Solution to Problem 2.19

The service facility for this case is the same as the one given in Fig. 1.9. Using this, the state transition diagram for this system may be drawn as shown in Fig. 1.13.

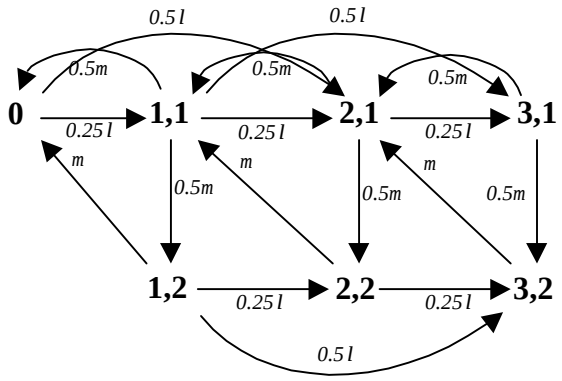

Figure 1.13. State Transition Diagram

For finding the state probabilities, the following balance equations may be used.

$$0.75\lambda p_0 = 0.5\mu p_{11} + \mu p_{12}$$
$$p_{12} + p_{22} + p_{32} = p_{11} + p_{21} + p_{31}$$
$$p_{12}(0.75\lambda + \mu) = 0.5\mu p_{11}$$
$$p_{22}(0.25\lambda + \mu) = 0.5\mu p_{21} + 0.25\lambda p_{12}$$
$$\mu p_{31} = 0.25\lambda p_{21} + 0.5\lambda p_{11}$$
$$p_{11}(0.75\lambda + \mu) = 0.25\lambda p_0 + \mu p_{22} + 0.5\mu p_{21}$$

These may be used along with the normalisation condition

$$(p_0 + p_{11} + p_{12} + p_{21} + p_{22} + p_{31} + p_{32}) = 1$$

to get the desired state probabilities.

The probability that a batch is refused entry may then be found as $0.75(p_{31}+p_{32})+0.5(p_{21}+p_{22})$ using the state probabilities obtained earlier.