Robust Beam Forming in Presence of Noise

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Abstract

The effect of additive noise at the antenna array signal input on the beam-forming is studied in this paper. Two techniques, viz., the optimization using reference signal and the optimal beam-forming are considered in this work. Methods based on singular value decomposition and orthogonal polynomial approximation are presented to combat the effect of noise on beam-forming.

1. Introduction

Wireless communication is becoming increasingly widespread. In high density use areas, there will be the need to extract as much utilization as possible from a given bandwidth by using multiple access technique. The multiple access technique used or being introduced at present, consisting of time division multiple access (TDMA), frequency division multiple access (FDMA) and code division multiple access (CDMA), each can be augmented by space division multiple access (SDMA) [1]. The SDMA technique employs an adaptive antenna array at the base station and perhaps at the mobile unit as well [2]. This technique uses the information of the location/position of the users.

An adaptive array is a system consisting of an array of sensor elements and a real time adaptive signal receiver processor that can be used to select one received signal out of many, and the terms desired signal and interfering signals are used to distinguish them [3,4]. A condition for this spatial selection is that the desired signal must have some spatial characteristic that distinguishes it from interferers. In the case of plane wave reception it could be direction of arrival (DOA). Interference signal suppression is obtained by appropriately steering beam pattern nulls and reducing sidelobe levels in the direction of interference sources, while desired signal reception is maintained by preserving desirable mainlobe (or signal beam) features [4,5].

By changing the value of weights of antenna elements (sensors), beam can be formed in the desired direction and null can be formed in the direction of interference. It is called as beam-forming. There are various types of beam-forming methods. Beam-forming techniques are classified, mainly, in two categories:
(1) Narrow band beam-forming
(2) Broad band beam-forming
In this paper we are concentrating on narrow band beam-forming only. In narrow band beam-forming several techniques are used. Two most important techniques are:
(1) Optimal Beam Former
(2) Optimization using Reference Signal

The existing sample matrix inversion (SMI) technique as applied to the beam-forming method seems to be too sensitive to noise input [5]. The autocorrelation matrix is always symmetric and very close to singular matrix (ill condition matrix). Consequently, when input error due to noise occurs, it may greatly affect the inverse of autocorrelation matrix and subsequently the weights of the beam former.

We use the Principal Component Solution (PCS) method to reduce the effect of noise [6]. When the rank of matrix is very much less than the number of diagonal elements of the matrix, this method will give satisfactory results. But, if the rank of matrix is not very much less than the number of diagonal elements of the matrix, this method will give substantial variation in results due to noise effect. We shall present a method based on the Orthogonal Polynomial Approximation (OPA) [7]. It has the ability to reduce the effect of noise with the help of the minimum error variance criterion [8].

Our proposed method which is based on the combination of the OPA and the Singular Value Decomposition (SVD) will give better results when the rank of matrix is not very much less than the number of diagonal elements of the matrix compared to the existing SVD based PCS method.

2. Signal Model and Beam-forming Techniques

Consider an array of L omni directional elements immersed in a homogeneous media in the far field of M uncorrelated sinusoidal point sources of frequency f_0 . For a linear array of equispaced elements with element spacing d aligned with the x - axis such that the first element situated at the origin, the time taken by a plane wave arriving from the i^{th} source in direction θ_i, and measured from the l^{th} element to the origin is given by

$$\tau_l(\theta_i) = \frac{d}{c}(l-1)cos\theta_i \quad (1)$$

where c is the speed of propagation of the plane wave front.

The signal induced on the reference element due to the i^{th} source is normally expressed in complex notation as

$$m_i(t)e^{j2\pi f_0 t} \quad (2)$$

with $m_i(t)$ denoting the complex modulating function. The structure of the modulating function reflects the particular modulation used in communications system.

Let x_l denote the total signal induced due to all M directional sources and background noise on the l^{th} element. Then it is given by

$$x_l = \sum_{i=1}^{M} m_i(t)e^{j2\pi f_0(t + \tau_l(\theta_i))} + n_l(t) \quad (3)$$

where $n_l(t)$ is a random noise component on the l^{th} element, which includes background noise and electronic noise generated in the l^{th} channel. It is assumed to be temporally white with zero mean and variance equal to σ_n^2.

