on earth's surface is around 1 muon per cm^2 [8]. When muon enters the volume their entry and exit points are detected and recorded. These points are used to construct entry and exit trajectories of a muon. Our proposed algorithm helps in the reconstruction of these trajectories. If the entry and exit trajectories are different, we infer that the muon has scattered inside and changed its original track. Using the reconstruction algorithm the point(s) of interaction and angle(s) of deflection is determined. The reconstruction is done by using these data points i.e., point of interaction that corresponds to certain voxel and the angle at that point. The angle of deflection is dependent on the atomic number of the material. The SNMs and shielding materials like lead have high atomic number and can be easily distinguished.

Two reconstruction algorithms are currently used: point of closest approach (POCA) and expectation maximization (EM). These reconstruction algorithms were developed at Los Alamos National Laboratory. The POCA algorithm is simple yet efficient algorithm. It takes the muon's entry and exit points, constructs the entry and exit trajectory, then calculates the line of shortest distance between them and the midpoint of that line is taken to be where scattering took place. This point is given the name POCA point.

EM reconstruction uses an iterative method. It takes both scattering angle and linear deviation as input. Then it distributes the scattering location along the POCA track. It then determines the maximum likelihood of scattering over voxels.

In this work, the POCA algorithm is modified and made iterative. This modification is an attempt to employ the effect of multiple scattering more significantly. The goal is again to find the POCA point but it is found after many iterations, after the convergence criteria is met. The code is written in MATLAB. This work is expected to more closely incorporate the real muon interaction. This is the beginning of our work, our future work will bring more modifications in algorithm and verification using simulations.

Iterative "1/3" POCA algorithm

The entry trajectory of a muon is determined using 2 or 3 muon entry points given by the top detectors and similarly, the exit trajectory is determined by the muon exit points given by the bottom detectors.

The trajectory can be completely specified by a point *d(x, y, z)* on it and the vector **u**. There are three possibilities for the entry and exit trajectories:

1. The two vectors are same.
2. The two vectors meet at a point (so they are coplanar).
3. The two vectors do not meet at any point (i.e., the vectors are skew).

Case 1, the two vectors are same, signifies that the muon went without any interaction within the volume. In this case, the points that lie on this vector do not contain any material, so the voxels corresponding to those points store this information. There is no POCA point for this case.

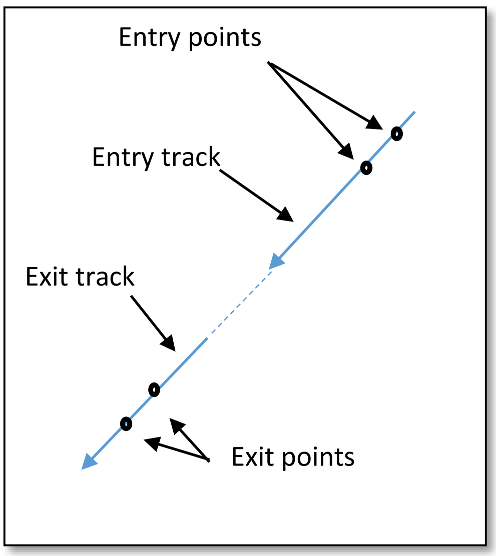

Fig. 1: Muon entry and exit tracks are same

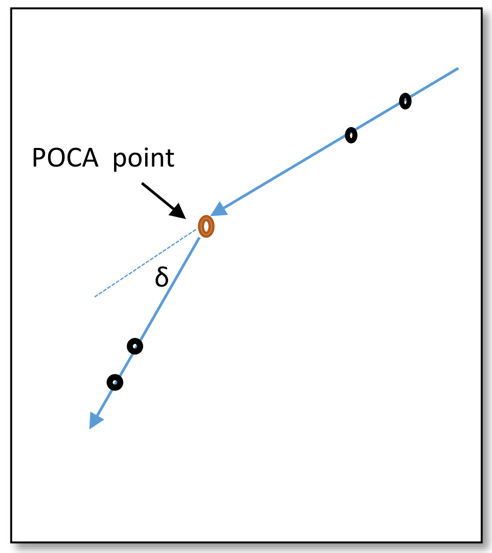

Fig. 2: The two tracks meet at a point

Case 2, when the two trajectories meet at some point. That point is taken to be the POCA point (though it is not actually the POCA point, it is more accurate than POCA point since here the two trajectories actually meet). The angle that exit trajectory makes with the entry trajectory is the angle of deflection, δ. This angle is stored for the voxel corresponding to the POCA point.

Case 3, in which the two vectors are skewed is the most probable case that can arise due to multiple scattering. Here the points on entry and exit trajectories are found that corresponds to the shortest distance between two vectors. Let that point on entry vector be P and that on exit vector be Q. Then, PQ is the shortest distance.

Now, points P' and Q' are located on PQ such that

$$PP' = P'Q' = Q'Q = \frac{PQ}{3}$$

That is, point P' is at one-third the length PQ from the point P on from entry track and point Q' is at one-third the length PQ from the point Q on exit track. Having found the two points, two new tracks are constructed. Let d_1 and d_2 be the entry points and d_3 and d_4 be the exit points. The entry track, thus, is through d_1 and d_2 and the exit vector is through d_3 and d_4.

