Solution to Problem 3.5

Case I: Customer finishing service decides to go for another service immediately with probability *p* or leaves with probability *(1-p)* and this continues until the eventual departure of the customer.

Let $L^*_B(s)$ be the L.T. of the pdf of the effective service time seen by a customer. (This would include all the repetitions of service that it has to go through.). Then, we can see that

$$L_B^*(s) = (1-p)\sum_{i=1}^{\infty} p^{i-1}[L_B(s)]^i = \frac{(1-p)L_B(s)}{1-pL_B(s)}$$

The correponding results for the LCFS or FCFS M/G/1 queues may then be used with $L^*_B(s)$ as the distribution of the effective service time as seen by a customer.

Case II: In this case, the LCFS model is the same as the LCFS model for *Case I* and the same results will hold. For the FCFS model, we may take the approach that the service time duration remains as $L_B(s)$ per customer arrival but that the customer arrival is itself increased from the original value of l to l^* which may be obtained as

$$l^* = l + l^* p \quad or \quad l^* = \frac{l}{1-p}$$

The problem is that this net arrival process with average rate l^* is not Poisson - this can be seen by choosing $(1-p)<<1$ which then corresponds to arrivals in burts. However, if we do approximate this to be also a Poisson process then the (FCFS) queue may be solved in exactly the same manner as a normal M/G/1 queue with l^* rather than l as the average arrival rate of customers.