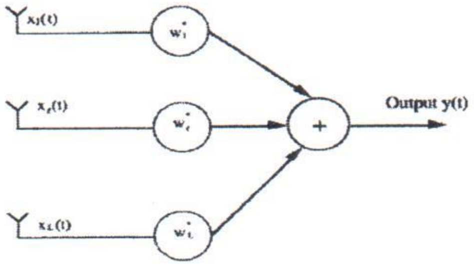

Figure 1: Narrow-band beam former structure

Consider a narrowband beam former, as shown in Fig. 1, where signals from each element are multiplied by weight functions and summed to form the array output. It follows from the figure that an expression for the array output is given by

$$y(t) = \sum_{i=1}^{L} w_l^* x_l(t) \quad (4)$$

where * denotes a complex conjugate. Denoting the weights of beam former as

$$\underline{w} = [\underline{w}_1,\ \underline{w}_2,\ \dots\dots\dots\ \underline{w}_L] \quad (5)$$

and signal induced on all elements as

$$\underline{x}(t) = [x_1(t),\ x_2(t)\ \dots\dots\dots x_L(t)] \quad (6)$$

The output of the beam former becomes

$$y(t) = \underline{w}^H \underline{x}(t) \quad (7)$$

where superscripts T and H, respectively, denote the transpose and complex conjugate transpose of a vector or matrix.

If the components of the array signal vector $\underline{x}(t)$ can be modeled as zero mean stationary processes, then for a given array weight vector $\underline{w}$, the mean output power of the processor is given by

$$p(\underline{w}) = E[y(t)\ y^H(t)] = \underline{w}^H R \underline{w} \quad (8)$$

where E[.] denotes the expectation operator. The array correlation matrix is defined by

$$R = E[\underline{x}(t)\ \underline{x}^H(t)] \quad (9)$$

The elements of this matrix denote the variation between various elements. For example R_{ij} denotes the correlation between i^{th} and j^{th} element of the array. Denote the steering vector associated with the direction θ_i or the i^{th} source by a L- dimensional complex vector $\underline{s}_i$ as

$$\underline{s}_i = [exp(j2\pi f_0 \tau_1(\phi_i,\theta_i)), \dots, exp(j2\pi f_0 \tau_L(\phi_i,\theta_i)]^T \quad (10)$$

Algebraic manipulation using equations (4), (7) and (10) leads to the following expression for R

$$R = \sum_{i=1}^{M} p_i \underline{s}_i \underline{s}_i^H + \sigma_n^2 I \quad (11)$$

where I is the identity matrix and p_i denotes the power of the i^{th} source measured at one of the elements of the array. It should be noted that p_i is the variance of the complex modulating function $m_i(t)$ when it is modeled as a zero mean low pass random process, as mentioned previously.

Using matrix notation, the correlation matrix R may be expressed in the following compact form:

$$R = SPS^H + \sigma_n^2 I \quad (12)$$

where columns of the L and M matrix S are made up of steering vectors, i.e.,

$$S = [\underline{s}_1, \underline{s}_2,\ \dots\dots\dots, \underline{s}_M] \quad (13)$$

and M by M matrix P denotes the source correlation. For uncorrelated sources, it is a diagonal matrix with

$$P_{ij} = \begin{cases} p_i & \text{i=j} \\ 0 & \text{else} \end{cases} \quad (14)$$

Denoting the Leigenvalues of R in descending order by $\lambda_l, l = 1,...$L.and their corresponding unit norm eigenvectors by $\underline{u}_l$, $l = 1,..L$, the matrix takes the following form:

$$R = U\Lambda U^H \tag{15}$$

with a diagonal matrix

$$\Lambda = \begin{bmatrix} \lambda_1 & & & & \\ & . & & 0 & \\ & & \lambda_l & & \\ & 0 & & . & \\ & & & & \lambda_L \end{bmatrix} \tag{16}$$

and

$$U = [\underline{u}_1, \cdots, \underline{u}_L] \tag{17}$$

This representation is refered to as the spectral decomposition of R. Using the fact that the eigenvectors form an orthonormal set, this leads to the following equation for R

$$R = \sum_{i=1}^{M} \lambda_l \underline{u}_l \underline{u}_l^H + \sigma_n^2 I. \tag{18}$$