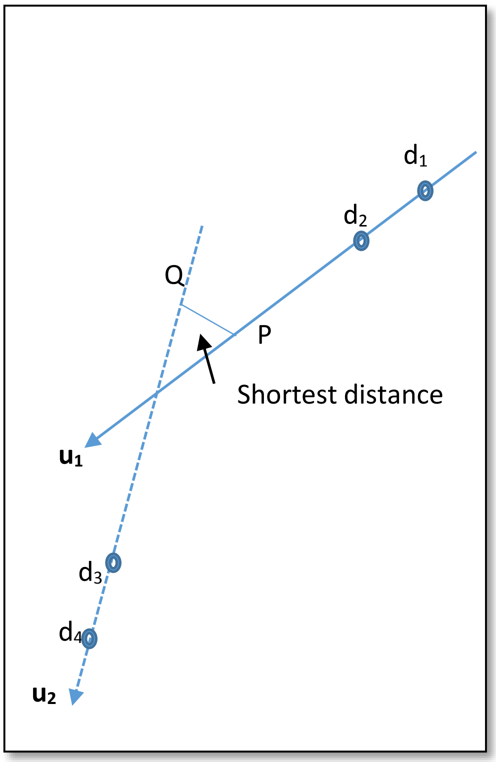

Fig. 3: The two tracks are skewed

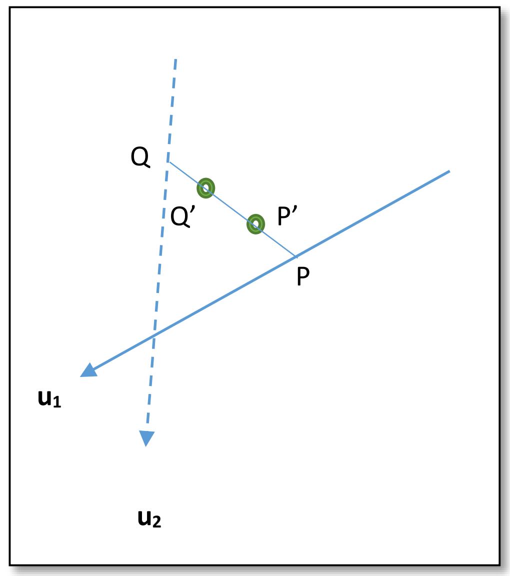

Fig. 4 Finding point P' and Q'

The new track (next track after the entry of muons) is now constructed and this passes through points d_2 and P'. Similarly, the new track (previous track before the exit of muons) is constructed and this passes through Q' and d_3.

So, again we have a pair of new tracks. The similar procedure is now applied to these new tracks/vectors i.e., the shortest distance between two tracks is found and points on two tracks corresponding to that distance (say P_1 and Q_1) is determined. Here again, the points P_1' and Q_1' are found such that

$$PP_1' = P_1'Q_1' = Q_1'Q = P_1Q_1/3$$

Once again, two new tracks are constructed to pass through P_1' and Q_1' respectively. This process is repeated till the shortest distance between two tracks becomes less than L (where distance L can be

specified according to the precision required, say half the dimension of a voxel). In the code, we have hard-coded this value to be 0.01 cm.
After the iteration ends, we have the shortest distance between the two tracks which is of the order of L. The midpoint of this line is taken to be the POCA point. The voxel corresponding to this point is updated with the scattering angle at that point.

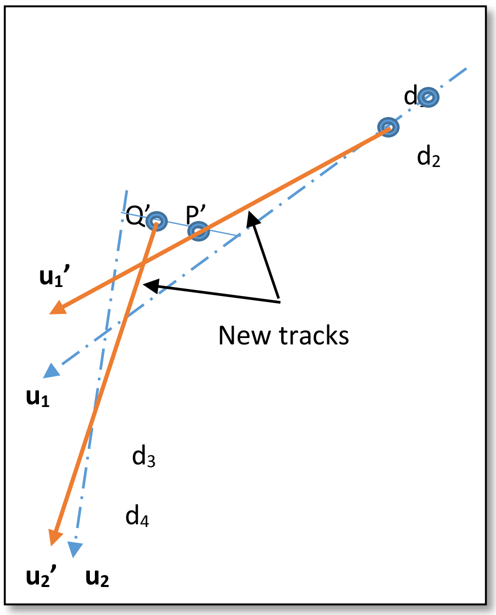

Fig. 5 Construction of new tracks

Conclusion

This algorithm takes an iterative path to finally reach a single point at which the interaction is assumed to happen. It more closely realizes the region where the interaction could have happened. However, the current work just proposes an algorithm that does take the multiple scattering into account and incorporate more accuracy. The actual behavior can only be inferred after the simulation data is analyzed by this algorithm (which is the future part of the work).

Future work

Next, we will perform the actual reconstruction using the proposed algorithm. The data points will be taken from Geant4 simulation of the experiment. Also, the algorithm will be modified. The intermediate points that were found (located at one third of shortest line between two tracks), that were used to modify muon tracks; all those points will also be analyzed and the voxels corresponding to those points will be updated with the angle of deflection formed at those points. That algorithm, then, will closely realize the multiple scattering. Further, reconstruction based on this algorithm will be done using simulation data.

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