Solution to Problem 3.1

If the residual life approach is used, the mean queueing delay will be given by Eq. (3.2) as

$$W_q = \frac{l\,\overline{X^2}}{2(1 - l\overline{X})}$$

where the first and second moments of the service time *X* may be found using the moment generating property of the L.T. $L_B(s)$ as given. This yields

$$\overline{X} = -\left.\frac{dL_B(s)}{ds}\right|_{s=0} = \left.\left[\frac{0.5m_1}{(s+m_1)^2} + \frac{0.5m_1 m_2(2s + m_1 + m_2)}{[(s+m_1)(s+m_2)]^2}\right]\right|_{s=0}$$

$$= \left(\frac{1}{m_1} + \frac{0.5}{m_2}\right)$$

$$\overline{X^2} = \left.\frac{d^2 L_B(s)}{ds^2}\right|_{s=0}$$

$$= \left.\left[\frac{m_1}{(s+m_1)^3} - \frac{0.5m_1 m_2(m_1 + m_2)}{[(s+m_1)(s+m_2)]^2} + \frac{0.5m_1 m_2(2s + m_1 + m_2)^2}{[(s+m_1)(s+m_2)]^3}\right]\right|_{s=0}$$

$$= \frac{1}{m_1^2} + 0.5\left(\frac{1}{m_1} + \frac{1}{m_2}\right)\left(\frac{1}{m_1} + \frac{1}{m_2} - 1\right)$$

These may be substituted in the expression for W_q to get the mean queueing delay.

For the imbedded Markov Chain approach, the generating function of the number in the system may be obtained by substituting this $L_B(s)$ in Eq. (3.14)

$$P(z) = \frac{(1-r)(1-z)L_B(l - lz)}{L_B(l - lz) - z}$$

and the L.T. of the pdf of the total time spent in system from Eq. (3.15) as

$$L_T(s) = \frac{s(1-r)L_B(s)}{s - l + lL_B(s)}$$

As in Eq. (3.16), the L.T. of the pdf of the queueing delay will be given by

$$L_Q(s) = \frac{L_T(s)}{L_B(s)} = \frac{s(1 - r)}{s - l + lL_B(s)}$$