Solution to Problem 4.10

Actually one can consider a more general version of this problem where both classes of customers have service times which are generally distributed. We can then assume that the high priority customer (Class 2) requires a service time with distribution $L_{B2}(s)$ and the lower priority customer (Class 1) requires a service time with distribution $L_{B1}(s)$.

For the Class 1 customers, we can actually apply the imbedded Markov Chain analysis for this as given in Section 4.5.3. Using Eq. (4.60) and the notation of Section 4.5.3, which is also applicable to this case, the distribution of Class 1 customers will be given as

$$P_1(z) = \frac{p_0[A(z) - z\hat{A}(z)]}{A(z) - z} \qquad \text{with } \; p_0 = \frac{1 - \overline{a}}{1 - \overline{a} + \overline{\hat{a}}}$$

Since only one Class 2 customer can be in the system at a time, the "server unavailable" intervals for the Class 1 queue will have the distribution $L_{B2}(s)$. [Note that each busy period of the Class 2 queue will now only be one Class 2 service time long]. The number of service interruptions during the service time of a Class 1 customer will still have a Poisson distribution with parameter $l_2 x_1$, where x_1 is the actual service time of a Class 1 customer.

We can then obtain (using the same notation as in Section 4.5.3) that

$$L_T(s) = L_{B1}(s + l_2 - l_2 L_{B2}(s))$$
$$A(z) = L_T(l_1 - l_1 z)$$
$$L_{WQ1}(s) = \frac{l_1}{l_1 + l_2 - l_2 L_{B2}(l_1)} + \frac{l_1 l_2 [L_{B2}(s) - L_{B2}(l_1)]}{[l_1 + l_2 - l_2 L_{B2}(l_1)](l_1 - s)}$$
$$L_{T*}(s) = L_T(s) L_{WQ1}(s)$$
$$\hat{A}(z) = L_{T*}(l_1 - l_1 z)$$

Once $P_1(z)$ is obtained using the above the required distribution of the time $D*$ spent in system by an arriving customer may be obtained as

$$L_{D*}(l_1 - l_1 z) = P_1(z) \quad or \quad L_{D*}(s) = P_1\left(1 - \frac{s}{l_1}\right)$$

Note that $L_T(s)$ is the required pdf asked for in the problem.