Course: BSO 203 Instructor: S. Raychaudhuri

SESSION 2002-2003, SEMESTER I

PROBLEM SET I

Blackbody Radiation

1. (a) Assuming a small temperature difference, show that the Stefan- Boltzmann law reduces to Newton's law of cooling.

   (b) Calculate the value of τ in terms of the Stefan constant σ for a radiating cube of side a, uniform density ρ and uniform specific heat C_V.

   (c) Now consider a copper cylinder, with height and diameter 10 cm, which is heated to 52^0 C and find how much time it will take to reach equilibrium with its surroundings at 27^0 C. You may assume that equilibrum has been reached when the temperature difference drops below 1^0 C.

   Search the literature and find out the values of the absolute constants you need for this problem.

2. (a) Assuming the surface of a star emits radiation like a blackbody, use Wien's displacement law to determine the surface tenperature of

   - a blue star with peak wavelength 3000 A;
   - a yellow star with peak wavelegth 6000 A;
   - a red star with peak wavelength 7800 A.

   For this you will need the value of Wien's constant $b = 0.2898$ cm-K.

   (b) What temperature must a body attain in order to emit X-rays of wavelength 1 A? Given that the ionisation potential of hydrogen is 13.6 eV, determine the material constituting a body at this temperature.

   (c) Using Wien's displacement law, find the peak wavelength of radiation at a temperature of 2.7 K. This is the temperature of the radiation left over from the creation of the Universe. What size of telescope would you need to detect this radiation? Search the literature and find out how this radiation was detected and by whom.

   (d) If you measure the radiation coming from all parts of the sky today, you will not find the wavelength to be the same in all directions. Explain this phenomenon. What is the maximum difference in temperature you would predict by applying Wien's displacement law to this anisotropy?

3. (a) Wien's distribution law can be used to arrive at both the Stefan-Boltzmann law and Wien's displacement law, provided we note that

$$\frac{\partial^2 Q}{\partial \mathcal{A} \partial t} = \int_0^\infty d\lambda \; e_b(\lambda, T) \; .$$

Prove this and determine the constants A and B in

$$e_b(\lambda, T) = \frac{A}{\lambda^5}\, e^{-\frac{B}{\lambda T}}$$

in terms of Stefan's constant σ and Wien's constant b.

(b) Put in the experimental numbers for σ and b and obtain the numbers for A and B. Choose convenient units. You now have the complete form of Wien's displacement law.

(c) Consider the Sun, whose peak wavelength for emitted radiation is around 5500 A. Treating it as a blackbody and assuming the truth of Wien's distribution formula, calculate the fraction of its energy emitted in the visible spectrum 3500-8000 A.

4. We have derived Planck's distribution formula in the class using the idea of molecular oscillators emitting discrete quanta of energy.

   (a) Assume that the energy is continuous rather than discrete and show that instead of the Planck formula, we shall get the classical value for the average energy $\bar{\epsilon} = k_B T$.

   (b) Go on to show that if the energy is continuous, then we get the Rayleigh-Jeans distribution formula instead of Planck's distribution formula.

   To obtain these results, you will only need to replace the energy summation by an integration. The results should convince you that the really new feature of Planck's theory is the use of energy quanta.

5. Starting from Planck's law,

$$e_b(\lambda, T) = \frac{2\pi h}{\lambda^5} \frac{1}{e^{hc/\lambda k_B T} - 1}$$

   (a) Show that it reduces to the Rayleigh-Jeans formula in the infrared limit.

   (b) Show that it reduces to the Wien formula in the ultraviolet limit. Hence determine the constants A and B in terms of h, c, and k_B.

   (c) Show that the distribution is peaked with a maximum obeying Wien's displacement law, and hence determine the constant b in terms of h, c, and k_B.

   (d) Find the total emissivity over all wavelengths and show that it is consistent with the Stefan-Boltzmann law. Hence calculate Stefan's constant σ in terms of h, c, and k_B.

   (e) Use the experimental results on σ, b and c to determine the numerical values of Planck's constant h and Boltzmann's constant k_B. Compare these with the standard values available in the literature or on the internet.