Time: 1 hour | Course: BSO 203 | Marks: 40

SESSION 2002-2003, SEMESTER I

MID-SEMESTER EXAMINATION I

1. An improved version of Wien's distribution formula has the form

$$e_b(\lambda, T) = \frac{A}{\lambda^5}\left[e^{-\frac{B}{\lambda T}} + e^{-\frac{2B}{\lambda T}}\right] .$$

   (a) Show that this distribution obeys Wien's displacement law and calculate Wien's constant b in terms of B. (You need to know that the solution of the transcendental equation $(5-x)e^x + (5-2x) = 0$ is $x \simeq 4.9656$).

   (b) Show that the total emissivity obeys the Stefan-Boltzmann law and calculate Stefan's constant in terms of A and B.

   (c) Use the experimental values of b and σ to calculate the constants A and B.

4 + 4 + 2

2. (a) A beam of photons of wavelength λ is incident head-on with a beam of electrons of kinetic energy 30 keV. Find the maximum wavelength change of the scattered radiation.

   (b) The $\lambda = 4686$ Å line of He^+ arises from the transition $n = 4 \rightarrow n = 3$. Assuming that these lines are split according to Sommerfeld's formula, determine how many lines are expected in the fine structure. Apply the selection rule $\Delta\ell = \pm 1$ and determine how many lines survive.

6 + 4

3. Re-work Bohr's theory of the hydrogen atom assuming that the force between the electron and the nucleus is purely gravitational.

   (a) Calculate in this case the 'Bohr radius', the ionisation potential of hydrogen, and the 'Rydberg constant' for hydrogen.

   (b) What should the mass of the proton be if these values are to equal their observed values? What is the physical significance of this result?

6 + 4

4. (a) A Gaussian wave-profile $\psi(x, 0) = (2\pi\sigma)^{-1/2}e^{-x^2/2\sigma}$ is allowed to evolve with time according to the de Broglie dispersion relation $w(k) = \hbar k^2/2m$. Find its oscillation frequency.

   (b) Derive an equation of continuity from the one-dimensional Schrödinger equation when the potential has the form $V(x) = V_0 \exp(-i\Gamma/\hbar)$.

6 + 4

Some important formulae:

- Relativistic energy-momentum relation:

$$E = T + m_o c^2 \qquad E^2 = p^2c^2 + m_0^2 c^4 \qquad p = \frac{m_0 v}{\sqrt{1 - v^2/c^2}}$$

- Wien's distribution formula:

$$e_b(\lambda, T) = \frac{A}{\lambda^5}\ e^{-\frac{B}{\lambda T}}$$

- Rayleigh-Jeans formula:

$$e_b(\lambda, T) = \frac{2\pi k_B T}{\lambda^4}$$

- Planck distribution formula:

$$e_b(\lambda, T) = \frac{2\pi h c^2}{\lambda^5}\ \frac{1}{e^{hc/k_B \lambda T} - 1}$$

- Change of wavelength in Compton scattering:

$$\lambda_s - \lambda_0 = \lambda_C (1 - \cos\theta_s) \qquad \lambda_C = h/m_e c \simeq 386.16 \text{ fm}$$

- Rydberg formula:

$$\bar{\nu}_{mn} = R\left(\frac{1}{m^2} - \frac{1}{n^2}\right)$$

- Bohr's formula for the Rydberg constant in a hydrogenic atom of atomic number Z:

$$R_Z = \frac{Z^2 e^4 \mu_e}{4\pi c \hbar^3} \qquad \mu_e = \frac{m_e}{1 + m_e/m_N}$$

where m_N is the nuclear mass.

- Bohr radius in Gaussian units:

$$a_0 = \frac{\hbar^2}{e^2 m_e} \simeq 0.53\ \mathring{A}$$

- Atomic energy levels according to Sommerfeld's theory:

$$E_{n,\ell} = -R_H\left[\frac{Z^2}{n^2} + \frac{Z^4\alpha^2}{n^3}\left(\frac{1}{\ell + 1} - \frac{3}{4n}\right)\right]$$

where $\alpha = e^2/\hbar c \simeq 1/137.035$ and $\ell = 0, 1, \ldots, (n-1)$.

- Bragg's law for spacing d between lattice planes:

$$2d\cos\theta_n = n\lambda$$

- Fourier transform:

$$f(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dk\ g(k)\ e^{ikx} \qquad g(k) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dx\ f(x)\ e^{-ikx}$$

- Schrödinger equation:

$$\frac{-\hbar^2}{2m}\nabla^2\Psi + V(\vec{x}, t)\Psi = i\hbar\frac{\partial\Psi}{\partial t}$$

- Gamma function:

$$\Gamma(s) = \int_0^{\infty} dx\ x^{s-1} e^{-x}$$

$$\Gamma(s+1) = s\Gamma(s) \qquad \Gamma(n) = (n-1)! \qquad \Gamma(\frac{1}{2}) = \sqrt{\pi}$$

- Gaussian integral:

$$\int_{-\infty}^{\infty} dx \, \frac{1}{\sqrt{2\pi\sigma}} \, e^{-x^2/2\sigma} = 1$$

- Relativistic Doppler shift:

$$\nu' = \nu\sqrt{\frac{c+v}{c-v}}$$

for observer approaching source.

Some important constants:

Velocity of light in vacuo	$c = 2.998 \times 10^{10}$ cm/s
Gravitational constant	$G_N = 6.67 \times 10^{-11}$ m^3/Kg/s^2
Planck's constant	$h = 6.626 \times 10^{-27}$ erg-s
Reduced Planck constant	$\hbar = 1.05 \times 10^{-27}$ erg-s
Electron charge	$e = 4.8 \times 10^{-10}$ statCoulomb
Electron mass	$m_e = 9.1 \times 10^{-31}$ Kg
Proton mass	$m_p = 1.67 \times 10^{-27}$ Kg
Electron rest energy	$m_e c^2 = 511$ keV
Bohr radius	$a_0 = 0.529$ Å
Boltzmann constant	$k_B = 1.38 \times 10^{-23}$ J/K
Wien's constant	$b = 2.898 \times 10^{7}$ ÅK
Stefan's constant	$\sigma = 5.67 \times 10^{-8}$ W/m^2/K^4
Electron volt	1 eV $= 1.6 \times 10^{-19}$ J
Electron volt $/c^2$	1 ev$/c^2 = 1.782 \times 10^{-33}$ gm