2.1 Optimal Beam Former

Let a L dimensional complex vector $\underline{w}$ represent the weight of the beam former, which maximizes the output SNR. For an array that is not constrained, an expression for $\underline{w}$ is given by [3]

$$\underline{w} = \mu R_n^{-1} \underline{s}_0 \tag{19}$$

where R_n array correlation matrix of the noise alone, that is, it does not contain any signal arriving from look direction θ_0, steering vector $\underline{s}_0$, and μ is a constant. For an array considered to have a unit response in the look direction, this constant becomes equal to

$$\frac{1}{\underline{s}_0^H R_n^{-1} \underline{s}_o} \tag{20}$$

leading to the following expression for the weight vector.

$$\underline{w} = \frac{R_n^{-1} \underline{s}_0}{\underline{s}_0^H R_n^{-1} \underline{s}_0} \tag{21}$$

In practice, when the estimate of the noise alone matrix is not available, the total R (signal plus noise) is used to estimate the weights. An expression for the weights for this case is given by

$$\underline{w} = \frac{R^{-1} \underline{s}_0}{\underline{s}_0^H R^{-1} \underline{s}_o} \tag{22}$$

2.2 Optimization using Reference Signal

A narrowband beam-forming structure that employs a reference signal to estimate the weights of the beam former is shown in Figure 2. The array output is subtracted from an available reference signal $r(t)$ to generate an error signal $\varepsilon(t) = r(t) - \underline{w}^H \underline{x}(t)$, which is used to control the weights. Weights are adjusted such that the MSE between the array output and the reference signal is minimized. The MSE is given by

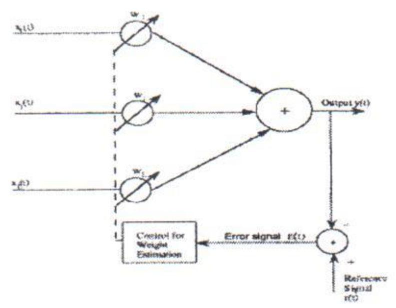

Figure 2: Structure of narrowband beam former using a reference signal

$$MSE = E[|\epsilon(t)|^2]$$

$$= E[|r(t)|^2] + \underline{w}^H R \underline{w} - 2\underline{w}^H \underline{z} \tag{23}$$

where

$$\underline{z} = E[\underline{x}(t)\, r^H(t)] \tag{24}$$

is the correlation between the hermitian of reference signal and the array signal vector $\underline{x}(t)$.

The MSE surface is a quadratic function of $\underline{w}$ and is minimized by setting is gradient with respect to $\underline{w}$ equal to zero, yielding the well known Wiener-Hoff equation for the optimal weight vector

$$\underline{w}_{MSE} = R^{-1} \underline{z} \tag{25}$$

2.3 Equation for Array Gain

For a linear array of L equispaced sensor elements, the overall array response may be found by considering the phasor sum of signal contribution from each array element as

$$y(t) = \sum_{i=1}^{L} w_i x(t) e^{j(i-1)\varphi} \tag{26}$$

where

$$\varphi = 2\pi(\frac{d}{\lambda}) cos\theta \tag{27}$$

the directional pattern in a plane containing the array may therefore be found by considering the array factor

$$A(\theta) = \sum_{i=1}^{L} w_i e^{j(i-1)\varphi} \tag{28}$$

and normalized directional pattern is given by

$$G(\theta) = 10\, log_{10}\left\{\frac{|A(\theta)|^2}{L^2}\right\} \quad (29)$$

3. Techniques to Reduce Effect of Noise on Beam-Forming

In optimal beam former and optimization using reference signal method, for finding the weight vector inverse of autocorrelation matrix is required. We know that autocorrelation matrix is close to singular. Because of this ill-conditioning of matrix if there is a small change in the autocorrelation matrix due to the noise effects, inverse of it will drastically change and consequently the weight vector. Change in the weight vector will change the beam pattern so that the interference may behave as desired signal and vice versa. So it is essential to reduce the effect of noise on weight vector. Basically there are two techniques to reduce the effect of noise:
[1] Principal Component Solution (PCS) method,
[2] Orthogonal Polynomial Approximation (OPA) with minimum error variance criterion.
Our proposed method is the combination of above two which will give significant advantage in the noisy enviornment.

