Solution to Problem 4.9

Even though we can write a Markov Chain description for this system at the departure instants of the jobs, this will not be very useful. The reason is that we cannot apply PASTA to this queue as the job arrival process is not Poisson and neither is Kleinrock's principle applicable (i.e. because of the batch arrivals). Therefore, the system state distribution at the departure instants will not hold for the general ergodic case. It is therefore more practical to use a residual life approach to get the mean queueing delays.

As before, given the generating function *b(z)* of the number in a batch and the L.T. $L_B(s)$ of the pdf of the service time *X* for a job, the batch service time distribution may be found to be $L_{B^*}(s) = b(L_B(s))$. This may be used to find the moments of the batch service time X^* to be

$$\overline{X^*} = \overline{X}\,\overline{r} \qquad \overline{X^{*2}} = \overline{X^2}\,\overline{r} + (\overline{X})^2(\overline{r^2} - \overline{r})$$

Normal Vacations

Given the moments $\overline{V}$ and $\overline{V^2}$ of the vacation interval, the mean queueing delay W_{qb} for service to start to a batch for the normal vacation case will be

$$W_{qb} = \frac{l\,\overline{X^{*2}}}{2(1-r)} + \frac{\overline{V^2}}{2\overline{V}} \quad \text{with} \quad r = l\overline{X}\,\overline{r}$$

as given in Section 4.1. The mean queueing delay W_2 within the batch may be found using the approach of Section 4.4 to be

$$W_2 = \frac{\overline{X}(\overline{r^2} - \overline{r})}{2\overline{r}}$$

Using these, the overall queueing delay $W_q = W_{qb} + W_2$ may then be found.

Only One Vacation on Idle

In this case, the batch queueing delay W_{qb} may be found using the result of Eq. (4.15) in Section 4.2 to be

$$W_{qb} = \frac{l\,\overline{X^{*2}}}{2(1 - l\overline{X}\,\overline{r})} + \frac{\overline{V^2}}{2\left(\overline{V} + \frac{1}{l}L_V(l)\right)}$$

where $L_V(s)$ is the Laplace Transform of the pdf of the vacation length. Combining this with the mean ing delay within the batch W_2 as given earlier, we can get the overall queueing delay as

$$W_q = \frac{l\,\overline{X^{*2}}}{2(1 - l\bar{X}\,\bar{r})} + \frac{\overline{V^2}}{2\left(\bar{V} + \frac{1}{l} L_V(l)\right)} + \frac{\bar{X}(\overline{r^2} - \bar{r})}{2\bar{r}}$$