Solution to Problem 5.3

Applying flow balance, we get

\[ 0.6\lambda_1 = \lambda \]
\[ 0.5\lambda_1 = \lambda_3 \]
\[ 0.5\lambda_1 + \lambda = \lambda_2 \]
\[ 0.8\lambda_2 + 0.2\lambda_3 = \lambda_4 \]

which yields \( \lambda_1 = 3.3333\lambda, \lambda_2 = 2.6667\lambda, \lambda_3 = 1.6667\lambda, \lambda_4 = 2.4667\lambda \) as the throughputs of the individual queues \( Q_1, Q_2, Q_3, \) and \( Q_4. \)

Defining \( \rho = \frac{\lambda}{\mu} \), the corresponding values of the traffic offered to the four queues are \( \rho_1 = 3.3333\rho, \rho_2 = 5.3334\rho, \rho_3 = 1.6667\rho \) and \( \rho_4 = 4.9334\rho \). Obviously, the system will be stable only if the largest value of the traffic offered to a queue is less than unity. This implies that the maximum value of \( \lambda \) for which the queueing network will be stable will be \( 0.1875\mu \) corresponding the traffic offered to \( Q_2. \)

For the specific values of \( \lambda = 0.1 \) and \( \mu = 1 \), we get \( \rho = 0.1 \) and the state distribution will be given by

\[
P(n_1, n_2, n_3, n_4) = (0.66667)(0.46666)(0.83333)(0.50666)(0.33333)^{n_1}
\]
\[
(0.53334)^{n_2}(0.16667)^{n_3}(0.49334)^{n_4}
\]

or

\[
P(n_1, n_2, n_3, n_4) = (0.13135)(0.33333)^{n_1}(0.53334)^{n_2}(0.16667)^{n_3}(0.49334)^{n_4}
\]

The mean number of customers (waiting and in service) in each of the queues \( Q_1, Q_2, Q_3, \) and \( Q_4, \) may then be found to be \( 0.5, 1.14286, 0.2 \) and \( 0.973684 \), respectively with the mean number of jobs in the overall system as \( 2.81654 \).

The mean time spent in the system by a customer will then be \( 2.81654/0.1 = 28.1654 \).