EE 679, Queueing Systems (2000-01F)
Solutions to Test -2

1. (a) $l_k = l e^{-\frac{a}{km}}$ for k=0, 1, 2, ...

$m_k = m$ for k=1, 2, ...

Therefore, $p_k = p_0 r^k \prod_{i=0}^{k-1} e^{-\frac{a}{m}i} = p_0 r^k e^{-\frac{a}{m}\frac{k(k-1)}{2}}$

where p_0 would have to be obtained by using the normalization condition,

i.e. $$p_0 = \frac{1}{\left[1 + \sum_{k=1}^{\infty} r^k e^{-\frac{a}{m}\frac{k(k-1)}{2}}\right]}$$

(b) Let p_r =P{arriving customer sees *r* in system (before joining the system)}

ΔE = event of an arrival in (t, t+Δt) which actually joins the system

and E_i = event of the system being in state i

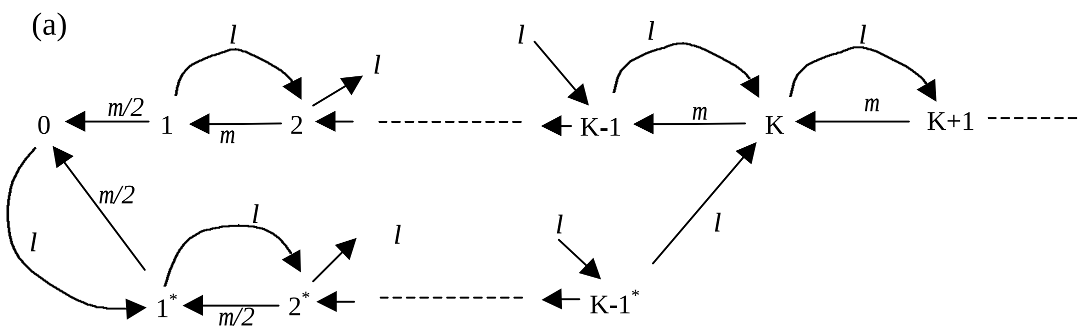
Then $$p_r = P\{E_r \mid \Delta E\} = \frac{P\{E_r\}P\{\Delta E \mid E_r\}}{P\{\Delta E\}} = \frac{P\{E_r\}P\{\Delta E \mid E_r\}}{\sum_{i=0}^{\infty} P\{E_i\}P\{\Delta E \mid E_i\}}$$

Therefore $$p_r = \lim_{\Delta t \to 0} \frac{p_r\, l(\Delta t) e^{-\frac{a}{m}r}}{\sum_{i=0}^{\infty} p_i\, l(\Delta t) e^{-\frac{a}{m}i}} = \frac{r^r e^{-\frac{a}{m}\frac{r(r+1)}{2}}}{\sum_{i=0}^{\infty} r^i e^{-\frac{a}{m}\frac{i(i+1)}{2}}}$$

2. Let n = number in system when both Prof. Joshi and Mr. Gupta are working

n^* = number in system when only Prof. Joshi is working

(a)



(b) $p_2 = (p_1 + p_{1^*})r \quad \& \quad (p_1 + p_{1^*}) = 2p_0 r$

$p_n = r^{n-2} p_2 = 2r^n p_0 \qquad$ for *n=3, 4, 5, ...μ*

Therefore, using the Normalization Condition -

$$p_0 = [1 + 2r + 2r^2 + 2r^3 + ...\infty]^{-1} = \left[1 + 2r\,\frac{1}{1-r}\right]^{-1}$$

or $\quad p_0 = \dfrac{1-r}{1+r}$

But $\quad p_1 = 2p_{1^*} r \quad \Rightarrow \quad p_{1^*} = \dfrac{2p_0 r}{1+2r}$

Therefore with $p_0 = \dfrac{1-r}{1+r}$, the state probabilities are

$$p_{1^*} = \frac{2r}{1+2r}p_0 \qquad p_1 = \frac{4r^2}{1+2r}p_0 \qquad p_2 = 2r^2 p_0$$

and $\quad p_n = 2r^n p_0 \qquad$ for *n=3, 4, 5, ...μ*

(c) Mean Number of Students in Conf. Room $= p_1 + p_{1^*} + \sum_{n=2}^{\infty} np_n$

$$= \frac{2r}{1-r^2}$$

(d) P{Prof. Joshi is working}

$$= 1 - p_0 - 0.5p_1 = \frac{2r}{1+r} - \frac{2r^2(1-r)}{(1+2r)(1+r)}$$

$$= \frac{2r^2(1+r+r^2)}{(1+r)(1+2r)}$$

(e) P{Mr. Gupta is working}

$$= \frac{1}{2}p_1 + \sum_{i=2}^{\infty} p_i = p_0\left(\frac{2r^2}{1+2r} + \frac{2r^2}{1-r}\right) = p_0\,\frac{2r^2(2+r)}{(1-r)(1+2r)}$$

$$= \frac{2r^2(2+r)}{(1+r)(1+2r)}$$

Note that in (d) and (e), we have made the natural assumption that when only one student is being registered in State 1, it could be either Prof. Joshi or Mr. Gupta who is doing the registering - these would be equally likely to happen.