Solution to Problem 2.18

We consider separately the case for the PBAS and WBAS approaches

Partial Batch Acceptance Strategy (PBAS)

The state transition diagram for this case is given in Fig. 1.11. This may be used to write the corresponding balance equations, which may be solved to get the state probabilities as

$$p_0\, l\,(b_1 + b_2) = m p_1$$
$$p_1\, l\,(b_1 + b_2) + p_0\, l\, b_2 = m p_2$$
$$p_2\, l\,(b_1 + b_2) + p_1\, l\, b_2 = m p_3$$
$$p_3\, l\,(b_1 + b_2) + p_2\, l\, b_2 = m p_4$$

These may be solved to obtain the state probabilities. Note that the normalisation condition is needed to get the value of p_0.

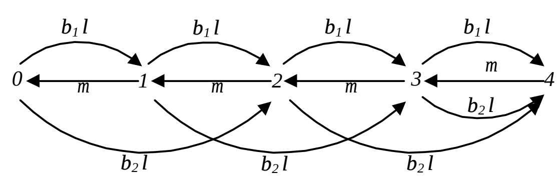

Figure 1.11. PBAS State Transition Diagram

$$p_1 = p_0 f \qquad \textit{where } r = l/m,\ f = r(b_1 + b_2)$$
$$p_2 = p_0(f^2 + r b_2)$$
$$p_3 = p_0(f^3 + 2 f r b_2)$$
$$p_4 = p_0(f^4 + 3 f^2 r b_2 + (r b_2)^2)$$
$$\textit{with } p_0 = \left[1 + f + f^2 + f^3 + f^4 + (r b_2)(1 + 2f + 3f^2) + (r b_2)^2\right]^{-1}$$

Using the state probabilities as given above, the mean number in the system may be found as

$$N = \sum_{i=1}^{\infty} p_i$$

Note that the mean flow rate of jobs offered to the system is $l(b_1+2b_2)$. Of the total flow rate of jobs offered, the flow rate of jobs refused entry into the system are $l[\,p_4(b_1+2b_2) + p_3 b_2\,]$. Therefore, the fraction of jobs refused entry to the system will be given by

$$P_{Blocked\,Jobs} = \frac{(b_1 + 2b_2)}{p_4(b_1 + 2b_2) + p_3 b_2}$$

Whole Batch Acceptance Strategy (WBAS)

The state transition diagram for this case is given in Fig. 1.12. Note the difference between this and the PBAS case of Fig. 1.11.

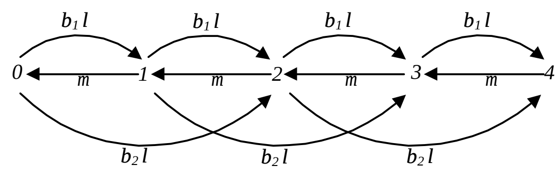

Figure 1.12. WBAS State Transition Diagram

The balance equations for this case are

$$p_0 l(b_1 + b_2) = mp_1$$
$$p_1 l(b_1 + b_2) + p_0 l b_2 = mp_2$$
$$p_2 l(b_1 + b_2) + p_1 l b_2 = mp_3$$
$$p_3 l b_1 + p_2 l b_2 = mp_4$$

Solving these, we get

$$p_1 = p_0 f \qquad\qquad where\ r = l/m, f = r(b_1 + b_2)$$
$$p_2 = p_0(f^2 + rb_2)$$
$$p_3 = p_0(f^3 + 2frb_2)$$
$$p_4 = p_0(f^3 rb_1 + f^2 rb_2 + 2fr^2 b_1 b_2 + (rb_2)^2)$$
$$with\ p_0 = \begin{bmatrix} 1 + f(1 + 2r^2 b_1 b_2) + f^2(1 + rb_2) + f^3(1 + rb_1) + \\ (rb_2)(1 + 2f) + (rb_2)^2 \end{bmatrix}^{-1}$$

For the WBAS case, the probability that a batch will be refused entry will be $[p_3 b_2 + p_4(b_1+b_2)]$