3.1 Principal Component Solution

When extended-order modelling is used, matrix R with dimension $L \times L$, rank$(R) = M$ where $M << L$ has M principal eigenvalues (large value) and remaining $L - M$ non-principal eigenvalues are close to zero. If we consider only M principal eigenvalues to compute the inverse of the matrix, then the effect of remaining $L-M$ non-principal eigenvlaues can be neglected [9].

If the number of sensors L is much large than the number of sources M, the PCS method will perform well in noisy enviornment. However, when the number of sensors is only two times or three times the number of sources, this method will give substantial variation in the weight vector under noisy condition.

3.2 Orthogonal Polynomial Approximation

By using functional approximating in terms of polynomial $p_j(x)$, which constitute an orthogonal set over the collection of sampled points x_i, the function $f(x_i)$ is expressed as

$$f(x_i) = \sum_{i=1}^{lf} c_j p_j(x_i) \quad (30)$$

where

$$c_j = \frac{\mu_j}{\delta_j}, \quad (31)$$

$$\mu_j = \sum_{i=1}^{k} f_i p_j(x_i), \quad (32)$$

$$\delta_j = \sum_{i=1}^{k} [p_j(x_i)]^2, \quad (33)$$

and f_i are sampled values of the function at x_i which are not necessarily at uniform spacing. Note that k is the number of samples. The polynomials can be evaluated from the recursive relation

$$p_j = (x - \alpha_j)p_{j-1} - \gamma_{j-1}p_{j-2} \quad (34)$$

where $j \geq 1,\ p_0 = 1,\ p_{-1} = 0$; and

$$\alpha_j = \frac{1}{\delta_{j-1}} \sum_{i=1}^{k} x_i [p_{j-1}(x_i)]^2 \quad (35)$$

$$\gamma_j = \frac{\delta_j}{\delta_{j-1}} \quad (36)$$

In equation (3.1), the order of approximation lf is chosen such that the error variance

$$\sigma_{lf}^2 = \frac{\sum_{i=1}^{k}[f_i - \sum_{i=1}^{lf} c_j p_j(x_i)]^2}{k - lf - 1} \quad (37)$$

is either minimum or does not decrease appreciably any further with increase lf.

When the minimum error variance criterion is used to get the polynomial degree of the approximation, The reconstruction using equation (30) ensures maximum noise rejection at the sampled set of data values.

4. Proposed Method for Reducing Effect of Noise

Simulation results of autocorrelation matrix show that the effect of white Gaussian noise predominates on diagonal elements of R and its effect is very less on off diagonal elements. Ideally there should not be any effect on the off diagonal elements of autocorrelation matrix due to noise as shown in equation (11). The OPA will decrease the noise effect in diagonal elements but, simultaneously, it will add some noise in the off diagonal elements. Because when white Gaussian noise is passed through the OPA, it will be converted into coloured noise. So it will affect off diagonal elements of autocorrelation matrix. In our proposed method we have taken diagonal elements from the matrix which is passed through the OPA (means after filtering operation) and off diagonal elements from the original matrix (matrix generated by the original signal which consists of signal and noise).

4.1 Procedure for Optimal Beam-Forming Technique

(1) Find steering vector for desired signal by using equation (10) with the knowledge of DOA corresponding to desired signal.
(2) Find x_l using equation (3), where $l = 1, 2, \ldots, L$.

It is the input received signal at l^{th} sensor. Received signal at the input of each sensor will be sum of signal transmitted by desired signal as well as all undesired signals or interferences.

(3) First consider the noiseless case and find autocorrelation matrix R (by taking expectation of input signal of each sensor with its hermitian). In practice the received signal will always have some noise. Add white Gaussian noise with the signal input and find autocorrelation matrix with noise R_1.

(4) Apply the OPA on the input of each sensors which consist of noisy signal (signal + noise).

(5) Reconstruct the input of each sensor using algorithm of minimum order of error variance and find autocorrelation matrix R_2 with the help of this reconstructed signal.

(6) Get new autocorrelation matrix R_n by considering diagonal elements from reconstructed autocorrelation matrix R_2 and off diagonal elements from noisy autocorrelation matrix R_1.

