Solution to Problem 4.8

Assume that the generating function of the number in a batch is given by

$$b(z) = \sum_{r=1}^{\infty} b_r z^r$$

where b_r=P{batch size = r} $r=1,2,$....... Using this, the L.T. of the pdf of the batch service time may be obtained as

$$L_{B*}(s) = \sum_{r=1}^{\infty} b_r [L_B(s)]^r e^{-s\Delta} = e^{-s\Delta} b(L_B(s))$$

Once this is calculated, the rest of the procedure will be the same as given in Sec. 4.4 and used in the earlier problem.

$$L'_{B*}(s) = -\Delta e^{-s\Delta} b(L_B(s)) + e^{-s\Delta} b'(L_B(s)) L'_B(s)$$
$$L''_{B*}(s) = \Delta^2 e^{-s\Delta} b(L_B(s)) - 2\Delta e^{-s\Delta} b'(L_B(s)) L'_B(s)$$
$$+ e^{-s\Delta} b''(L_B(s)) \left[L'_B(s)\right]^2 + e^{-s\Delta} b'(L_B(s)) L''_B(s)$$

Using the above, we can get

$$\overline{X^*} = \bar{r}\,\overline{X} + \Delta$$
$$\overline{X^{*2}} = \Delta^2 + \bar{r}\left[\overline{X^2} - \left(\overline{X}\right)^2 + 2\Delta\overline{X}\right] + \overline{r^2}\left(\overline{X}\right)^2$$

The mean queueing delay W_{qb} before service can start to a batch is

$$W_{qb} = \frac{l\,\overline{X^{*2}}}{2(1-r)} \quad \text{with} \quad r = l\,\overline{X^*} = l\left(\overline{X}\,\bar{r} + \Delta\right)$$

The mean queueing delay W_2 within a batch will be

$$W_2 = \left[\frac{\overline{r^2} - \bar{r}}{2\bar{r}}\right]\overline{X} + \Delta$$

Using these the mean queueing delay W_q for a job may be calculated as $W_{qb}+W_2$.