

Solution to Problem 2.7

$$p_k = p_0 \prod_{i=0}^{k-1} \frac{l_i}{m_{i+1}} = p_0 \mathbf{r}^k \mathbf{a}^{\binom{k-1}{i=1}} = p_0 \mathbf{r}^k \mathbf{a}^{\frac{k(k-1)}{2}} \quad \text{with } \mathbf{r} = \frac{l}{m}, k=1,2,\dots \text{ \textyen}$$

and $p_0 = \frac{1}{1 + \sum_{k=1}^{\infty} \mathbf{r}^k \mathbf{a}^{\frac{k(k-1)}{2}}}$ from the normalisation conditions

For ensuring the existence of the equilibrium solution, we need

$$\mathbf{a} = \sum_{k=0}^{\infty} \left[\prod_{i=0}^{k-1} \frac{l_i}{m_{i+1}} \right] = \sum_{k=0}^{\infty} \mathbf{r}^k \mathbf{a}^{\frac{k(k-1)}{2}} < \infty$$

$$\mathbf{b} = \sum_{k=0}^{\infty} \frac{1}{l_k \prod_{i=0}^{k-1} \frac{l_i}{m_{i+1}}} = \frac{1}{l} \sum_{k=0}^{\infty} \mathbf{r}^k \mathbf{a}^{\frac{k(k-1)}{2}} = \infty$$