Solution to Problem 5.2

Applying flow balance conditions, we get the following.

$$l_1 = l + 0.1 l_2$$
$$l_2 = l_1 + 0.55 l_2 + l_3$$
$$l_3 = 0.3 l_2$$

Solving these, we get $l_1=3l$, $l_2=20l$, and $l_3=6l$ as the throughputs of the queues Q_1, Q_2 and Q_3. (Note that the overall flow balance to the system also implies that $l_2=0.05l$.) The traffic on the three queues will be given by

$$r_1 = \frac{3l}{m_1} \quad r_2 = \frac{20l}{m_2} \quad r_3 = \frac{6l}{m_3}$$

It is important to note that the queueing network will be stable only if the traffic to each queue is less than unity, i.e. max$\{r_1, r_2, r_3\} < 1$. The mean number of jobs (waiting and in-service) in the three queues will then be

$$N_1 = \frac{r_1}{1-r_1} \quad N_2 = \frac{r_2}{1-r_2} \quad N_3 = \frac{r_3}{1-r_3}$$

with the mean of the total number of jobs in the system given by

$$N = N_1 + N_2 + N_3$$

The mean total delay (mean time spent in the system by a job) may then be found using Little's result to be $W=N/l$.