Solution to Problem 4.6

As in the case of Problem 4.4, we need to first evaluate the generating function *b(z)* of the batch arrivals. The results of Section 4.4 may then be used directly as before.
In this case, we get that

$$b(z) = \sum_{r=0}^{\infty} (1-q)q^r z^r = \frac{1-q}{1-qz}$$

with $b'(z) = \frac{q(1-q)}{(1-qz)^2}$, $b''(z) = \frac{2q^2(1-q)}{(1-qz)^3}$

and $b'(1) = \frac{q}{1-q}$, $b''(1) = \frac{2q^2}{(1-q)^2}$

Using these, the moments of the batch sizes will be

$$\bar{r} = \frac{q}{1-q} \qquad \overline{r^2} = \frac{2q^2}{(1-q)^2} + \bar{r} = \frac{q(1+q)}{(1-q)^2}$$

The L.T. $L_{B*}(s)$ of the pdf of the *batch service time* may then be obtained as

$$L_{B*}(s) = b(L_B(s)) = \frac{1-q}{1-qL_B(s)}$$

with its moments given by

$$\overline{X^*} = \overline{X}\,\bar{r} = \frac{q\overline{X}}{1-q}$$

$$\overline{X^{*2}} = \overline{X^2}\,\bar{r} + (\overline{X})^2(\overline{r^2} - \bar{r}) = \overline{X^2}\,\frac{q}{1-q} + (\overline{X})^2 \frac{2q^2}{(1-q)^2}$$

Using Eq. (4.33), the mean batch queueing delay W_{qb} will be given by

$$W_{qb} = \frac{l}{2(1-r)}\overline{X^{*2}}$$

with l as the mean arrival rate of batches to the queue and $r = l\bar{r}\overline{X}$ as the offered traffic.

Using Eq. (4.38), the mean queueing delay W_2 for a job within a batch will be given by

$$W_2 = \frac{\overline{X}(\overline{r^2} - \bar{r})}{2\bar{r}}$$

This leads to the mean queueing delay for a job as

$$W_q = W_{qb} + W_2 = \frac{l}{2(1-r)}\overline{X^{*2}} + \frac{\overline{X}(\overline{r^2} - \bar{r})}{2\bar{r}}$$

The results of Section 4.4 may also be used to get the L.T. of the pdf of the queueing delay encountered by a job entering the system.