Solution to Problem 2.21

This is the same server model as the one considered in Problem 2.20, except that arrivals now come from a batch process where a batch will have either one or two jobs with probability 0.5 each.

With this modification, the state transition diagram of Fig. 1.15 may be modified to give the new state transition diagram shown in Fig. 1.16

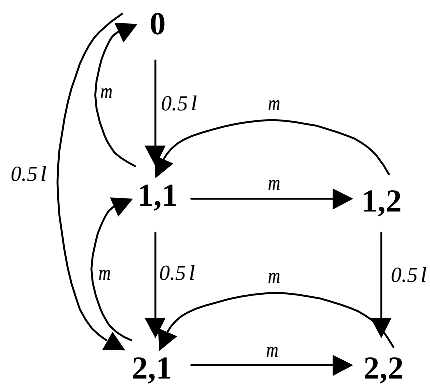

Figure 1.16. State Transition Diagram

The balance equations will then be

$$l p_0 = m p_{11} = (0.5 l + m) p_{12}$$
$$0.5 l (p_{11} + p_{12} + p_0) = m p_{21}$$
$$p_{22} + p_{12} = p_{11} + p_{21}$$

which gives

$$p_{11} = r p_0 \quad p_{12} = \frac{2r}{2+r} p_0$$

$$p_{21} = \frac{r(2+5r+r^2)}{2(2+r)} p_0 \quad p_{22} = \frac{r(2+7r+r^2)}{2(2+r)} p_0$$

Applying the normalisation condition to this, we get

$$p_0 = \frac{2+r}{2+7r+7r^2+r^2}$$

In this case, the total job flow offered will be $0.5l + (0.5)(2l) = 1.5l$

Of this, the total flow blocked will be $[(p_{11}+p_{12})(0.5)(2l)+(p_{21}+p_{22})\{(0.5)l+(0.5)(2l)\}]=l[p_{11}+p_{12}+1.5(p_{21}+p_{22})]$.

Therefore the ratio of blocked jobs will be $[p_{11}+p_{12}+1.5(p_{21}+p_{22})]/1.5$.