Solution to Problem 2.12

We can rewrite $L_B(s)$ as

$$L_B(s) = \frac{1}{2} \frac{\mu_1}{s + \mu_1} + \frac{1}{2} \frac{\mu_2}{s + \mu_2}$$

which implies that the service centre looks like the following.

![Service Facility Diagram](image)

*Figure 1.4. Equivalent Service Center Model (individual services are exponentially distributed)*

Customers entering the service centre choose either of the two exponential servers. A new customer does not enter the service facility until the previous customer has departed. Let the state of the system be denoted by $\{n, j\}$ where $n$ is the number in the system and $j$ is the server currently used by the customer in the service centre. The state transition diagram may then be drawn as follows.

![State Transition Diagram](image)

*Figure 1.5. State Transition Diagram*

The system can now be completely solved by writing the appropriate balance equations and solving them for the individual state probabilities. In particular, we will find that
\[ p_{12} = p_0 \frac{\lambda(\lambda + \mu_1)}{\lambda(\mu_1 + \mu_2) + 2\mu_1\mu_2} \quad \text{and} \quad p_1 = p_{11} + p_{12} \]
\[ p_{11} = p_0 \frac{\lambda(\lambda + \mu_2)}{\lambda(\mu_1 + \mu_2) + 2\mu_1\mu_2} \quad = p_0 \frac{\lambda(2\lambda + \mu_1 + \mu_2)}{\lambda(\mu_1 + \mu_2) + 2\mu_1\mu_2} \]

We can similarly find that

\[ p_2 = p_0 \left( \frac{2\lambda^2 \left( \lambda + \mu_1 \right)^2 + (\lambda + \mu_2)^2}{\lambda(\mu_1 + \mu_2) + 2\mu_1\mu_2} \right) \]