Course: BSO 203 Instructor: S. Raychaudhuri

Session 2002-2003, Semester I

PROBLEM SET III

The Quantum Atom and Matter Waves

1. (a) Show that replacing the mass of the electron by the reduced mass

$$\frac{1}{\mu} = \frac{1}{m_e} + \frac{1}{m_n}$$

in Bohr's formula for the Rydberg constant leads to the ratio

$$\frac{R_{He^+}}{R_H} = 4.00163\ .$$

(b) Assuming that a sample of pure hydrogen consists of an admixture of normal hydrogen ${}_1\mathrm{H}^1$ and 'heavy hydrogen' or deuterium ${}_1H^2$, find the consequent splitting in the H_α, H_β, H_γ and H_δ lines of the Balmer series. (It was the observation of these splittings that led to the discovery of deuterium by Urey and collaborators in 1932; nowadays it is called an *isotope shift*.

2. (a) Calculate the velocity of an electron in a circular Bohr orbit of principal quantum number n in a hydrogenic atom of atomic number Z. For what values of n and Z does this equal the speed of light c?

(b) In the neighbourhood of a heavy nucleus (which can take some of the recoil momentum), a hard γ-ray photon can annihilate, creating an electron-positron pair. Calculate the maximum wavelength such a photon can have.

3. The de Broglie wavelength of a particle is given by

$$\lambda = \frac{h}{p}$$

where p is the momentum of the particle. Find the de Broglie wavelengths of the following objects:

(a) An electron of kinetic energy 10 eV;

(b) A proton of kinetic energy 1 MeV;

(c) A nucleus of mass 206 MeV/c^2 with kinetic energy 1 GeV.

(d) An electron of energy 100 GeV.

(e) A cricket ball of mass 163 g, travelling at a speed of 40 m/s.

4. Establish the following properties of the Dirac delta function:

   (a) $\delta(x) = \lim_{\lambda\to\infty} \frac{\sin \lambda x}{\pi x}$.

   (b) $\delta(x) = \lim_{\lambda\to 0} \frac{1}{2\lambda}\theta(x+\lambda)\theta(\lambda - x)$.

   (c) $\delta[f(x)] = \sum_i \frac{x - x_i}{|f'(x_i)|}$ where $f(x_i) = 0$.

   (d) $\int dx f(x)\delta'(x) = -f'(0)$ where a prime denotes a derivative.

5. (a) Prove Parseval's relation

$$\int dx \; |\psi(x)|^2 = \int dk \; |\phi(k)|^2$$

   where $\psi(x)$ and $\phi(k)$ are Fourier transforms of each other.

   (b) Taking the dispersion relation $\omega = \hbar k^2/2m$, establish that for a wave-profile in k-space given by

$$\phi(k) = \phi_0 \delta(k - k_0)$$

   the phase velocity of the wave is half of the particle velocity.

   (c) Taking a relativistic dispersion relation, investigate the spreading of a Gaussian wave-packet.