Solution to Problem 2.14

The system's server may be represented by the service facility shown in Fig. 1.7. Here stages 1 and 2 serve with exponentially distributed service times with means m^{-1} and $(2m)^{-1}$, respectively.

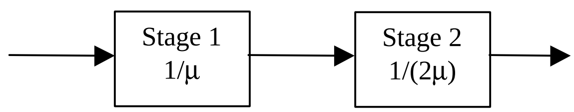

Figure 1.7. Service Facility Model

As usual, we represent the system state by (*n*, *m*) where *n* is the number in the system and *m* is the stage at which the currently served customer may be found. The corresponding state transition diagram is given in Fig. 1.8.

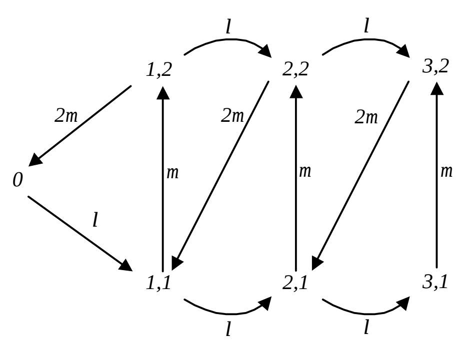

Figure 1.8. State Transition Diagram

The balance equations may be written as follows.

$$l p_0 = 2m p_{12} \qquad (l+2m) p_{12} = m p_{11} \qquad l p_{21} = m p_{31}$$

$$(l+m) p_{21} = l p_{11} + 2m p_{32} \qquad 2m p_{32} = l p_{22} + m p_{31}$$

$$(l+m) p_{11} = l p_0 + 2m p_{22} \qquad (l+2m) p_{22} = l p_{12} + m p_{21}$$

These may be solved for the state probabilities from which the individual state probabilities may be found as

$$P\{n=0\} = p_0 \qquad P\{n=1\} = p_{11} + p_{12}$$
$$P\{n=2\} = p_{21} + p_{22} \qquad P\{n=3\} = p_{31} + p_{32}$$

The probability that an arrival is blocked and leaves without service is the same as finding the system in state 3 with probability ($p_{31} + p_{32}$).