EE 679, Queueing Systems (2000-01F)
Solutions to Test -6

1. We can write the flow balance equations and solve them. In this case, it is easy to get the following by inspection -

$$0.6 l_3 = l \quad \Rightarrow \quad l_3 = 1.6667 l$$
$$0.5 l_1 = l_3 \quad \Rightarrow \quad l_1 = 3.3333 l$$
$$0.5 l_1 + l = l_2 \quad \Rightarrow \quad l_2 = 2.6667 l$$
$$0.8 l_2 + 0.2 l_3 = l_4 \quad \Rightarrow \quad l_4 = 2.4667$$

Therefore $\tilde{l} = (3.3333 l, 2.6667 l, 1.6667 l, 2.4667 l)$

and $\tilde{r} = (3.3333 r, 5.3334 r, 1.6667 r, 4.9334 r)$ with $r = \frac{l}{m}$

Maximum Value of l for which the queuing network will be stable = *0.1875m*

For $l=0.1$, $m=1$, we get $r=0.1$

State Distribution is -

$$P(\tilde{n}) = (0.66667)(0.46666)(0.83333)(0.50666)(0.33333)^{n_1}(0.53334)^{n_2}(0.16667)^{n_3}(0.49334)^{n_4}$$

or $$P(\tilde{n}) = (0.13135)(0.33333)^{n_1}(0.53334)^{n_2}(0.16667)^{n_3}(0.49334)^{n_4}$$

Mean Numbers in the various queues are (*0.5, 1.14286, 0.2, 0.973684*)
Mean of Total Number in Network = *2.81654*
Mean Time Spent in System by a customer = *2.81654/0.1=28.1654*

2. The *Flow Balance* equations are $\bar{l}_1 = 0.8\bar{l}_2 + 0.4\bar{l}_3$ & $\bar{l}_3 = 0.5\bar{l}_1$

Solving with $\bar{l}_1 = 1$, we get $\bar{l}_2 = 1$ and $\bar{l}_3 = 0.5$

Therefore, $u_1 = 1 \quad u_2 = 2 \quad u_3 = 1$

The following table may be constructed for ***g(n, k)***

k=	*1*	*2*	*3*	
n=0	*1*	*1*	*1*	
n=1	*1*	*3*	*4*	
n=2	*1*	*7*	*11*	
n=3	*1*	*15*	*26*	
n=4	*1*	*31*	*57*	***G(4)=57***

For n_1, n_2, n_3 ³ 0 and $n_1+n_2+n_3 = 4$, we get $P(n_1, n_2, n_3) = \frac{1}{57}(2)^{n_2}$

Actual Throughputs are $l_1 = 0.45614 \quad l_2 = 0.45614 \quad l_3 = 0.22807$

The mean numbers in each queue are $\bar{N}_1 = 0.7368 \quad \bar{N}_2 = 2.5263 \quad \bar{N}_3 = 0.7368$