Solution to Problem 2.11

(a) Let "*0*" represent the state when the system is empty. Let {*1, 2,....., ¥*} be the system states when the system is working like a normal M/M/1 queue and let {*1*,....., K**} be the states when the server is not working - for both these cases, the state *i* or *i** represents the situation when there are *i* customers in the system. For this, the system's state transition diagram may be drawn as shown in Fig. 1.3.

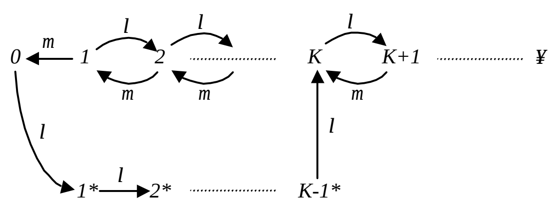

Figure 1.3. State Transition Diagram

(b) From flow balance conditions, for the case *K=2*, we can get the following using $r = l/m$

$$l p_0 = l p_{1*} \qquad p_{1*} = p_0$$
$$l p_0 = m p_1 \qquad p_1 = r p_0$$
$$(l + m) p_1 = m p_2 \qquad p_2 = (1 + r) p_1 = r(1 + r) p_0$$
$$i = 3,4,........\infty \qquad p_i = r^{i-2} p_2$$

Using the normalisation condition, we can find

$$p_0 \left[2 + r + r(1 + r) \frac{1}{(1 - r)} \right] = 1$$

$$p_0 = \frac{1 - r}{2}$$

$$P\{k \text{ users in the system}\} = \begin{cases} \dfrac{1-r}{2} & k=0 \\ \dfrac{1-r^2}{2} & k=1 \\ \dfrac{r^{k-1}(1-r^2)}{2} & k \geq 2 \end{cases}$$

This may be used to find that the mean number in the system is = $\dfrac{(1+r)}{2(1-r)}$

Note that the probability of the server being busy = $1 - p_0 - p_{1^*} = (1-r)$
This is the same as experienced in a normal M/M/1 queue so the server does not gain anything by following this modified approach - it still works equally hard. This will be true even if K is larger than two .