Solution to Problem 3.2

Imbedding the Markov Chain at the customer departure instants, we can write the Markov Chain as

$$\begin{aligned} n_{i+1} &= K + a_{i+1} - 1 \qquad n_i = 0 \\ &= n_i + a_{i+1} - 1 \qquad n_i \geq 1 \end{aligned} \quad \text{or} \quad n_{i+1} = n_i + a_{i+1} - 1 + K[1 - U(n_i)]$$

Under equilibrium conditions, taking expectations of both sides of the above, we can get the probability of the system being empty as

$$p_0 = \frac{1 - l\overline{X}}{K} \qquad \overline{X} = \text{mean service time}$$

The generating function *P(z)* of the number in the system may be found by taking the expectation of z^n as

$$P(z) = \frac{(1-r)(1-z^K)A(z)}{K(A(z)-z)} \qquad \text{with } A(z) = L_B(l - lz)$$

where $L_B(s)$ as the L.T. of the pdf of the service duration.