Solution to Problem 4.5

The service facility may be modelled as shown in Fig. 4.1.

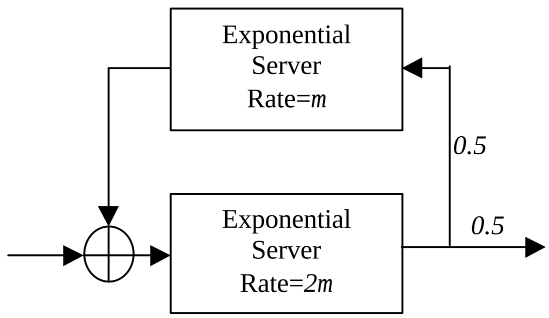

Figure 4.1. Service Facility Model

The L.T. of the effective overall service time distribution may be found to be

$$L_B(s) = 0.5\left(\frac{2m}{s+2m}\right)\sum_{i=0}^{\infty}\left[0.5\left(\frac{2m}{s+2m}\right)\left(\frac{m}{s+m}\right)\right]^i$$
$$= \frac{m(s+m)}{(s^2+3ms+m^2)}$$

which is the same as that in Problem 4.4

We give next the solution for the special case where the queue is of type $M^{[X]}/-/1/2$ following a *partial batch acceptance* strategy. Note that this queue has already been solved for the *whole batch acceptance strategy* in Problem 2.21. The case of the infinite buffer queue of this type would be a straightforward generalisation of these solutions and may be solved as well.

$M^{[X]}/-/1/2$, *Partial Batch Acceptance Strategy:*

For this, the state transition diagram will be as shown in Fig. 4.2.

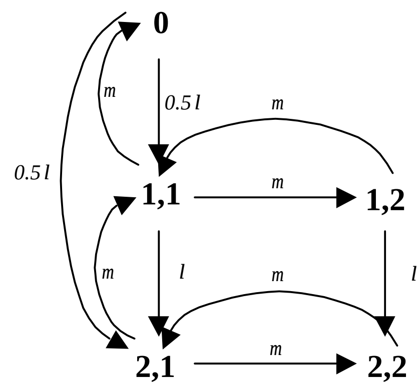

Figure 4.2. State Transition Diagram

Note that the flow from state (1,1) to (2,1) is l, even though one job will be lost in this case if a batch of size 2 actually arrives. (If a whole batch acceptance strategy was followed then the whole batch of size 2 would have been lost!) The same comment holds for the flow from state (2,1) to (2,2).

The flow balance equations for this case may be written as follows

$$l p_0 = m p_{11} = (l + m) p_{12}$$
$$(2m + l) p_{11} = m p_{12} + m p_{21} + 0.5 l p_0$$
$$p_{22} + p_{12} = p_{11} + p_{21}$$

These may be solved to get the state probabilities as

$$p_{11} = r p_0$$
$$p_{12} = p_0 \frac{r}{1 + r}$$
$$p_{21} = \frac{3}{2} r p_0$$
$$p_{22} = p_0 \frac{r(5r + 3)}{2(1 + r)}$$

with p_0 obtained from the normalisation condition to be

$$p_0 = \frac{1}{1 + 5r}$$

Note that the mean flow rate of jobs offered to the queue will be $1.5l$. The mean flow rate of jobs accepted in the queue will be $(1.5p_0+p_{11}+p_{12})l$ with $(1.5p_{21}+1.5p_{22}+0.5\, p_{11}+0.5p_{12})l$ as the mean flow rate of jobs denied entry into the queue. The blocking probability $P_{B,(job)}$ may the be found as

$$P_{B(job)} = \frac{1}{3}(p_{11} + p_{12}) + (p_{21} + p_{22})$$