EE 679, Queueing Systems (2000-01F)
Solutions to Test -5

Note: *The notation used below is the standard notation that has been used in the text*

1. The group service time is $\hat{X}$ is either X or $(2X+D)$, each with probability 0.5 and its LST given by -

$$\tilde{\hat{B}}(s) = 0.5\tilde{B}(s)[1+\tilde{B}(s)e^{-s\Delta}]$$

and moments $\overline{\hat{X}} = 1.5\overline{X} + 0.5\Delta$

and $\overline{\hat{X}^2} = 2.5\overline{X^2} + 2\overline{X}\Delta + 0.5\Delta^2$

Using this, the mean queuing delay $\overline{W_{qb}}$ before service starts to a group will be -

$$\overline{W_{qb}} = \frac{l\,\overline{\hat{X}^2}}{2(1-r)} \qquad \text{with } r = l\overline{\hat{X}}$$

The mean queuing delay within the group will be -

$$\overline{W_2} = \frac{\overline{r^2}-\bar{r}}{2\bar{r}}\overline{X} = \frac{2.5-1.5}{2(1.5)}\overline{X} = \frac{1}{3}\overline{X}$$

or

$$\overline{W_2} = \frac{P\{batch\,size \geq 2\}}{Mean\ Batch\ Size}\overline{X} = \frac{0.5}{1.5}\overline{X}$$

Therefore, the total mean queuing delay $\overline{W_q}$ will be given by -

$$\overline{W_q} = \frac{l[2.5\overline{X^2} + 2\overline{X}\Delta + 0.5\Delta^2]}{2(1-1.5l1\overline{X} - 0.5l\Delta)} + \frac{1}{3}\overline{X}$$

2. LST of Class 2 busy period duration = $\tilde{F}_{B_2}(s) = e^{-sX_2}$

$$E\{e^{-sT} \mid u = X_1, n\} = e^{-s(u+nX_2)}$$

$$E\{e^{-sT} \mid u = X_1\} = e^{-su}\sum_{n=0}^{\infty} e^{-snX_2}\frac{(l_2 u)^n}{n!}e^{-l_2 u} = e^{-u(s+l_2-l_2\exp(-sX_2))}$$

Therefore, $\tilde{F}_T(s) = \exp\left(-X_1(s+l_2-l_2e^{-sX_2}\right)$

3.

$$\overline{W_2} = X_2 + \frac{R_2}{(1-r_2)} \quad \text{with } R_2 = \frac{1}{2}l_2X_2^2 \text{ and } r_2 = l_2X_2$$

Since $\overline{W_1} = X_1 + \dfrac{R_1}{(1-r_2-r_1)} + X_2 l_2\overline{W_1}$ with $R_1 = \dfrac{1}{2}l_1X_1^2 + \dfrac{1}{2}l_2X_2^2$

& $r_1 = l_1X_1$

Therefore

$$\overline{W_1} = \frac{X_1}{1-r_2} + \frac{R_1}{(1-r_1-r_2)(1-r_2)}$$