(7) Find weight vector using autocorrelation matrix with noise R_1 and new autocorrelation matrix R_n as equation (22). Compare this two weight vector. Here we will use singular value decomposition for both cases and will consider $'M'$ number of diagonal components out of $'L'$ components.

After comparisons we can say that noise effect in weight vector by using new autocorrelation matrix R_n is less compared to weight vector by using autocorrelation matrix with noise R_1. It is giving very good advantage for $L = 2M$ and $L = 3M$ over the existing SVD based technique. Simulation results in the next section will prove this statement.

4.2 Procedure for Optimization using Reference Signal

In optimal beam-forming technique, the steering vector for the desired signal is required to find the weight vector, while in this method the reference signal is required. We need not required to do Step 1. But in Step 2 while finding the autocorrelation matrix by taking expectation of input signal of each sensor with its hermitian, we have to take another expectation of the reference signal with hermitian of the input signal of each sensor for finding z as shown in equation (23). Finding the reference signal is critical issue in this method. Desired signal and undesired signal must have discrimination at the receiver end. Here we have used different carrier frequencies for the desired signal and interferences so that we can easily reconstruct the desired signal at the receiver end. This is only the additional step required in this method. Then follow Step 3 to Step 6 as described in the optimal beam-forming technique. In step 7 use equation (24) for finding the weight vector.

5. Simulation Results

In this simulation, we have considered two sources, one is the desired signal having direction of arrival (DOA) of 0^0 and the other is the undesired signal having DOA 30^0. Each source is modulated by phase-shift keying (PSK) (digital modulation) having carrier frequency(F_0)100 MHz, symbol frequency(F_d) 10 MHz, and sampling frequency (F_s) of a modulated signal 1000 MHz. Where $F_s > F_0$, $\frac{F_s}{F_d}$ must be integer.

In this simulation we have considered four sensors having interelment spacing of $\frac{\lambda}{3}$. Results for autocorrelation matrix without noise R, with noise(SNR = 10 dB) R_1, and autocorrelation matrix after applying OPA on input signal of each sensor R_2 are shown in Tables 1-3.

2.0000 - 0.0000i	-0.7406 + 1.8366i	-1.3842 - 1.3331i	1.6661 - 0.7458i
-0.7406 - 1.8366i	2.0000 - 0.0000i	-0.7406 + 1.8366i	-1.3842 - 1.3331i
-1.3842 + 1.3331i	-0.7406 - 1.8366i	2.0000 + 0.0000i	-0.7406 + 1.8366i
1.6661 + 0.7458i	-1.3842 + 1.3331i	-0.7406 - 1.8366i	2.0000 + 0.0000i

Table 1: Autocorrelation matrix for Noiseless case: (R)

2.6360 + 0.0000i	-0.7667 + 1.8279i	-1.3499 - 1.3984i	1.6673 - 0.7017i
-0.7667 - 1.8279i	2.6214 + 0.0000i	-0.7803 + 1.8215i	-1.3576 - 1.3262i
-1.3499 + 1.3984i	-0.7803 - 1.8215i	2.6254 - 0.0000i	-0.7132 + 1.8367i
1.6673 + 0.7017i	-1.3576 + 1.3262i	-0.7132 - 1.8367i	2.6278 + 0.0000i

Table 2: Autocorrelation matrix for 5 dB SNR: (R_1)

1.9873 + 0.0000i	-0.4112 + 1.1020i	-1.0135 - 0.8838i	0.8895 - 0.5473i
-0.4112 - 1.1020i	1.7915 + 0.0000i	-0.4184 + 1.1215i	-0.8072 - 0.6232i
-1.0135 + 0.8838i	-0.4184 - 1.1215i	1.9581 - 0.0000i	-0.4080 + 1.2020i
0.8895 + 0.5473i	-0.8072 + 0.6232i	-0.4080 - 1.2020i	1.8783 + 0.0000i

Table 3: Autocorrelation matrix after applying OPA: (R_2)

For simulation purpose we have considered two sources-four sensors and two sources-eight sensors cases.

One source acts as the desired source and the other as interference (the undesired source). We have considered the DOAs of the desired and the undesired signals to be 0^o and 30^o respectively.. For all cases we have found the weight vector using the optimal beam-forming method and by optimization using the reference signal. In all cases we got same weight vector in noiseless case by using the two methods. we have shown the beam patterns for 5 dB and 10 dB SNR levels.

