Solution to Problem 2.12

We can rewrite $L_B(s)$ as $L_B(s) = \frac{1}{2}\frac{m_1}{(s+m_1)} + \frac{1}{2}\frac{m_2}{(s+m_2)}$ which implies that the service centre looks like the following.

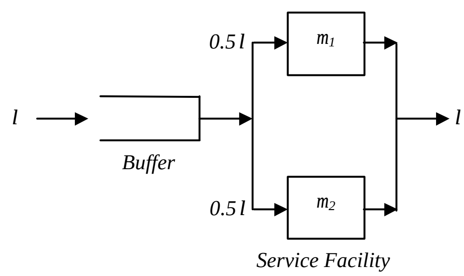

Figure 1.4. Equivalent Service Center Model (individual services are exponentially distributed)

Customers entering the service centre choose either of the two exponential servers. A new customer does not enter the service facility until the previous customer has departed. Let the state of the system be denoted by $\{n, j\}$ where n is the number in the system and j is the server currently used by the customer in the service centre. The state transition diagram may then be drawn as follows.

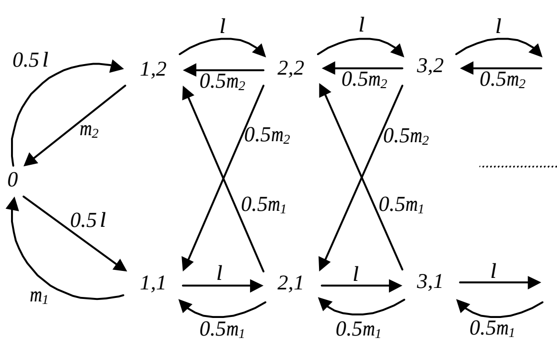

Figure 1.5. State Transition Diagram

The system can now be completely solved by writing the appropriate balance equations and solving them for the individual state probabilities. In particular, we will find that

$$p_{12} = p_0 \frac{l(l + m_1)}{l(m_1 + m_2) + 2m_1 m_2}$$

$$p_{11} = p_0 \frac{l(l + m_2)}{l(m_1 + m_2) + 2m_1 m_2}$$

and

$$p_1 = p_{11} + p_{12}$$
$$= p_0 \frac{l(2l + m_1 + m_2)}{l(m_1 + m_2) + 2m_1 m_2}$$

We can similarly find that

$$p_2 = p_0 \left(\frac{2l^2 \left[(l + m_1)^2 + (l + m_2)^2 \right]}{[l(m_1 + m_2) + 2m_1 m_2]^2} \right)$$