Solution to Problem 4.4

The generating function *b(z)* of the batch sizes will be given by

$$b(z) = 0.5(z + z^2)$$

with moments $E\{r\} = \bar{r} = 1.5$ and $E\{r^2\} = \overline{r^2} = 2.5$

Using Eq. (4.27), the pdf $b^*(t)$ of the batch service time X^* will have L.T. of

$$L_{B^*}(s) = 0.5(L_B(s) + L_B^2(s))$$

with moments $\overline{X^*} = 1.5\bar{X}$ and $\overline{X^{*2}} = 1.5\overline{X^2} + (\bar{X})^2$ where $\bar{X}$ and $\overline{X^2}$ are the first and second moments of the service time of a particular job.

Using Eq. (4.33), the mean batch queueing delay W_{qb} will be given by

$$W_{qb} = \frac{l}{2(1-r)}\left(1.5\overline{X^2} + (\bar{X})^2\right)$$

with l as the mean arrival rate of batches to the queue and $r = l\bar{r}\bar{X}$ as the offered traffic.

Using Eq. (4.38), the mean queueing delay W_2 for a job within a batch will be given by

$$W_2 = \frac{\bar{X}}{3}$$

This leads to the mean queueing delay for a job as

$$W_q = W_{qb} + W_2 = \frac{l}{2(1-r)}\left(1.5\overline{X^2} + (\bar{X})^2\right) + \frac{\bar{X}}{3}$$

The results of Section 4.4 may also be used to get the L.T. of the pdf of the queueing delay encountered by a job entering the system.