EE 679, Queueing Systems (2001-02F)
Solutions to Test -6

1. Using flow balance conditions, we get the following equations.

$$l_3 = 0.5 l_1$$
$$l_1 = l + 0.2 l_2 + 0.2 l_3$$
$$l = 0.6 l_3 + 0.5 l_4$$
$$l_2 = 0.5 l_1 + 0.5 l_4$$

which may be solved to get

$$(l_1, l_2, l_3, l_4) = (1.395 l, 1.279 l, 0.698 l, 1.162 l)$$
$$(r_1, r_2, r_3, r_4) = (1.395 r, 1.279 r, 1.396 r, 2.324 r) \quad r = \frac{l}{m}$$

(a) Network stable for $2.324 r < 1 \quad \Rightarrow \quad l < 0.43m$

(b) In this case, $(r_1, r_2, r_3, r_4) = (0.14, 0.128, 0.14, 0.23)$

Therefore

$$P(n_1, n_2, n_3, n_4) = 0.497(0.14)^{n1}(0.128)^{n2}(0.14)^{n3}(0.23)^{n4}$$

(c) Using the result $N_i = \dfrac{r_i}{1 - r_i}$

we get $N_1=0.163$ $N_2=0.147$ $N_3=0.162$ $N_4=0.303$

(d) $N=N_1+N_2+N_3+N_4=0.775$

Therefore $W = \dfrac{N}{l} = 7.75$

2. The flow balance equations for this case are

$$l_1 = 0.6 l_2$$
$$l_3 = 0.5 l_1 + 0.2 l_2 + 0.4 l_3$$

which gives $l_2=1.667 l_1$ and $l_3=1.389 l_1$

Choosing *Q1* as the reference queue, we get the visit ratios to be

$V_1=1$ $V_2=1.667$ $V_3=1.389$

(a) Using the above, the steps of the MVA algorithm will be as follows.

(1) $m=0$ $N_1=0$ $N_2=0$ $N_3=0$

(2) $m=1$ $W_1=2$ $W_2=1$ $W_3=2$

$$\lambda = \frac{1}{2 + 1.667 + (2)(1.389)} = 0.155$$

$N_1=0.31$ $N_2=0.258$ $N_3=0.431$

(3) $m=2$ $W_1=2.62$ $W_2=1.258$ $W_3=2.862$

$$\lambda = \frac{2}{2.62 + (1.667)(1.258) + (1.389)(2.862)} = 0.23$$

$N_1=0.603$ $N_2=0.482$ $N_3=0.914$

(4) $m=3$ $W_1=3.206$ $W_2=1.482$ $W_3=3.828$

$$\lambda = \frac{3}{3.206 + (1.667)(1.482) + (1.389)(3.828)} = 0.273$$

$N_1=0.875$ $N_2=0.674$ $N_3=1.451$

The mean number in each queue will be

$N_1=0.875$ $N_2=0.674$ $N_3=1.451$

(b) Note that if we choose $\lambda_1=\mu_1=0.5$, then $u_1=1$, $u_2=0.833$, $u_3=1.389$ will be the relative utilizations of the three queues. From this, it is evident that as M becomes large, i.e. $M\to\infty$, the queue that will get bottlenecked will be Q_3. This also implies that when M is sufficiently large, there will always be one or more users in Q_3 and hence the departure rate for this queue will approach its service rate $\mu_3=0.5$.

Therefore, for large M, we will have $\lambda_3=\mu_3=0.5$
Using this and flow balance, we get $\lambda_1=0.5/1.389=0.36$
and $\lambda_2=(1.667)(0.36)=0.6$

With large M, i.e. $M\to\infty$, we then have

Actual Throughput = $(\lambda_1, \lambda_2, \lambda_3)$ = $(0.36, 0.60, 0.50)$
Actual Utilization = (ρ_1, ρ_2, ρ_3) = $(0.72, 0.60, \rho_3\to 1)$
Note that, as expected, the actual utilization of Q_3 tends toward unity as $M\to\infty$

Using the result $$N_i = \frac{\rho_i}{1-\rho_i}$$

we get $N_1=2.57$ $N_2=1.5$ $N_3=(M-4.07)$

Note that this will be how the M users will get distributed between the three queues.