Solution to Problem 5.1

Applying flow balance conditions, we get

$$l_1 = l_1 p + l$$
$$l_2 = l_2 q + l_1(1-p)$$

These may be solved to get the flows (i.e. throughputs) of queues Q_1 and Q_2 to be

$$l_1 = \frac{l}{1-p}$$
$$l_2 = \frac{l}{1-q}$$

Note that since the queues Q_1 and Q_2 are single server ones with service rates given to be m_1 and m_2 , we can also write their joint state distributions and their individual state distributions as follows.

$$r_1 = \frac{l_1}{m_1} = \frac{l}{(1-p)m_1} \qquad r_2 = \frac{l_2}{m_2} = \frac{l}{(1-q)m_2}$$
$$P(n_1, n_2) = r_1^{n1}(1-r_1)\, r_2^{n2}(1-r_2)$$
$$p_{n1} = r_1^{n1}(1-r_1) \qquad p_{n2} = r_2^{n2}(1-r_2)$$

The mean numbers in the two queues will be $N_1 = \frac{r_1}{1-r_1}, N_2 = \frac{r_2}{1-r_2}$

The total number of jobs in the system will be given by

$N=N_1+N_2$.

Applying Little's result to the overall system, we can also find the mean total time W spent by a customer in this system of two queues to be

$$W = \frac{N_1 + N_2}{l} = \frac{r_1}{l(1-r_1)} + \frac{r_2}{l(1-r_2)}$$

Note that of the total delay W as given above, the first component is the mean time spent by an arriving customer in Q_1 while the second component is the mean time spent in Q_2.