For optimal beam-forming we have considered the carrier frequency F_0 for the desired signal and the interference to be 100 MHz, the symbol frequency F_D 10 MHz and the sampling frequency F_s 1000 MHz. We will get 100 samples per symbol as the ratio of sampling frequency to symbol frequency is 100. Here we have considered two symbols so we will get 200 samples on each sensor. Binary phase shift keying (BPSK) is used for modulation purpose at the transmitter end. For optimization using the reference signal, we used the 100 MHz carrier frequency for the desired signal and 50 MHz for the undesired signal. Here different frequencies are required to discriminate the desired signal at receiver end. Remaining all data are same for both the methods. For each combination, we have plotted array beam pattern for the noiseless case (solid line '-'), with 5/10 dB SNR for the PCS method (dashed line '–') and for the proposed method (dash dot line '-.').

5.1 Array Beam Pattern for Four Sensors

Here we have considered two sources and four sensors. It is a case of $L = 2M$(number of sensors is twice to number of sources). As we have discussed in Section 3, the PCS method will give satisfactory results in noisy enviornment when $L >> M$. Simulation results show that in all cases the proposed method is giving better results compared to the PCS method. In each plot solid line ('-') represents the beam pattern for the noiseless case, dashed ('–') line shows the beam pattern by the PCS method, and dash-dot ('-.') line represents the beam pattern by using the proposed method under same noisy enviornment. In most of the cases the beam pattern by the proposed method is close to that of the noiseless case. It means that our method is robust to the effect of noise. In Tables 4-7, we show the norm of the error in the weight vector. This norm can be taken as an index of performance.

5.2 Array Beam Pattern for Eight Sensors

Here we have considered two sources and eight sensor case. Among these two sources one is the desired source and the other is the interference. We have shown the array beam pattern for the DOAs of 0^o and 30^o for the desired signal and interference respectively. It is a case of $L = 4M$ (the number of sensors is four times the number of sources). In this case the PCS method will give better results than when $L = 2M$. If we compare beam patterns of the eight sensors case with that of the four sensors case for same configuration, we can say that variation in beam pattern due to noise in former case is much less than that in the later case. However in some cases our proposed method is still giving better results. For example when the SNR is 5 dB, the beam pattern by the PCS method will not give sufficient null in the direction of interference but beam pattern by using the proposed method is giving accurate results as shown in Figures 9 and 10.

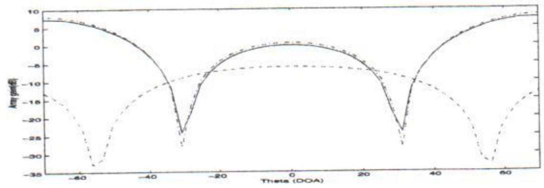

Figure 3: Beam pattern of 0^o- 30^o by using Reference signal with 5 dB SNR

Noiseless case	PCS method	Proposed method
0.3472 + 1.0258i	0.1433 + 0.1285i	0.2858 + 0.8456i
0.2276 - 0.3078i	-0.0168 - 0.1235i	0.5620 - 0.6454i
0.2276 + 0.3078i	0.0103 + 0.1436i	0.4126 + 0.3991i
0.3472 - 1.0258i	0.1242 - 0.0950i	0.3701 - 1.0982i
Norm (MSE):	1.3895	0.5571

Table 4: Weights for Fig 3 by using PCS and Proposed methods

6. Conclusion

As the number of sensors increases, the noise effect reduces because the PCS method gives satisfactory results when $L >> M$. In practice it is very difficult to have condition $L >> M$. For $L = 2M$, the PCS method will not give stable results in noisy enviornment. Our method based on the OPA gives very good results under noisy environment when the number of sensors is not very large ($L = 2M$ or $L = 3M$) compared to the number of sources. In this paper, we have consider only narrowband beam-forming. Similar work can be extended for broadband beam-forming as well.

