# Solution to Problem 2.2

Let $t_1$ = Arrival time of the *first* student
$t_2$ = Arrival time of the *second* student

Since the arrival process is Poisson, $f_{(t_2-t_1)}(t) = l e^{-l t}$ for $t$ ³ $0$

(a) When $X = c$ (fixed), then

P{second student does not wait}= $P\{t > c\} = \int_c^{\infty} l e^{-l t}\, dt = e^{-l c}$

Average waiting time for the second student = $\int_0^c (c-t)\, l e^{-l t}\, dt = (c - \frac{1}{l}) + \frac{1}{l} e^{-l c}$

(b) When $X$ is exponentially distributed with parameter $m$, then

P{second student does not wait | $X$=$x$} = $\int_x^{\infty} l e^{-l t}\, dt = e^{-l x}$

P{second student does not wait} = $\int_0^{\infty} e^{-l x} m e^{-m x}\, dx = \frac{m}{l+m}$

E{waiting time for the second student | $X$=$x$} $= (x - \frac{1}{l}) + \frac{1}{l} e^{-l x}$

E{waiting time for the second student} $= \int_0^{\infty} \left( (x - \frac{1}{l}) + \frac{1}{l} e^{-l x} \right) m e^{-m x}\, dx = \frac{1}{m} - \frac{1}{l} + \frac{m}{l(m+l)}$

$= \frac{l}{m(l+m)}$