Solution to Problem 2.14

The system’s server may be represented by the service facility shown in Fig. 1.7. Here stages 1 and 2 serve with exponentially distributed service times with means $\mu^{-1}$ and $(2\mu)^{-1}$, respectively.

![Figure 1.7. Service Facility Model](image)

As usual, we represent the system state by $(n, m)$ where $n$ is the number in the system and $m$ is the stage at which the currently served customer may be found. The corresponding state transition diagram is given in Fig. 1.8.

![Figure 1.8. State Transition Diagram](image)

The balance equations may be written as follows.

$$
\begin{align*}
\lambda p_0 &= 2\mu p_{12} \\
(\lambda + \mu) p_{21} &= \lambda p_{11} + 2\mu p_{32} \\
(\lambda + \mu) p_{11} &= \lambda p_0 + 2\mu p_{22} \\
\lambda p_{21} &= \mu p_{31} \\
2\mu p_{32} &= \lambda p_{22} + \mu p_{31} \\
(\lambda + 2\mu) p_{22} &= \lambda p_{12} + \mu p_{21}
\end{align*}
$$
These may be solved for the state probabilities from which the individual state probabilities may be found as

\[
\begin{align*}
P(n = 0) &= p_0 \\
P(n = 1) &= p_{11} + p_{12} \\
P(n = 2) &= p_{21} + p_{22} \\
P(n = 3) &= p_{31} + p_{32}
\end{align*}
\]

The probability that an arrival is blocked and leaves without service is the same as finding the system in state 3 with probability \((p_{31} + p_{32})\).