Course: BSO 203 Instructor: S. Raychaudhuri

Session 2002-2003, Semester I

PROBLEM SET II

Blackbody Radiation and Photons

1. (a) Consider the following situation: two small black plates, of areas δS and $\delta S'$, respectively, are placed a distance r apart. The inward normal to each plate makes an angle θ and θ', respectively, with the line joining the two plates. Argue that the radiation per unit time emitted by each and intercepted by the other is given by

$$\frac{e_b \; \delta S \; \delta S' \; \cos\theta \; \cos\theta'}{4\pi r^2} \; .$$

This is known as the *mutual radiation* between two black plates. If we wish to find the same quantity but only in the range λ to $\lambda + d\lambda$, we simply replace e_b with $e_b(\lambda, T)d\lambda$.

(b) Consider a small volume δV inside a large cavity filled with diffuse radiation, which is in equlilibrium through multiple reflections from the walls. Assuming that the walls have emissivity K, establish that the energy density u is given by

$$u = \frac{2K}{c} \; .$$

(c) Consider a small hole of area δA in the walls of a cavity containing diffuse radiation. Show that the radiation escaping per unit time per unit area (of the hole) is just $K/2$. Hence relate the emissivity of such a hole to the energy density of the black radiation.

2. The molecules of an ideal gas have only kinetic energy given by

$$E = \frac{1}{2}m \; \left(v_x^2 + v_y^2 + v_z^2\right) \; .$$

where $\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$.

If the gas is at a temperature T and the molecules obey the Maxwell-Boltzmann distribution

$$N(E) = N(0)e^{-E/k_BT}$$

find the number of molecules having speeds between v to $v + dv$.

(b) Using the above distribution find

- the average speed of the molecules
- the rms speed of the molecules
- the most probable speed of the molecules

and show that these obey the ratio 1.13:1.22:1 respectively.

3. (a) Following Einstein's derivation of Planck's distribution formula, calculate the ratio of coefficients A_{21}/B_{12} by taking the limit $\nu \to 0$ and comparing with the Rayleigh-Jeans formula.

   (b) Consider a distribution of a large number of oscillators having different epochs

$$\psi(t) = \psi_0 \cos(\omega t + \varepsilon)$$

   where the number of oscillators whose epochs lie between ε and $\varepsilon + d\varepsilon$ is given by

$$N(\varepsilon) = N(0)e^{-\varepsilon^2/2\sigma}$$

   where σ is a constant. Assuming that the amplitudes of these oscillators are summed principle of linear supersposition), calculate the total amplitude for N oscillators. This is a model of *incoherent* light. Compare this value with the case when $\varepsilon = 0$ for all the oscillators.

4. (a) In a Compton scattering experiment, calculate the recoil velocity of the electron as a function of the scattering angle.

   (b) Show that in the Compton scattering experiment, it is not possible for the electron to completely absorb the photon.

5. Consider the following semi-classical model of the Helium atom: the two electrons move in such a way that the nucleus is always in the centre of the line joining them.

   (a) Following Bohr's theory of the hydrogen atom, calculate the first and second ionization potentials of Helium. Compare them with standard values.

   (b) Considering one electron only, show that the ionisation potential can be interpreted as a *screening* of the nuclear charge. Determine the ratio of the screened nuclear charge to its actual value.

   (c) Determine Rydberg's constant for Helium and compare it with the standard value.