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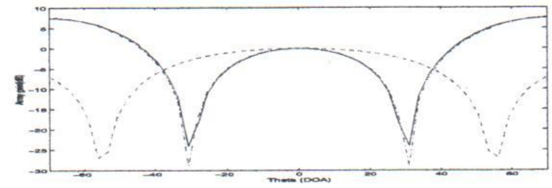

Figure 4: Beam pattern of 0^o- 30^0 for Optimal beam former with 5 dB SNR

Noiseless case	PCS method	Proposed method
0.3472 + 1.0258i	0.2855 + 0.2560i	0.2663 + 0.7880i
0.2276 - 0.3078i	-0.0334 - 0.2461i	0.5237 - 0.6014i
0.2276 + 0.3078i	0.0206 + 0.2860i	0.3844 + 0.3719i
0.3472 - 1.0258i	0.2473 - 0.1893i	0.3449 - 1.0233i
Norm (MSE):	1.1922	0.5154

Table 5: Weights for Fig 4 by using PCS and Proposed methods

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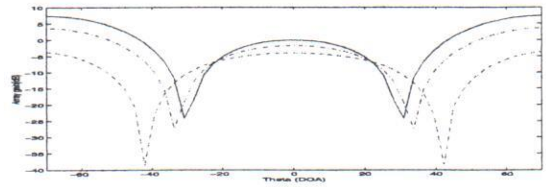

Figure 5: Beam pattern of 0^o- 30^o by using Reference signal with 10 dB SNR

Noiseless case	PCS method	Proposed method
0.3472 + 1.0258i	0.1745 + 0.3210i	0.2462 + 0.5618i
0.2276 - 0.3078i	0.0235 - 0.1793i	0.2202 - 0.3409i
0.2276 + 0.3078i	0.0397 + 0.1740i	0.0407 + 0.1794i
0.3472 - 1.0258i	0.1841 - 0.3097i	0.2531 - 0.7981i
Norm (MSE):	1.0851	0.5820

Table 6: Weights for Fig 5 by using PCS and Proposed methods

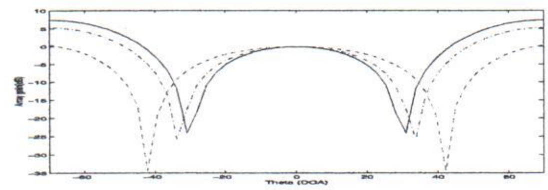

Figure 6: Beam pattern of 0^o- 30^o by using Optimal beam former with 10 dB SNR

Noiseless case	PCS method	Proposed method
0.3472 + 1.0258i	0.2756 + 0.5072i	0.3004 + 0.6854i
0.2276 - 0.3078i	0.0371 - 0.2833i	0.2687 - 0.4160i
0.2276 + 0.3078i	0.0627 + 0.2748i	0.0496 + 0.2189ii
0.3472 - 1.0258i	0.2909 - 0.4893i	0.3089 - 0.9738i
Norm (MSE):	0.7939	0.4186

Table 7: Weights for Fig 6 by using PCS and Proposed methods

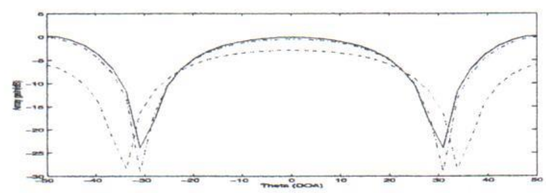

Figure 7: Beam pattern of 0^o- 30^o by using Reference signal with 5 dB SNR

Noiseless case	PCS method	Proposed method
0.1968 + 0.2385i	0.1308 + 0.1174i	0.1989 + 0.2219i
0.0918 - 0.2111i	0.0283 - 0.1297i	0.0791 - 0.2035i
-0.1486 + 0.0231i	-0.0834 + 0.0369i	-0.1469 + 0.0235i
0.0720 + 0.0401i	0.0573 + 0.0187i	0.0596 + 0.0411i
-0.0708 - 0.0423i	-0.0502 - 0.0460i	-0.0704 - 0.0424i
0.0543 + 0.1402i	0.0135 + 0.0935i	0.0565 + 0.1396i
0.1369 - 0.1851i	0.0948 - 0.0881i	0.1452 - 0.2266i
-0.3049 - 0.0512i	-0.1608 - 0.0539i	-0.2195 - 0.0527i
Norm (MSE):	0.2665	0.0987

Table 8: Weights for Fig 7 by using PCS and Proposed methods

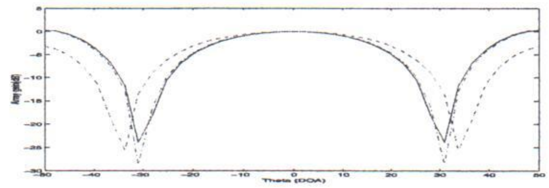

Figure 8: Beam pattern of 0^o- 30^o for Optimal beam former with 5 dB SNR

Noiseless case	PCS method	Proposed method
0.1968 + 0.2385i	0.1814 + 0.1629i	0.2085 + 0.2326i
0.0918 - 0.2111i	0.0393 - 0.1799i	0.0829 - 0.2133i
-0.1486 + 0.0231i	-0.1156 + 0.0512i	-0.1540 + 0.0246i
0.0720 + 0.0401i	0.0794 + 0.0260i	0.0625 + 0.0430i
-0.0708 - 0.0423i	-0.0696 - 0.0638i	-0.0738 - 0.0444i
0.0543 + 0.1402i	0.0187 + 0.1297i	0.0592 + 0.1463i
0.1369 - 0.1851i	0.1314 - 0.1222i	0.1522 - 0.2374i
-0.3049 - 0.0512i	-0.2230 - 0.0748i	-0.2300 - 0.0552i
Norm (MSE):	0.1578	0.0952

Table 9: Weights for Fig 8 by using PCS and Proposed methods

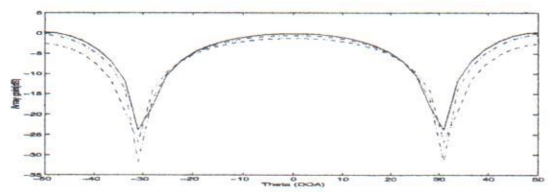

Figure 9: Beam pattern of 0^o- 30^o by using Reference signal with 10 dB SNR

Noiseless case	PCS method	Proposed method
0.1968 + 0.2385i	0.1480 + 0.1679i	0.1855 + 0.2091i
0.0918 - 0.2111i	0.0712 - 0.1734i	0.0854 - 0.2091i
-0.1486 + 0.0231i	-0.1330 + 0.0255i	-0.1302 + 0.0286i
0.0720 + 0.0401i	0.0899 + 0.0276i	0.0668 + 0.0575i
-0.0708 - 0.0423i	-0.0591 - 0.0388i	-0.0744 - 0.0416i
0.0543 + 0.1402i	0.0155 + 0.1314i	0.0433 + 0.1254i
0.1369 - 0.1851i	0.1008 - 0.1486i	0.1503 - 0.2309i
-0.3049 - 0.0512i	-0.2422 - 0.0461i	-0.2239 - 0.0503i
Norm (MSE):	0.1351	0.1045

Table 10: Weights for Fig 9 by using PCS and Proposed methods

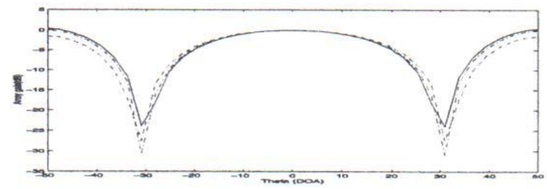

Figure 10: Beam pattern of 0^o- 30^o by using Optimal beam former with 10 dB SNR

Noiseless case	PCS method	Proposed method
0.1968 + 0.2385i	0.1876 + 0.2009i	0.1803 + 0.2118i
0.0918 - 0.2111i	0.1055 - 0.1596i	0.0959 - 0.2329i
-0.1486 + 0.0231i	-0.1559 + 0.0494i	-0.1321 + 0.0256i
0.0720 + 0.0401i	0.0833 + 0.0401i	0.0734 + 0.0398i
-0.0708 - 0.0423i	-0.0822 - 0.0981i	-0.0665 - 0.0431i
0.0543 + 0.1402i	0.0483 + 0.1460i	0.0396 + 0.1326i
0.1369 - 0.1851i	0.1153 - 0.1652i	0.1392 - 0.1790i
-0.3049 - 0.0512i	-0.2803 - 0.0451i	-0.3051 - 0.0543i
Norm (MSE):	0.1001	0.0459

Table 11: Weights for Fig 10 by using PCS and Proposed methods